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Notes on Finite Maximal Infix Codes[1]

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Abstract: The fact that every finite infix code is contained in a finite maximal infix code was given in 1994 by Ito and Thierrin. In this paper, we construct two kinds of maximal infix codes from a finite maximal infix code. Then we show that a finite infix code is a finite intersection of some finite maximal infix codes.

Keywords: Infix Codes, Maximal Infix Code, Prefix Code, Maximal Prefix Code

Mathematics Subject Classifications: 68R15; 68Q45

1 Introduction

Prefix codes are widely used in information theory and computer science, for example: in encoding and decoding, data compression and transmission, DES and Huffman's algorithms. Infix codes is a special kind of prefix codes, but there exists a prefix code which is not an infix code, for example, $A=\{b,aba\}$, where the alphabet is $\{a, b\}$. In [1], the authors show that every finite infix code is contained in a finite maximal infix code, and in [2], every finite prefix code is contained in a finite maximal prefix code.

There are lots of finite infix codes. But a finite infix code may not be a maximal infix code. For example, if the alphabet is $\{a, b\}$, then $A=\{a^2, b^2\}$ is a finite infix code which is not a maximal finite infix code. In this paper, we obtain some finite maximal infix codes from a finite

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maximal infix code, and prove the following results.

(1) If a word or a set of some kind of words is in a maximal infix code, then we find another maximal infix code which doesn't contain the word or the set.
(2) Every finite infix code is written as the intersection of finite numbers of finite maximal infix codes.
(3) A finite maximal infix code contains some powers of any letter.

In this paper, we construct two kinds of maximal infix codes and prove that a finite infix code is the intersection of finite numbers of finite maximal infix codes. Then for every infinite infix code, there exists an infinite maximal infix code containing it by Zorn's Lemma. At the end of the paper, we show that every maximal infix code contains a special kind of words. The paper is organized as follows. In section 2, we quote some definitions and properties on theory of formal languages which will be used in the paper. In section 3, we give all infix codes when $|X| \geq 1$. In section 4, we prove the above results on finite maximal infix codes.

Finite prefix codes and infix codes have important applications in combination mathematics, coding theory, computer science and biology. The field of formal language theory studies the purely syntactical aspects of such languages that is, their internal structural patterns. Formal language theory sprang out of linguistics, as a way of understanding the syntactic regularities of natural languages. In computer science, formal languages are often used as the basis for defining programming languages and other systems in which the words of the language are associated with particular meanings or semantics. In computational complexity theory, decision problems are typically defined as formal languages, and complexity classes are defined as the sets of the formal languages that can be parsed by machines with limited computational power. In logic and the foundations of mathematics, formal languages are used to represent the syntax of axiomatic systems, and mathematical formalism is the philosophy that all of mathematics can be reduced to the syntactic manipulation of formal languages in this way. Prefix codes are important subjects in formal language theory and they can be widely used in computer science, control theory etc (see [3-7]).

2 Preliminaries

Let X be a non-empty finite set of letters, which is called an alphabet. Let $|X|$ be the number of letters in X. Any finite string over X is called a word. For example, $w = abab^2a$ is a word over the alphabet $X = \{a, b\}$. The word which contains no letter is called the empty word, denoted by 1. The set of all words over X is denoted by X^*, which is a free monoid with concatenation. For example, the product of two words $x = ab^2$ and $x = ab^3a^2$ is the word $xy = ab^2ab^3a^2$. Let $X^+ = X^* \setminus \{1\}$, which is a free semigroup. For any word w in X^+, let $\lg(w)$ be the number of letters that occur in w and $\lg(1) = 0$. Then $\lg(w) = 6$ for the former word $w = abab^2a$. For any $w \in X^+$, let $w^0 = 1$ and $w^1 = w$, we call a $n+1$ power of w is $w^{n+1} = w^n \cdot w$ for $n \geq 1$. For instance, if $w = aaaa$ where $a \in X$, we call it the fourth power of a and its length is 4. For $u, v \in X^*$, u is called a prefix of v, denoted by $u \leq_p v$, if $v = ux$ for some $x \in X^*$. If $x \neq 1$ and $u \neq 1$, then u is a proper prefix of v, denoted by $u <_p v$. Similarly, $u \leq_s v$ and $u <_s v$ are defined.

