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Optimal Combinations of Character of a Superior and His/Her Subordinate in View of Transforming Business-front Information into Value

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Abstract: This paper firstly formulates a mathematical framework to analyze the effect of different combinations of self-loyal character of a superior (B) and his subordinate (C) on the process of transforming business-front information C brings into a value. Here, the self-loyal character is described as the character that makes a person tend to stick to his own value base so long as the issue is an important matter of his pride. In the formulation, our consideration is given to the fact that C and B are likely to confront a psychological game with respect to differences of their assessments on the information value and with respect to differences of their self-loyal character, and we formulate character-dependent utility functions on which the game develops. Secondly proposing a specific model, we derive a Nash equilibrium of the assessments for the game between C and B, and when the process is repeated, via simulation we derive optimal combinations of their self-loyal character from a viewpoint of human resource management for management (A) that seeks the organizational efficiency and stability of each business line. It is shown that A highly assesses the combinations with B's self-loyal character being strong.

JEL Classification: M50, M12, M51, M54

Keywords: mathematical model for superior and subordinate relation, transformation of business front information into value, optimal combination of self-loyal character, Nash equilibrium, human resource management.

Abbreviations: bf = business-front, PVCIT = Process of Value-creating Information Transformation, MPACC = Management Process of Assessing Character-Combination

1. Introduction

Naturally, firms need to adapt their management processes to evolving business environments by using effective information. Among others, those subordinate employees (C's) who directly contact a business front play an important role in routinely collecting relevant business-front information (abbreviated as bf-information below), streaming it into their business lines and transforming it into knowledge for valuable implementation. The bf-information includes information on clients' or customers' needs, suppliers' movements, industry tendency and competitors' activities such as services and marketing. If C's direct superior (boss for simplicity) B

can well implement such bf-information together with C, it will enable B and C to understand changes in the external environment, to improve the efficiency of current processes for strengthening the competitiveness of their business line and to sometimes help B and managers develop new strategies that can lead to important improvements or changes in products and services. Consequently this information-transforming process has a potential to increase sales volume and/or reduce cost, e.g., by modifying or improving current procurement or current sales or marketing processes, through which firm's value increases.

In this paper we formulate a general mathematical framework to analyze the effect of combinations of character of a direct boss B and a subordinate C of his on the process of their transforming bf-information, which is routinely collected by C, into a decision for implementing it for an increase of firm value. In this process, the character we focus on is the self-loyalty that makes a person tend to stick to his own value base so long as the issue is an important matter of his pride or intellectual identity. And some optimal combinations of character are found for B and C via simulation by introducing the utility of management body A.

We will not aim to directly treat the relation between our information-transforming process and firm value or profit, but the effectiveness of the process between B and C in view of character-dependent relationship from a perspective of management A. It is assumed that the intellectual capabilities of B and C are not very different though their experiences and knowledge may be different. In addition, we only treat the case of two-person game with a single subordinate C and his boss B with the game being repeated. However, in Section 3 where a specific model is assumed, when B is strongly self-loyal, the process of value-creating information transformation is shown to be effective for any subordinate.

1.1 PVCIT and MPACC

To state our problem, we define the two processes; 1) Process of Value-creating Information Transformation (abbreviated as PVCIT), and 2) Management Process of Assessing Character-Combination (abbreviated as MPACC). These two processes form the structure of our problem to mathematically formulate and the MPACC is a control process for management A to optimally control a combination of character of B and C.

Definition 1. A *PVCIT* is defined to be a process of the following three sub-processes.

1) Sub-process (1) of prior assessments: Subordinate C meets bf-information, assesses its potential value, and transforms it into knowledge with his prior view (distribution) on the success of implementation for a value. Then C selectively reports it to his direct boss B. The selection rule depends on the mean of his prior and his self-loyal character. Then B forms a prior view on the success of possible implementation for a value after he reads C's report.
2) Sub-process (2) of Nash assessments: They communicate and discuss about its potential value of the bf-information by sharing their views on the probability of success for a valuable implementation, and then they form their posterior views (distributions) that depend on their self-loyal character. Finally, when they express their final assessments based on their expected character-dependent utility functions, they confront a game-theoretic situation in stating their assessments, which leads them to finding their optimal (Nash) assessments of the success probability for implementation.
3) Sub-process (3) of B's decision making: After they exchange their Nash assessments, B takes an action on whether the bf-information should be implemented for materialization or not. The decision rule for taking an action or no action is based on B's expected character-dependent utility function, which is different from the one in Sub-process (2).

The above PVCIT is one time process for each piece of information but it is routinely and daily repeated as a business process between C and B for improving efficiency and seeking for

future cash flows. In the definition of MPACC, the repetition of PVCIT is assumed because the information generation process in business front is stochastic.

Definition 2. The *MPACC* owned by management A is defined to be the process of evaluating the average performances of the PVCIT between C and B under repetitions and deriving optimal combinations of the self-loyal character of C and B for more effective PVCIT as a human resource management along their empowerment hierarchy, where the evaluation is based on A's expected utility function and optimality is defined relative to its control parameter in the utility function.

In the PVCIT, C daily collects pieces of information in his business front, assesses them and only passes to B what he thinks important in possibly adding a value to their business. However, this PVCIT process will include a potential game-theoretic problem between C and B that arises due to disagreement on their assessment about the potential value, which may be due to the asymmetry of information and/or due to the difference of their character. Hence it will be a character-dependent psychological game. In fact, even after discussions and exchanges of opinions, B does not necessarily agree with C about the potential value of the bf-information that C finds valuable and passes. And when the information is rejected by B even after C explains fully about his reasoning, C tends to feel devalued or disgraced in his pride and intellectual capability. Here it is noted that C's confidence on the evaluation may follow partly from the reason that he thinks he knows better about his business front and partly from the reason that the information passed to B has been filtered by his judgment and intellectuality. In addition, a strong self-loyalty character associated with intellectual capability will often govern a person over any other character including leniency.

The MPACC in Definition 2 is a control process on the PVCIT. The management A concerns the problem on whether B and C cooperate effectively in the PVCIT and so when the PVCIT is repeated over a certain period, A evaluates an average effectiveness of the process and tries to find an optimal combination of the character of B and C based on A's expected utility function, where optimality is defined relative to its control parameter in the utility function.