Let $X^+ = X^* \setminus \{1\}$. Any non-empty subset of X^+ is called a language. In particular, we consider $\{1\}$ as a language. A language P is called a prefix code if $P \cap PX^+ = \varnothing$. A language A is called an infix code if there exist words $x, y \in X^*, u \in A$ such that $xuy \in A$ implies that $xy = 1$. Every infix code is a prefix code. An infix code A is called a maximal infix code if and only if for any $x \in X^+ \setminus A$, the language $A \cup \{x\}$ is not an infix code. In the following, we cite some properties of infix codes and maximal infix codes which will be used later.

Lemma 2.1.[1] Let A be a finite infix code. Then there exists a finite maximal infix code containing A.

Lemma 2.2.[8] Let A be a language. Then the following two statements are equivalent:

(1) A is an infix code;

(2) $(A \cap X^* AX^+) \cup (L \cap X^+ AX^*) = \varnothing$.

Many mathematicians are interested in infix codes (see [9-15]). For example: in [9], the set of all infix codes is not a free monoid; but the set of all e-convex infix codes is a free monoid in [10]; in [11], an infix code is the intersection of a right semaphore code and a left semaphore code; in [7], every solid code is an infix code; in [14], the authors characterized comma-free codes by using infix codes. Definitions which are used in the paper but not stated here can be found in [8, 16-19].

3 Maximal Infix Codes over One Letter Alphabet

Proposition 3.1. Let $X = \{a\}$. Then $A = \{a^i\}$ is a maximal infix code for any $i \geq 1$.

Proof. Since A only contains one word, then it is an infix code. Let $B = \{a^j, a^{j+m_1}, \cdots\}$ such that $|B| \geq 2$, where $j < j + m_1 < \cdots$ and $m_1 \geq 1$. Then $a^{j+m_1} = 1 \cdot a^j \cdot a^{m_1}$. So B is not an infix code. Thus any language containing more than one word is not an infix code. Therefore, for all $i \geq 1$, $A = \{a^i\}$ are all kinds of maximal infix code when $X = \{a\}$.

So when $|X| = 1$, for a infix code, only is itself containing it.

4 Finite Infix Codes and Maximal Infix Codes over an Alphabet with More Than One Letters

In this section, we always let $|X| \geq 2$. The set of all infix codes over an alphabet X will be denoted by $I(X)$ and the set of all maximal infix codes over X will be denoted by $MI(X)$. By the following theorem, we will obtain two distinct kinds of maximal infix codes from a maximal infix code.

Theorem 4.1. Let $\{1\} \neq \mathrm{M} \in MI(x)$.

(1) If $y \in M$, then $(M \setminus \{y\}) \cup yX \cup Xy \in MI(x)$.

(2) If $yX \cup Xy \subseteq M$, then $[M \setminus (yX \cup Xy)] \cup \{\mathrm{y}\} \in MI(x)$.

Proof. (1) Let $(M \setminus \{y\}) \cup yX \cup Xy = M_1$. Then

$M_1 \cap X^* M_1 X^+$
$= [(M \setminus \{y\}) \cup yX \cup Xy] \cap X^*[(M \setminus \{y\}) \cup yX \cup Xy]X^+$
$= [(M \setminus \{y\}) \cap X^*(M \setminus \{y\})X^+] \cup [(M \setminus \{y\}) \cap X^* yXX^+] \cup [(M \setminus \{y\}) \cap X^* XyX^+] \cup$
$[yX \cap X^*(M \setminus \{y\})X^+] \cup [yX \cap X^* yXX^+] \cup [yX \cap X^* XyX^+] \cup [Xy \cap X^*(M \setminus \{y\})X^+] \cup$
$[Xy \cap X^* yXX^+] \cup [Xy \cap X^* XyX^+]$.

First, we can see that $(M \setminus \{y\}) \cap X^*(M \setminus \{y\})X^+ = \varnothing$, $yX \cap X^* yXX^+ = \varnothing$, $yX \cap X^* XyX^+ = \varnothing$, $Xy \cap X^* yXX^+ = \varnothing$ and $Xy \cap X^* XyX^+ = \varnothing$. Suppose there exists a word $u \in (M \setminus \{y\}) \cap X^* yXX^+$. Then there exist $x_2, y_2 \in X^+$ and $b \in X$ such that $u = x_2 yby_2 \in M$. Since $y \in M$ and $M \in I(x)$, we have a contradiction. Thus $(M \setminus \{y\}) \cap X^* yXX^+ = \varnothing$. In the same way, we can prove $(M \setminus \{y\}) \cap X^* XyX^+ = \varnothing$.

Suppose $yX \cap X^*(M \setminus \{y\})X^+ \neq \varnothing$, then there exist $u_1 \in yX \cap X^*(M \setminus \{y\})X^+$. Then there exist $a \in X, u_2 \in \cap M \setminus \{y\}, x_1 \in X^*$ and $x_2 \in X^+$ such that $u_1 = ya = x_1 u_2 x_2$. Since $\lg(y) \geq \lg(x_1 u_2)$, then there exists $x_3 \in X^*$ such that $y = x_1 u_2 x_3$. Since y, $u_2 \in M$ and M is a maximal infix code, we have a contradiction. Thus $yX \cap X^*(M \setminus \{y\})X^+ = \varnothing$. Similarly, we have $Xy \cap X^*(M \setminus \{y\})X^+ = \varnothing$. Therefore $M_1 \cap X^* M_1 X^+ = \varnothing$. In the same way, we can show $M_1 \cap X^+ M_1 X^* = \varnothing$. So $M_1 \cap (X^* M_1 X^+ \cup X^+ M_1 X^*) = \varnothing$. Hence $M_1 \in I(x)$ by Lemma 2.2.
Suppose $M_1 \notin MI(x)$. Then there exists $f \in X^+ \setminus M_1$ such that $M_1 \cup \{f\} \in I(x)$.

(I) If $f = y$, then $M_1 \cup \{f\} = M_1 \cup \{y\} = (M \setminus \{y\}) \cup yX \cup Xy \cup \{y\} = M \cup yX \cup Xy$. For any $a \in X$, we have $ya \in yX \subseteq M_1 \cup \{y\}$, which contradicts $M_1 \cup \{f\} \in I(x)$, because $y \in M_1 \cup \{f\}$.

(II) If $f \neq y$, since $f \in X^+ \setminus M_1$ and $M \in MI(x)$, then $M \cup \{f\} \notin I(x)$.

(II-1) If there exist $m_1 \in M$ and $x_1, x_2 \in X^+$ such that $f = x_1 m_1 x_2$, then we think about m_1 and y. If $m_1 \neq y$, then $m_1 \in M \setminus \{y\} \subseteq M_1 \cup \{f\}$. So $f = x_1 m_1 x_2 \in M_1 \cup \{f\}$, which contradicts $M_1 \cup \{f\} \in I(x)$. If $m_1 = y$, let $x_1 = a_1 a_2 \cdots a_k$ where $a_1 a_2 \cdots a_k \in X$. Then $f = x_1 m_1 x_2 = a_1 a_2 \cdots a_k m_1 x_2 = (a_1 a_2 \cdots a_{k-1})(a_k m_1) x_2$, because $a_k m_1 \in Xy \subseteq M_1 \cup \{f\}$. Hence $f \in M_1 \cup \{f\}$, which contradicts $M_1 \cup \{f\} \in I(x)$.

(II-2) If there exist $m_2 \in M$ and $x_3, x_4 \in X^+$ such that $m_2 = x_3 f x_4$, then we think about m_2 and y. If $m_2 = y$, then for any $a \in X$ we have $m_2 a = ya = x_3 f x_4 a \subseteq M_1 \cup \{f\}$. Then $f \subseteq M_1 \subseteq M_1 \cup \{f\}$, which contradicts $M_1 \cup \{f\} \in I(x)$. If $m_2 \neq y$, since $m_2 \in M$, then $f \subseteq M_1 \subseteq M_1 \cup \{f\}$, which contradicts $M_1 \cup \{f\} \in I(x)$. Therefore $M \cup \{f\} \in MI(x)$.