It is remarked that in Japanese corporate system, when the disparity between B and C becomes large due to the differences of self-loyalty character along the repetitions of the process, the relationship often tends to be serious in a long run. This is because C's dissatisfaction could not be replaced by a bonus or income compensation as the salary system is inflexible and C's employment is protected by law.

We summarize our work. In Section 2, the PVCIT formulation is made, where for an analytical tractability, the character-dependent utility functions of C and B are assumed to be Cobb-Douglas type. And truncated normal distribution is assumed for C's and B's prior views (subjective distribution). In Sub-process (2), after communications and discussions, they form posterior views on [0,1], with respect to which C's and B's posterior expected utility functions are evaluated from their character-dependent utility functions. Then the Nash success probabilities for the character-dependent game between B and C are analytically derived. In Sub-process (3), B makes a decision on whether to take an action or no action based on another expected character-dependent utility function. The decision rule we formulate for a given piece of bf-information is shown to be optimal in Appendix when the PVCIT is repeated.

In Section 3 the MPACC as a control process on the PVCIT is formulated for finding optimal combinations of character of B and C. The management A tries to find an optimal solution for B and C to work cooperate effectively when the PVCIT is repeated over a period. With Cobb-Douglas type utility for A, the effectiveness of the PVCIT in the MPACC under his expected utility is evaluated via simulations and optimal combinations of self-loyal character of B and C are found out for the organizational efficiency and stability of each business line for firm value. Then when B is strongly self-loyal, the PVCIT is shown to be effective for any character of the subordinate or

equivalently for any subordinate. Hence this case will be a solution for controlling a PVCIT in view of A. Finally, Section 4 concludes our results and makes some concluding remarks.

1.2 Preceding Research

In the literature many problems on manager–employee relationships in views of such concepts as leadership, trust, incentive, teamwork, rapport, motivation etc. have been treated mostly in the areas of human resource management and organizational behavior and psychology. However, to our knowledge, it seems that there seems no such paper as formulating and considering character-dependent game-theoretic relationship of direct superior and subordinate in the PVCIT with character of self-loyalty deeply related to pride and intellectual identity.

In view of organizational economics, using a model in Mintzberg (1979), Gibbons, Matouschek and Roberts (2013) recently formalized an interesting broad model to treat a problem about how information sent by a sender will make an effect on the decision of its receiver with utility functions, where the decision process is focused on advice-selection in the process of information-advice-selection-action. This is related to our paper, but different from our superior-subordinate game via a combination of their self-royal character.

Concerning empowerment hierarchy in our terminology, Baker, Gibbons and Murphy (1999) assert that decision rights in organizations are not contractible: the boss can always overturn a subordinate's decision, so formal authority resides only at the top. But as the authors say, decision rights cannot be formally delegated, and the rights might be informally delegated through self-enforcing relational contracts. Also, to some extent, our argument is related to that of Willemyns, Gallois and Callan (2003) who examine employees' perceptions of trust, power and mentoring in manager-employee relationships in a variety of sectors, including health care, education, hospitality and retail.

Manzoni and Barsoux (2009) analyze the boss-employee relationship that leads to what they call the *setup-to-fail syndrome*, where employees —sometimes deliberately, sometimes subconsciously —take steps to sabotage their boss's success. Leaders make the situation worse by not understanding why an employee might act this way. Our paper may give a specific reason why a subordinate sometimes sets his boss up to fail in our context of bf-information transformation.

Leonard (2008) points out the importance of the role of managers' trust in subordinates. Payne and Clark (2003) analyzed factors that develop trust toward management, along with suggestions for developing more trust in an organization. In view of rapport, a big related topic is the leader-member exchange (LMX) and most LMX studies have examined how member characteristics affect leader perceptions of the member (Liden, Sparrowe and Wayne (1997)). In fact, LMX refers to the relationship quality a leader shares with members of his or her workgroup, typically described as differentiation in quality within the group as in the recent paper by Schyns, Maslyn and Weibler (2010), who suggest how LMX consensus and a high LMX level can be established even in large spans of control (see also Ilies, Nahrgang and Morgeson (2007)). The literature in this area demonstrates that the quality of this relationship is positively related to followers' attitudes and organizational outcomes, and proposes that the quality of possible relationships between the leader and the led will be affected by the number of employees directly reporting to the leader.

2. Process of Value-Creating Information Transformation (PVCIT) and Nash Equilibrium

In this section we mathematically formulate the PVCIT in Definition 1 and under the assumption that character-dependent utility function is Cobb-Douglas-type with self-loyalty parameters (α_C, α_B), we analytically derive a Nash equilibrium for the game concerning the final statements of C and B on the probability of success for valuable implementation of bf-information. Here (α_C, α_B) denotes the relative degree of self-loyalty in character of C and B, which is a central focus of this study, where α_C (or α_B) takes a value in the unit interval [0, 1]. Here the closer α_C (or α_B) is to 1, the greater the degree of self-loyalty is and so the stronger his tendency to adhere to his own assessment on each bf-information.

As in the Definition 1, the PVCIT consists of the three sub-processes:

1) Sub-process (1): When a subordinate C meets bf-information, he assesses it for a possible implementation, forms a prior subjective view (distribution F_C^0) on the interval [0,1] with mean view x_C^0 of success, on which he decides on whether he reports it to B or not. The decision depends on the degree of self-loyal character α_C.
2) Sub-process (2): When B gets a report from C, B forms a prior subjective view (distribution F_B^0) on [0,1] with mean view x_B^0 of success. Then after their communications and discussion about possible implementations they form their posterior views (distributions). And they will finally "state" their assessments of the probability of success with respect to their character-dependent utility functions, where importantly it entails a psychological game-theoretic nature in expressing or exchanging their final assessments. The game will lead them to the Nash equilibria of assessments.
3) Sub-process (3): After they exchange their Nash assessments, B decides on whether the bf-information should be implemented for materialization or not. The decision rule is based on B's another expected character-dependent utility function.

This argument of shifting from prior view to posterior view is very similar to a well-known Bayesian decision process that combines a prior view on unknown parameters and a distribution of data given the parameter to get a posterior distribution on the likelihood of parameter. However, our formulation is not the same as the Bayesian formulation because a subordinate and his boss seem likely to form posterior views more intuitively in routinely activities.