(2) Let $(M \setminus (yX \cup Xy)) \cup \{y\} = M_2$. Then

$$\begin{aligned}&M_2 \cap X^* M_2 X^+ \\ &= [(M \setminus (yX \cup Xy)) \cup \{y\}] \cap X^*[(M \setminus (yX \cup Xy) \cup \{y\}] X^+ \\ &= [(M \setminus (yX \cup Xy)) \cap X^*(M \setminus (yX \cup Xy)) X^+] \cup [(M \setminus (yX \cup Xy)) \cap X^* \{y\} X^+] \\ &\cup [\{y\} \cap X^*(M \setminus (yX \cup Xy)) X^+] \cup [\{y\} \cap X^* \{y\} X^+].\end{aligned}$$

First, we can see that $(M \setminus (yX \cup Xy)) \cap X^*(M \setminus (yX \cup Xy)) X^+ = \varnothing$ and $\{y\} \cap X^* \{y\} X^+ = \varnothing$. Suppose there exists $u \in (M \setminus (yX \cup Xy)) \cap X^* \{y\} X^+$. Then $u = x_1 y y_1$ for some $x_1 \in X^*$ and $y \in X^+$ Let $y_1 = b_1 b_2 \cdots b_k$ where $b_i \in X$ Then $u = x_1 (y b_1) b_2 \cdots b_k \subseteq M$, which contradicts $M \in MI(X)$, because $y b_1 \in yX \subseteq M$. Thus $(M \setminus (yX \cup Xy)) \cap X^* \{y\} X^+ = \varnothing$. Suppose there exists $u_1 \in \{y\} \cap X^*(M \setminus (yX \cup Xy)) X^+$. Then $u_1 = y = x_2 u_2 y_2$ for some $u_2 \in M \setminus (yX \cup Xy)$ and $x_2 \in X^*, y_2 \in X^+$. For any $a \in X$, we have $ya = x_2 u_2 y_2 a \in yX \subseteq M$ and $u_2 \in M$, which contradicts $M \in MI(X)$. Thus $\{y\} \cap X^*(M \setminus (yX \cup Xy)) X^+ = \varnothing$. From all above, we know $M_2 \cap X^* M_2 X^+ = \varnothing$. By the same way, we can show $M_2 \cap X^+ M_2 X^* = \varnothing$. Thus $M_2 \in I(X)$ by Lemma 2.2.

Suppose $M_2 \notin MI(X)$. Then there exists $g \in X^+ \setminus M_2$ such that $M_2 \cup \{g\} \in I(x)$. For any $a \in X$, we know $g \neq ya, g \neq ay$ and $g \notin M$. So $g \in X^+ \setminus M$. Since $M \in MI(X)$, then $M \cup \{g\} \notin I(x)$. We consider the following two cases.

(III) There exist x_3, $y_3 \in X^*, \lg(x_3 y_3) \geq 1$ and $u_3 \in M$ such that $g = x_3 u_3 y_3$. If $u_3 = ya$ where $a \in X$, then $g = x_3 u_3 y_3 = x_3 y a y_3 \in M_2 \cup \{g\}, y \in M_2 \subseteq M_2 \cup \{g\}$, which contradicts $M_2 \cup \{g\} \in I(X)$. If $u_3 = by$ where $b \in X$, similarly, we have $g = x_3 b y y_3 \in M_2 \cup \{g\}$, which contradicts $M_2 \cup \{g\} \in I(X)$. If $u_3 \notin (yX \cup Xy)$, then $u_3 \in M_2 \subseteq M_2 \cup \{y\}$, which contradicts $M_2 \cup \{g\} \in I(X)$.

(IV) There exist x_4, $y_4 \in X^*, \lg(x_4 y_4) \geq 1$ and $u_4 \in M$ such that $u_4 = x_4 g y_4$. If $u_4 = ya$ where $a \in X$, then $u_4 = ya = x_4 g y_4 = x_4 g y_5 a$. If $x_4 = 1$, then $ya = g y_5 a$ where $y = g y_5$. Then $y_5 \neq 1$ as $y \neq g$. So $y = x_4 g y_5 \in M_2 \cup \{g\}$, which contradicts $M_2 \cup \{g\} \in I(X)$. If $x_4 \neq 1$ then $y = x_4 g y_5 \in M_2 \cup \{g\}$, which contradicts $M_2 \cup \{g\} \in I(X)$. If $u_4 = ya$ where $a \in X$, we also have a contradiction. If $u_4 \notin (yX \cup Xy)$, then $u_4 \in M_2, u_4 = x_4 g y_4 \in M_2 \cup \{g\}$, which contradicts $M_2 \cup \{g\} \in I(X)$. From all the above, we can obtain $(M \setminus (yX \cup Xy)) \cup \{y\} \in MI(x)$.

Theorem 4.2. Let $A \in I(X)$ such that $Lg(A) = n$ and $x \notin A$ Then there exists $M \in MI(X)$ such that $Lg(M) \leq n+1$, $A \subseteq M$ and $x \notin M$.

Proof. If $A \in MI(X)$, let $M = A$. Now assume that $A \notin MI(X)$. Then, by Lemma 2.1, there exists $M \in MI(X)$ such that $A \subseteq M$ and $Lg(M) = n$. If $x \notin M$, then M is the required maximal infix code. If $x \in M$, then $(M \setminus \{x\}) \cup xX \cup Xx \in MI(X)$ by Theorem 4.1. Since $x \notin A$, then $A \subseteq (M \setminus \{x\}) \cup xX \cup Xx$ and $Lg[(M \setminus \{x\}) \cup xX \cup Xx] \leq n+1$.

Theorem 4.3. Let $A \in I(X)$ and $Lg(A) = n$. Then there exists finite number maximal infix codes

$M_1, M_2, \cdots, M_k$ such that $A = \cap_{i=1}^{k} M_i$, where $Lg(M_i) \le n+1$ and $i = 1,2,\cdots,k$.

Proof. Let $D = \{M \in MI(X) | A \subseteq M, Lg(M) \le n+1\}$. It is known that the set D is finite. Let $D = \{M_1, M_2, \cdots, M_k\}$. Then $A \subseteq \cap_{i=1}^{k} M_i$. Let $x \notin A$. By Theorem 4.2, there exists an $M^{'} \in MI(X)$ such that $A \subseteq M^{'}$ and $x \notin M^{'}$, where $Lg(M^{'}) \le n+1$. But $M^{'}$ is in D. So $x \notin \cap_{i=1}^{k} M_i$. Thus $A = \cap_{i=1}^{k} M_i$.

In [1], the authors show that for any finite infix code A, the language $\overline{A}$ is the maximal infix code containing A, which contains A and for any $u \in X_n$, where $X_n = \{u \in X^* \mid \lg(u) \le n\}$, there exist a word in $\overline{u} \in \overline{A}$ such that u is not an infix of $\overline{u}$ and $\overline{u}$ is not an infix of u. For example, Let $X = \{a,b\}$ and $A = \{a^2, b^3\}$. By calculation, we obtain that the finite maximal infix codes containing A are $\{a^2, b^3, aba, ab^2, b^2a, bab\}$ or $\{a^2, b^3, ab, ba\}$. So there are some maximal infix codes which contain an infix code. Hence the expression of A given in the above Theorem is, in general, not unique.

For every infinite infix code, we can prove that there exists a maximal infinite infix code containing it by Zorn's Lemma.

Proposition 4.4. Every infix code is contained in a maximal infix code.