For analytical tractability, we assume truncated normal distribution on [0, 1] for C's and B's prior views, denoted as $N^t(\mu, \sigma^2)$, where normal distribution $N(\mu, \sigma^2)$ with mean μ and variance σ^2 is truncated to unit interval [0, 1]. The pdf is given by

$$g(x:\mu,\sigma^2) = f(x:\mu,\sigma^2)/c(\mu,\sigma^2) \quad \text{with} \quad c(\mu,\sigma^2) = \int_0^1 f(u:\mu,\sigma^2)du \qquad (1)$$

where $0 \le x \le 1$ and $f(x:\mu,\sigma^2)$ is the pdf of $N(\mu,\sigma^2)$. In our case, note that the larger (or smaller) μ, the smaller σ^2 because when C's mean view x_C^0 is closer to 1 (or 0 respectively), he tends to be more confident in his assessment on the information. Hence it will be assumed that either $(1 \ge \mu \ge 0.5,\ 1-\mu \ge 3\sigma > 0)$ or $(0.5 > \mu \ge 0,\ \mu \ge 3\sigma > 0)$ holds. Under this assumption the denominator in equation (1) satisfies $0.9974 < c(\mu,\sigma^2) < 0.9986$, meaning that the truncated density is extremely close to normal density.

2.1. Sub-process (1): Business-front Information and Screening

Now when C gets a piece of bf-information with an idea of implementation based on his business-front knowledge and intellectual insight, C first conceives a mean probability, say x_C^0, of success for a valuable implementation of the information and then forms a prior subjective probability distribution function F_C^0 on the interval [0,1] with mean x_C^0, where each point in [0,1] corresponds to the probability of success for a valuable implementation.

Let $\tilde{x}_C^0$ denote a random variable following distribution F_C^0 and by C's prior view, we mean his prior distribution F_C^0, where $x_C^0 = E(\tilde{x}_C^0)$. Suppose that F_C^0 is assumed to be the following truncated normal distribution

$$\tilde{x}_C^0 \sim N^t\left(x_C^0\ ,\ \left(\sigma_C^0\right)^2\right) \text{with}\ \ \sigma_C^0 = \min\left(\frac{x_C^0}{3}\ ,\ \frac{1-x_C^0}{3}\right), \qquad (2)$$

where σ_C^0 is originally the standard deviation of $N(x_C^0,(\sigma_C^0)^2)$. But under F_C^0 in equation (2), we shall call it uncertainty parameter about the mean view $x_C{}^0$. For simplicity σ_C^0 is specified as a function of the mean view $x_C{}^0$ and the larger or the smaller $x_C{}^0$ is, the smaller σ_C^0 is, and the denser the mass is around $x_C{}^0$. This implies that the closer $x_C{}^0$ is to 1 (or 0), the more he is confident in his assessment for valuable (or for valueless respectively) implementation.

2.1.1 Character-Dependent Screening Rule

For each piece of bf-information that he meets, C assesses it by his business-front knowledge and experiences and reports his implementation idea or plan with his mean view to B only if his mean view or equivalently expected probability of success x_C^0 for a valuable implementation is greater than or equal to a certain threshold level d in [0,1];

$$C \text{ reports only if } \ x_C^0 \geq d \ \text{ and the PVCIT stops here if } \ x_C^0 < d \qquad (3)$$

In other words, only when $x_C^0 \geq d$, the bf-information with his implementation idea and mean view x_C^0 will be reported to B. When $x_C^0 < d$, nothing will be reported to B though C accumulates it in his knowledge. The value of the non-reported information will be often revealed to C later, and he thinks he is more knowledgeable about relations between bf-information and a possible value. Thus C will supposedly be in a relatively advantageous position to know about the information on his business-front movements, which will in turn make him tend to be more confident especially when he is strongly self-loyal to his pride on intellectuality. The threshold d will reflect "propensity to screen out bf-information relative to his pride on intellectuality" and it will be dependent on the C's degree of self-loyalty, i.e., $d = d(\alpha_C)$, where the stronger α_C is, the greater d is and the less he reports. If C is of weaker self-royal character α_C, he tends to report more to his boss for getting credits and having a good relation with his boss since he less sticks to his assessment x_C^0. We will specify

$$d = f(\alpha_C) = a + b\alpha_C \qquad \text{with} \qquad 0 \leq a,b \leq 1 \qquad (4)$$

In the simulation of finding optimal combinations of character in Section 3, we will assume $d = 0.5 + 0.13\alpha_C$ ($\geqq 0.5$). It will be reasonable to assume that even a subordinate of weaker self-royal character will not report unless the mean probability $x_C{}^0$ of success is larger or equal to 0.5. Under this assumption, it follows from equation (2) that

$$\sigma_C^0 = \left(1 - x_C^0\right)/3 \qquad (5)$$

When B receives from C a report on specific bf-information along with C's mean assessment (mean view) x_C^0, B conceives his mean probability, say x_B^0, of success for a valuable implementation and then form a prior subjective distribution F_B^0 on [0,1] on the basis of his knowledge and experiences. Let $\tilde{x}_B^0$ denote a random variable following F_B^0 with mean view $x_B^0 = E(\tilde{x}_B^0)$, where B's formation of x_B^0 is completely independent of C's formation of x_C^0, by which we mean that $\tilde{x}_C^0$ and $\tilde{x}_B^0$ are independent.

Here similarly to equation (2), for F_B^0 we assume the following truncated normal distribution:

$$\tilde{x}_B^0 \sim N^t\left(x_B^0, \left(\sigma_B^0\right)^2\right) \quad \text{with} \quad \sigma_B^0 = \min\left(\frac{x_B^0}{3}, \frac{1-x_B^0}{3}\right) \qquad (6).$$

2.2. Sub-process (2): Nash Assessment on the Business-front Information

After B gets a report from C, B starts to communicate with C. First, while B knows C's mean view x_C^0, B also states to C about his mean view x_B^0 before the discussion. Then B and C discuss about the potential value of the bf-information. Through the discussion B and C will share some more information, knowledge and idea, compare the other party's view to each own view via their experiences and expertise knowledge, and in the end they form posterior views F_C^1 and F_B^1, which are also respectively represented by random variables $\tilde{x}_C^1$ and $\tilde{x}_B^1$.

Each posterior distribution is assumed to be dependent on each own self-royal character. This means that they take the other party's assessments on the probability of success into consideration in forming posterior distributions. In other words, this posterior probability distribution of the likelihood for valuable implementation will be influenced by the other party's initial mean view x_B^0 or x_C^0, which are respectively expressed as

$$F_B^1(\cdot) \equiv F_B^1(\cdot : \alpha_B, x_B^0, x_C^0) \text{ and } F_C^1(\cdot) \equiv F_C^1(\cdot : \alpha_C, x_C^0, x_B^0) \qquad (7)$$

Here the posterior views are influenced by the other party's prior view and their own self-loyal character. Here for simplicity and analytical tractability, we assume that for C and B

$$\tilde{x}_i^1 \sim N^t(\mu_i^1, (\sigma_i^1)^2) \qquad (8)$$

with $\mu_i^1 = \alpha_i x_i^0 + (1-\alpha_i) x_j^0$ and $\sigma_i^1 = \min\left(\frac{\mu_i^1}{3}, \frac{1-\mu_i^1}{3}\right)$,

where i, j=C, B and $i \neq j$. These specifications exhibit self-evident implications; their posterior mean views μ_C^1 and μ_B^1 linearly combine the initial mean views of other party by their own self-loyalty parameters (α_C, α_B), and if the self-loyal character is stronger, they take less the other's assessment into account. For uncertainty parameters σ_C^1 and σ_B^1, the more the posterior mean views deviate from 0.5, the more confident they are about their posterior mean views.

To describe the potential game that C and B eventually confront, we introduce their utility functions for making their final statements on the probability x_C^* and x_B^* of success. Let C's and B's utility functions on [0,1]×[0,1]×[0,1] be respectively given by

$$U_C = U_C(w_{1C}, w_{2C(B)} : \alpha_C) \text{ and } U_B = U_B(w_{1B}, w_{2B(C)} : \alpha_B) \qquad (9)$$

where U_C is continuous, positive and differentiable in its interior points,

$$w_{1C} = \left|\tilde{x}_C^1 - x_C^*\right|, \quad w_{2C(B)} = \left|x_B^* - x_C^*\right| \qquad (10)$$

$$\partial U_C(w_1,w_2)/\partial w_1 < 0 \text{ and } \partial U_C(w_1,w_2)/\partial w_2 < 0$$

For the case of B, the suffixes C and B in equation (10) are respectively replaced by B and C. Note that U_C is bounded and takes values in a closed interval by its assumption. Here w_{1C} measures the gap between C's posterior view $\tilde{x}_C^1$ and his assessment x_C^* for C to choose as his statement. Since the utility function is assumed to be a decreasing function of w_{1C} given the remaining variables, the larger the deviation in w_{1C}, the smaller the utility is. While, $w_{2C(B)}$ measures the gap between the assessments that C and B state and since the utility is assumed to be a decreasing function of $w_{2C(B)}$ given the remaining variables, the larger the gap, the smaller the utility is. A key variable is the parameter α_C representing the degree of self-royal character that will make a balance between w_{1C} and $w_{2C(B)}$. If α_C is close to 1, the utility will be likely to be more sensitive to the variable w_{1C}. The same argument applies to the case of B.

In the sequel, for simplicity, the character-dependent utilities are specified by Cobb-Douglas type functions:

$$U_C = \exp\{-\alpha_C(\tilde{x}_C^1 - x_C^*)^2 - (1-\alpha_C)(x_C^* - x_B^*)^2\} \tag{11}$$

and

$$U_B = \exp\{-\alpha_B(\tilde{x}_B^1 - x_B^*)^2 - (1-\alpha_B)(x_B^* - x_C^*)^2\} \tag{12}$$

These utility functions are normalized to have maximum exp(0)＝1 and minimum exp(－1).

Now as a choice of x_C^*, C is assumed to maximize his expected utility function

$$E(U_C) = \int_0^1 U_C(w_{1C}(\tilde{x}_C^1), w_{2C(B)} : \alpha_C) dF_C^1(\tilde{x}_C^1) \equiv \bar{u}_C(x_C^*, x_B^*; \alpha_C) \tag{13}$$

for given x_B^*. The optimal choice of x_C^* is denoted by $x_C^{**} = x_C^{**}(x_B^*) \equiv x_C^{**}(x_B^*; \alpha_C)$ with

$$\max_{x_C^*} \bar{u}_C(x_C^*, x_B^*; \alpha_C) = \bar{u}_C(x_C^{**}(x_B^*), x_B^*; \alpha_C). \tag{14}$$

The maximization is carried out for the other party's assessment x_B^* given. The case of B is obtained simply by replacing the role of C by B. Hence a common solution of the maximizing variables $x_C^{**} = x_C^{**}(x_B^*) \equiv x_C^{**}(x_B^*; \alpha_C)$ and $x_B^{**} = x_B^{**}(x_C^*) \equiv x_B^{**}(x_C^*; \alpha_B)$ is a Nash equilibrium or equivalently to say, Nash assessment (x_C^n, x_B^n) in our context.

Note that x_B^n and x_C^n are functions of (x_C^0, x_B^0) and (α_C, α_B):

$$x_C^n \equiv x_C^n(x_C^0, x_B^0; \alpha_C, \alpha_B) \text{ and } x_B^n \equiv x_B^n(x_C^0, x_B^0; \alpha_C, \alpha_B). \tag{15}$$

For this argument to be effective, it is assumed that B and C both know

1) each other's character parameters α_B and α_C, and
2) each other's utility functions together with prior views in equation (2) and equation (6).

As a result, they finally choose a Nash equilibrium (x_B^n, x_C^n) for their statements.