Proof. Let $A \subseteq X^+$ be an infix code and $M = \{L \in I(X) | A \subseteq L\}$. Assume that $L_1 \subseteq L_2 \subseteq L_3 \subseteq \cdots$ be a ascending chain in M. Denote $\overline{L} = \bigcup_{i \ge 1} L_i$. Then $A \subseteq \overline{L}$. Now we show $\overline{L} \in I(X)$. For any $x, uxv \in \overline{L}$ where $u, v \in X^*$, there exist $i, j \ge 1$ (assume $i \ge j$) such that $x \in L_i, y \in L_j$. Since $i \ge j$, then we have $x, y \in L_i$. Since $L_i \in I(X)$, then $u = v = 1$. This implies that $\overline{L} \in I(X)$. Hence we have $\overline{L} \in M$. It implies that $\overline{L}$ is a upper bound of M. By the Zorn's Lemma, the set M has a maximal element, denoted by L'. Suppose there exists $w \in X^* \setminus L'$ such that $L' \cup \{w\} \in I(X)$. Since $A \subseteq L'$, then $A \subseteq L' \cup \{w\}$. So $L' \cup \{w\} \in M$ and $L' \subseteq L' \cup \{w\}$. Thus $L' = L' \cup \{w\}$, because L' is a maximal element in M. Therefore $w \in L'$, which contradicts $w \in X^* \setminus L'$. Hence L' is a maximal infix code containing A.

In the following, we give some properties on elements of finite maximal infix codes.

Theorem 4.5. Let A be a finite maximal infix code. Then for any $a \in X$ there exists $m \ge 0$ such that $a^m \in A$.

Proof. Suppose there exists $a \in X$ and for all $m \ge 1$ such that $a^m \notin A$. Let $Lg(A) = s$ then $A \cup \{a^{s+1}\} \in I(X)$. If $A \cup \{a^{s+1}\} \notin I(X)$, then there exists $u \in A$, then one of the following case holds. (1) $x_1 y u_1 = a^{s+1}$ where $x_1, y_1 \in X^*$ and $\lg(x_1 y_1) \ge 1$; (2) $u = x_2 a^{s+1} y_2 \in A$ where $x_2, y_2 \in X^*$ and $\lg(x_2 y_2) \ge 1$. If (1) holds, then $u = a^i$ for some $i \ge 1$, which contradicts the assumption. If (2) holds, then $x_2 a^{s+1} y_2 \in A$, which contradicts $Lg(A) = s$. So $A \cup \{a^{s+1}\} \in I(X)$, which contradicts $A \in MI(X)$. Thus for any $a \in X$, there exists $m \ge 1$ such that $a^m \in A$.

Proposition 4.6. Let ABC be a finite maximal infix code. Then for any $a \in X$, there exist

$m,n,s \ge 1$ such that $a^m \in A, a^n \in B, a^s \in C$.

Proof. Since ABC is a finite maximal infix code, then for any $a \in X$ there exists $k \ge 0$ such that $a^k \in ABC$ by Theorem 4.5. Therefore, there exist $m,n,s \ge 0$ and $m+n+s=k$ such that $a^m \in A, a^n \in B, a^s \in C$.

Proposition 4.7. Let ABC be a finite maximal infix code. Then $a^k x a^k \notin B$ for any $x \in B$, $a \in X$ and $k \ge 1$.

Proof. Since ABC is a finite maximal infix code, then there exist $m,n \ge 0$ such that $a^m \in A, a^n \in C$ by Proposition 4.6. Then $a^m x a^n \in ABC$ for any $x \in B$. Suppose there exists $a^k x a^k \in B$. Then $a^m (a^k x a^k) a^n \in ABC$. So $a^k (a^m x a^n) a^k \in ABC$, which contradicts $ABC \in MI(X)$. Thus $a^k x a^k \notin B$ for any $x \in B$, $a \in X$ and $k \ge 1$.

5 Conclusion

In [1], the authors proved that if A is a finite infix code, then there exists a finite maximal infix code containing it. For every infinite infix code, we proved that there exists a maximal infinite infix code containing it by Zorn's Lemma in theory. But we can't know the detailed construction of the maximal infix code. In the future, we indicate to give the detailed construction of the maximal infix codes for any infix code.

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