2.2.1 Nash Assessments Under Cobb-Douglas Type Utility Functions

Under the Cobb-Douglas type utilities in equation (11) and equation (12) we shall derive a Nash assessment for them to state, which is in fact unique. For $i = B,\ C$, let

$$\mathrm{M}_i = \frac{2\alpha_i\left(\sigma_i^1\right)^2 x_i^* + \mu_i}{2\alpha_i\left(\sigma_i^1\right)^2 + 1} \text{ and } \Upsilon_i = \frac{\sigma_i^1}{\sqrt{2\alpha_i\left(\sigma_i^1\right)^2 + 1}} \tag{16}$$

For normal distribution $N(\mu, \sigma^2)$, let its density be denoted by $f(x : \mu, \sigma^2)$, and let

$$c_i = c(\mathrm{M}_i, \Upsilon_i^2) \quad (i = C, B) \quad \text{with} \quad c(\mu, \sigma^2) = \int_0^1 f(x : \mu, \sigma^2)dx \tag{17}$$

Then the following proposition holds.

Proposition 2.1 The expected utility functions of C and B are respectively given by

$$\bar{u}_C(x_C^*, x_B^*; \alpha_C) = c_C D_C \exp\left[-\left\{\frac{\alpha_C(x_C^* - \mu_C^1)^2}{2\alpha_C\left(\sigma_C^1\right)^2 + 1} + (1-\alpha_C)(x_C^* - x_B^*)^2\right\}\right] \tag{18}$$

and

$$\bar{u}_B(x_B^*, x_C^*; \alpha_B) = c_B D_B \exp\left[-\left\{\frac{\alpha_B(x_B^* - \mu_B^1)^2}{2\alpha_B\left(\sigma_B^1\right)^2 + 1} + (1-\alpha_B)(x_B^* - x_C^*)^2\right\}\right], \tag{19}$$

where $D_i = 1/\sqrt{2\alpha_i\left(\sigma_i^1\right)^2 + 1} \quad (i = B,\ C)$.

Proof. The expected utility function of B is computed as

$$\begin{aligned}
\bar{U}_B &= \int_0^1 \exp\left(-\alpha_B(\tilde{x}_B^1 - x_B^*)^2 - (1-\alpha_B)(x_B^* - x_C^*)^2\right) f\left(\tilde{x}_B^1 : \mu_B^1, \left(\sigma_B^1\right)^2\right) d\tilde{x}_B^1 \\
&= \frac{1}{\sigma_B^1\sqrt{2\pi}} \int_0^1 \exp\left[-\left\{\mathrm{I}_B\left(\tilde{x}_B^1\right)^2 - 2\mathrm{II}_B\tilde{x}_B^1 + \mathrm{E}_B\right\}\right] d\tilde{x}_B^1 \\
&= \frac{1}{\sigma_B^1\sqrt{2\mathrm{I}_B}} \exp[-\mathrm{E}_B + \frac{(\mathrm{II}_B)^2}{\mathrm{I}_B}] \cdot \frac{\sqrt{2\mathrm{I}_B}}{\sqrt{2\pi}} \int_0^1 \exp\left[-\frac{2\mathrm{I}_B}{2}\left(\tilde{x}_B^1 - \frac{\mathrm{II}_B}{\mathrm{I}_B}\right)^2\right] d\tilde{x}_B^1 \\
&= c_B D_B \exp[-\mathrm{E}_B + \frac{(\mathrm{II}_B)^2}{\mathrm{I}_B}],
\end{aligned}$$

where

$$\mathrm{I}_B = \frac{2\alpha_B\left(\sigma_B^1\right)^2 + 1}{2\left(\sigma_B^1\right)^2}, \qquad \mathrm{II}_B = \frac{2\alpha_B\left(\sigma_B^1\right)^2 x_B^* + \mu_B^1}{2\left(\sigma_B^1\right)^2}, \qquad \frac{1}{\sigma_B^1\sqrt{2\mathrm{I}_B}} = D_B \quad \text{and}$$

$$\mathrm{E}_B = \left(\alpha_B(x_B^*)^2 + \frac{(\mu_B^1)^2}{2(\sigma_B^1)^2} + (1-\alpha_B)(x_B^* - x_C^*)^2\right). \tag{20}$$

Computing $-\mathrm{E}_B + (\mathrm{II}_B)^2 / \mathrm{I}_B$ yields the result. The case of C is similar. □

We shall confirm that $c_i = c(\mathrm{M}_i, \Upsilon_i^2) \quad (i = C, B)$ is close to 1 in order to show the validity for using the approximate expected utilities $\bar{U}_C / c_C$ and $\bar{U}_B / c_B$ for C and B. Note that c_i is the integral of $N(\mathrm{M}_i, \Upsilon_i^2)$ over [0,1].

Lemma 2.1. The mean plus/minus λ times standard deviations are evaluated;

(1) $\mathrm{M}_B + \lambda\Upsilon \le 1$ is maximized at $\lambda = 3\sqrt{18/19} \approx 2.92$.

(2) $\mathrm{M}_B - \lambda\Upsilon \ge 0$ is minimized at $\lambda = 3\sqrt{18/19} \approx 2.92$.

Hence $c_i = c(\mathrm{M}_i, \Upsilon_i^2) = P(0 \le Z \le 1) \ge 0.98 \ (i = B, C)$.

Proof. By our assumption we prove only the case $x_C^0 \ge 0.5$. For (1) set

$$F = \mu_B^1 - (1 - \mu_B^1) = -2g + 1 \ \text{with} \ g = 1 - \mu_B^1 = \alpha y_B + (1-\alpha)y_C \tag{21}$$

where $y_B = 1 - x_B^0,\ y_C = 1 - x_C^0$. Note that $\mathrm{M}_B = \mathrm{II}_B / \mathrm{I}_B$ and $\Upsilon_B = 1/\sqrt{2\mathrm{I}_B}$ by equations (16)

and (20), and so

$$Q=\frac{\mathrm{II}_B}{\mathrm{I}_B}+\lambda\frac{1}{\sqrt{2\mathrm{I}_B}}=\frac{2\alpha_B\left(\sigma_B^1\right)^2x_B^*+\mu_B^1+\lambda(\sigma_B^1)\sqrt{2\alpha_B\left(\sigma_B^1\right)^2+1}}{2\alpha_B\left(\sigma_B^1\right)^2+1}$$

$$=\frac{1}{J}\{2\alpha h^2(x_B^*-1)+J+(\mu_B^1-1)+\lambda h\sqrt{J}\}$$

$$=-k+1+\frac{1}{J}[-g+\lambda h\sqrt{J}],$$

where

$$h=\sigma_B^1,\quad \alpha=\alpha_B,\quad J=1+2\alpha h^2,\quad k=2\alpha h^2(1-x_B^*)/J.$$

All these variables are nonnegative. In particular since $0\le x_B^*\le 1$, $k\ge 0$. And $k=0$ only if $x_B^*=1$ or $2\alpha h^2$=0. Hence

$$Q=-k+1+\frac{1}{J}[-g+\lambda h\sqrt{J}]\le 1+\frac{1}{J}[-g+\lambda h\sqrt{J}] \tag{22}$$

Here if $[-g+\lambda h\sqrt{J}]$ is non-positive, then $Q\le 1$ and so we find the maximum value of $\lambda(\ge 0)$ for all (α,y_B,y_C) satisfying $-g+\lambda h\sqrt{J}\le 0$.

If $h=0$, this inequality always holds for any $\lambda(\ge 0)$, where $h=0$ holds if and only if $\mu_B^1=0$ *or* $\mu_B^1=1$. Here $\mu_B^1=0$ holds if and only if $\alpha=1,\ x_B^0=0$. On the other hand, $\mu_B^1=1$ holds if and only if $(x_B^0=1,\ x_C^0=1)$ or $(\alpha=1, x_B^0=1)$.

Now suppose that $h\ne 0$. Then $\lambda\le g/h\sqrt{J}$ follows. Hence we minimize the left side of this inequality.

Case (1) $0.5\ge g=\alpha y_B+(1-\alpha)y_C>0$: Since $x_C^0\ge 0.5$, note

$$0.5\ge y_C\ge 0 \text{ and } h\equiv\sigma_B^1=(1-\mu_B^1)/3=(\alpha y_B+(1-\alpha)y_C)/3=g/3.$$

Hence by equation (22)

$$g/h\sqrt{J}=3/\sqrt{1+2\alpha g^2/9}\ \ge 3/\sqrt{1+2\alpha/36}\ge 3\sqrt{18/19}\approx 2.92.$$

Note that the second term is decreasing in g and hence the first inequality in the third term holds for any (α_B,y_B,y_C) satisfying $g=\alpha y_B+(1-\alpha)y_C=1/2$. For example, it holds if $y_B=y_C=1/2$.

Case (2) $0.5\le g=\alpha y_B+(1-\alpha)y_C\le 1$: Then since $0.5\ge y_C\ge 0$, it needs that $0\le x_B^0\le 0.5$ or equivalently $1\ge y_B\ge 0.5$. Then as $h\equiv\sigma_B^1=\mu_B^1/3=[-g+1]/3$,

$$g/h\sqrt{J}=3g/[(1-g)\sqrt{1+2\alpha[(1-g)/3]^2}]\ \ge 3/\sqrt{1+2[1/6]^2}=3\sqrt{18/19}.$$

Note that the second term is increasing in g. Therefore the minimum value of $g/h\sqrt{J}$ is attained for any (α_B,y_B,y_C) satisfying $g=\alpha y_B+(1-\alpha)y_C=1/2$.

We prove (2) in Lemma 2.1. By the same viewpoint, in the equation

$$Q^*=\frac{1}{J}\{2\alpha h^2x_B^*+\mu_B^1-\lambda h\sqrt{J}\}=\frac{1}{J}\{2\alpha h^2x_B^*\}+\frac{1}{J}[\mu_B^1-\lambda h\sqrt{J}],$$

we shall find the maximum value of λ for which $[\mu_B^1-\lambda h\sqrt{J}]$ is non-negative or equivalently $\mu_B^1/h\sqrt{J}\ge\lambda$. If $\mu_B^1=0$, λ is arbitrary. Consider the case $0<\mu_B^1\le 1/2$. Then since $h\equiv\sigma_B^1=\mu_B^1/3$,

$$\mu_B^1 / h\sqrt{J} = 3/\sqrt{1+2\alpha(\mu_B^1/3)^2} \geq 3/\sqrt{1+2(1/36)} = 3\sqrt{18/19}.$$

Therefore we have $\lambda = 3\sqrt{18/19} \approx 2.92$.

Next consider the case $1/2 \leq \mu_B^1 \leq 1$. Then since $h \equiv \sigma_B^I = (1-\mu_B^I)/3$,

$$\mu_B^1 / h\sqrt{J} = 3\mu_B^1/(1-\mu_B^1)\sqrt{1+2\alpha[(1-\mu_B^1)/3]^2} \geq 3/\sqrt{1+2(1/36)} = 3\sqrt{18/19}.$$

Here since the inequality of the left side is non-decreasing of μ_B^1, when $\mu_B^1 = 1/2$, it is minimized. In any case for any (α_B, x_B^0, x_C^0) satisfying $\mu_B^1 = \alpha x_B^0 + (1-\alpha)x_C^0 = 1/2$ the inequality holds. The same proof holds for $c_C = c(\mathrm{M}_C, \Upsilon_C^2)$. □

This lemma will justify the normal approximation to prior and posterior distributions and the expected values of the utility functions can be considered even under the normal distributions, though we stick to our original formulation under the truncated normal distributions. It is noted that $c_i = c(\mathrm{M}_i, \Upsilon_i^2)$ >0.98 $(i = B, C)$ in Lemma 2.1 is dependent on self-loyalty α_i. But it is almost equal to 1 uniformly in α_i, implying that it is insensitive to changes of α_i. Confirming this fact, we assume that C and B respectively maximize $\bar{U}_C / c_c$ and $\bar{U}_B / c_B$ rather than $\bar{U}_C$ and $\bar{U}_B$.

To find their Nash assessments, let

$$\Delta_C = 2\alpha_C(1-\alpha_C)\left(\sigma_C^1\right)^2 + (1-\alpha_C), \quad \Delta_B = 2\alpha_B(1-\alpha_B)\left(\sigma_B^1\right)^2 + (1-\alpha_B)$$

and $G = \Delta_B\alpha_C + \Delta_C\alpha_B + \alpha_B\alpha_C$, (23)

and define

$$\varsigma(C:B) = [\Delta_B\alpha_C + \alpha_B\alpha_C]/G \quad \text{and} \quad \varsigma(B:C) = [\Delta_C\alpha_B + \alpha_B\alpha_C]/G \tag{24}$$

which are explicitly expressed respectively as

$$\varsigma(C:B) = \frac{\left(2\alpha_B(1-\alpha_B)\left(\sigma_B^1\right)^2+1\right)\alpha_C}{\left(2\alpha_B(1-\alpha_B)\left(\sigma_B^1\right)^2+1\right)\alpha_C + \left(2\alpha_C(1-\alpha_C)\left(\sigma_C^1\right)^2+(1-\alpha_C)\right)\alpha_B},$$

$$\varsigma(B:C) = \frac{\left(2\alpha_C(1-\alpha_C)\left(\sigma_C^1\right)^2+1\right)\alpha_B}{\left(2\alpha_C(1-\alpha_C)\left(\sigma_C^1\right)^2+1\right)\alpha_B + \left(2\alpha_B(1-\alpha_B)\left(\sigma_B^1\right)^2+(1-\alpha_B)\right)\alpha_C}. \tag{25}$$

Then the following theorem holds.

Theorem 2.1 The Nash assessments of C and B are respectively and uniquely given by

$$x_C^n = \varsigma(C:B)\mu_C^1 + (1-\varsigma(C:B))\mu_B^1$$
$$x_B^n = \varsigma(B:C)\mu_B^1 + (1-\varsigma(B:C))\mu_C^1 \tag{26}$$

Proof. For given the other party's variable C and B maximize the expected normalized utility functions $\bar{U}_C / c_C$, $\bar{U}_B / c_B$ by differentiating them with respect to x_C^* and x_B^*. Then we get

$$x_C^* = \frac{2\alpha_C(1-\alpha_C)\left(\sigma_C^1\right)^2 x_B^* + (1-\alpha_C)x_B^* + \alpha_C\mu_C^1}{2\alpha_C(1-\alpha_C)\left(\sigma_C^1\right)^2+1} \quad \text{and}$$

$$x_B^* = \frac{2\alpha_B(1-\alpha_B)\left(\sigma_B^1\right)^2 x_C^* + (1-\alpha_B)x_C^* + \alpha_B\mu_B^1}{2\alpha_B(1-\alpha_B)\left(\sigma_B^1\right)^2+1}.$$

By using equation (25) , solving for x_C^* and x_B^* gives the Nash assessments (x_C^n, x_B^n).□

By Theorem 2.1, using $\mu_B^1 = \alpha_B x_B^0 + (1-\alpha_B) x_C^0$ and $\mu_C^1 = \alpha_C x_C^0 + (1-\alpha_C) x_B^0$, the following corollary is easily obtained.

Corollary 2.1

(1) The difference of the Nash statements is given by

$$x_C^n - x_B^n = [\alpha_B \alpha_C / G](\mu_C^1 - \mu_B^1) = [\alpha_B \alpha_C / G]\Big((\alpha_B + \alpha_C - 1)\big(x_C^0 - x_B^0\big)\Big) \qquad (27)$$

(2) No matter what the initial mean views (x_C^0, x_B^0) may be, the Nash assessments are identically equal, i.e., $x_C^n \equiv x_B^n$ if their self-loyal character is completely complementary; $\alpha_B + \alpha_C = 1$.

(3) If x_C^0 and x_B^0 happen to be equal, $x_C^n = x_B^n$ follows for any character combination.

Proof. It is obvious from

$$\begin{aligned} x_C^n - x_B^n &= [\varsigma(C:B) - (1-\varsigma(B:C))]\mu_C^1 + [(1-\varsigma(C:B)) - \varsigma(B:C)]\mu_B^1 \\ &= [\alpha_B \alpha_C / G](\mu_C^1 - \mu_B^1) = [\alpha_B \alpha_C / G]\Big((\alpha_B + \alpha_C - 1)\big(x_C^0 - x_B^0\big)\Big) \end{aligned} \quad .□$$

In the case of $x_C^0 = x_B^0$, then $\mu_C^1 = \mu_B^1$, $\sigma_B^0 = \sigma_C^0$, $\sigma_B^1 = \sigma_C^1$ follow and so $x_C^n = x_B^n$ holds. Hence in this case C and B get along for any action that B takes. In other words, in this case, for the PVCIT with $x_C^0 = x_B^0$ however different their character may be, they agree to the decision on any action for implementation. Also, in the case of $\alpha_B + \alpha_C = 1$, B and C always get along in the PVCIT and any action for implementation taken by B will please C.

In Figure 1, we plot the Nash equilibriums of (x_C^n, x_B^n) for some combinations of α_C and α_B in {0.1,0.2,…,0.9}. In case of $\alpha_B + \alpha_C = 1$, when (x_C^0, x_B^0) moves over [0.5,1]×[0,1], the graphs with (α_C, α_B) =(0.9, 0.1), (0.5, 0.5) and (0.1, 0.9) are diagonal lines in the axes of (x_B^n, x_C^n) as they should be.

When $\alpha_B + \alpha_C \neq 1$ and $x_C^0 \neq x_B^0$ hold, the graphs of (x_B^n, x_C^n) become fatter as (α_C, α_B) deviates from $\alpha_B + \alpha_C = 1$ and as (x_C^0, x_B^0) deviates from $x_C^0 = x_B^0$. The sign of the difference in equation (27) depends on the sign of $\mu_C^1 - \mu_B^1$ as $G > 0$. In particular, when $\alpha_B \to 0.9$ and $\alpha_C \to 0.9$, the graphs expand to upper and below directions from the line $x_C^n = x_B^n$.

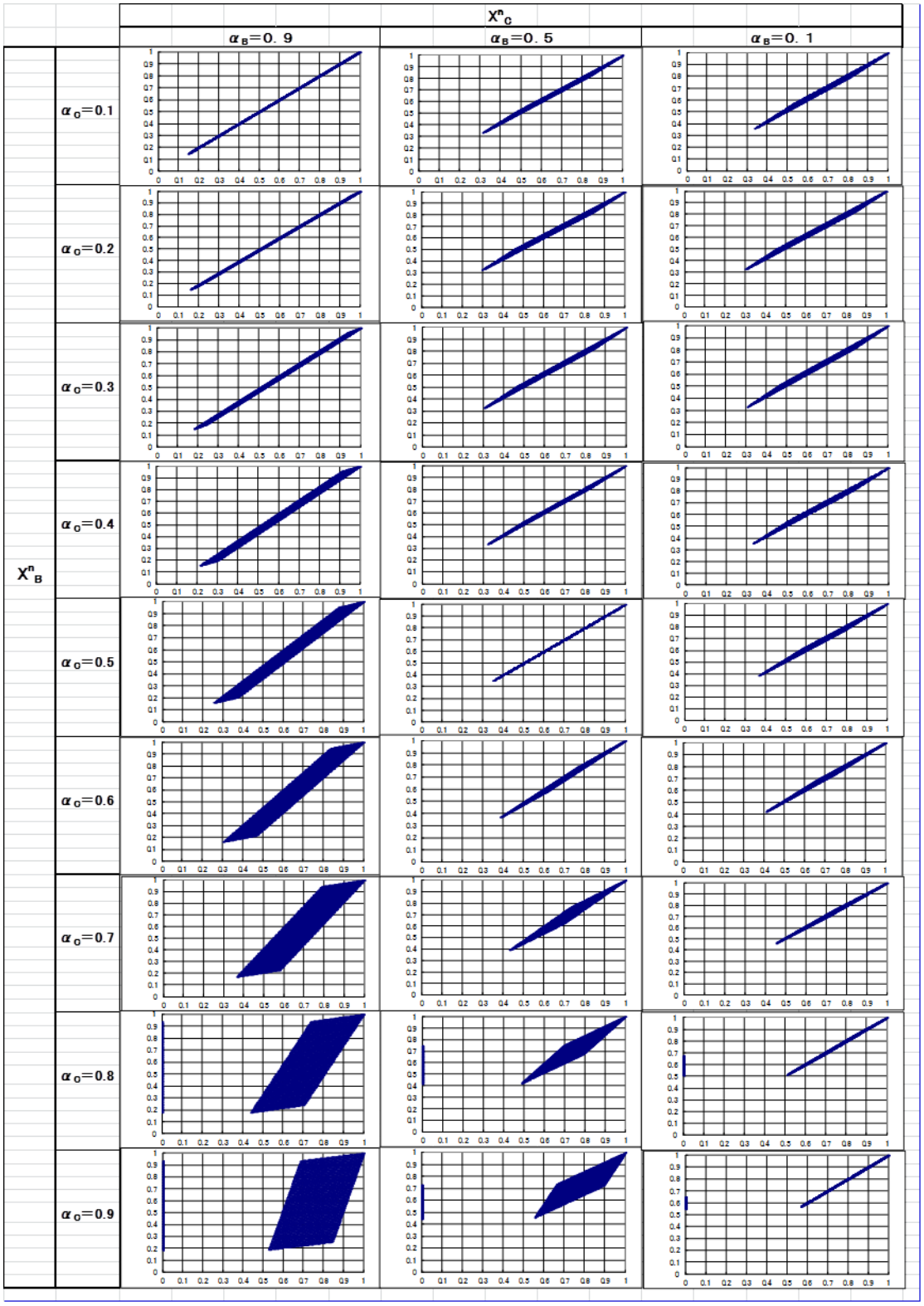

Figure 1. The graphs of the Nash equilibriums $\{(x_C^n, x_B^n) : (x_C^0, x_B^0) \in [0.5,1] \times [0,1]\}$ in each pair of (α_C, α_B), where the horizontal axis is x_C^n, and the vertical axis is x_B^n.

2.3. Sub-process (3): Decision for Action or no Action

When a Nash assessment (x_B^n, x_C^n) is stated by B and C, boss B has to make decision on whether an implementation should be carried out or not. We here formulate B's decision function via another utility function U_{B2}:

$$U_{B2} = U_{B2}\left(y_{1B}, y_{2B(C)}\right) : \alpha_B \quad \text{on} \quad [0,1]\times[0,1]\times[0,1] \tag{28}$$

where U_{B2} is continuous, positive and differentiable in its interior points,

$$y_{1B} = \left|\tilde{x}_B^1 - x_B^a\right|, \; y_{2B(C)} = \left|x_C^n - x_B^a\right|, \tag{29}$$
$$\partial U_{B2}(y_1, y_2)/\partial y_1 < 0, \text{ and } \partial U_{B2}(y_1, y_2)/\partial y_2 < 0.$$

Here x_B^a in $y_{1B} = \left|\tilde{x}_B^1 - x_B^a\right|$ is an action x_B^a for B to choose, where 0 and 1 respectively represent no action and an action for implementation. Hence y_{1B} measures the gap between B's posterior view $\tilde{x}_B^1$ with F_B^1 and his action x_B^a, and U_{B2} is decreasing in y_{1B}. On the other hand, $y_{2B(C)}$ measures the gap between C's Nash statement x_C^n and B's action x_B^a and U_{B2} is decreasing in $y_{2B(C)}$. Here the closer the self-loyalty parameter α_B is to 1, the greater weight will be given to $y_{1B} = \left|\tilde{x}_B^1 - x_B^a\right|$, and the less B tends to consider about C' s assessment. While, the closer it is to 0, the more weight will be given to $y_{2B(C)} = \left|x_C^n - x_B^a\right|$, and the more his action tends to be under C's influence. And B's decision function for taking an action is based on the expected utility

$$\begin{aligned} h\left(x_C^0, x_B^0 : x_B^a\right) &\equiv E\left(U_{B2}\right) = \int_0^1 U_{B2}\left(y_{1B}(\tilde{x}_B^1), y_{2B(C)} : \alpha_B\right) dF_B^1\left(\tilde{x}_B^1\right) \\ &\equiv \bar{u}_{B2}\left(x_B^a, x_C^n : \left(x_C^0, x_B^0\right), \alpha_B\right), \end{aligned} \tag{30}$$

where F_B^1 is B's posterior distribution and $x_C^n = x_C^n(x_C^0, x_B^0; \alpha_C, \alpha_B)$ as in equation (15). For given (x_C^0, x_B^0), the expected utility in equation (30) is the character-dependent decision-making function of B and B's action is affected by the degree of his self-royalty.

As B's decision rule, we assume that B chooses action $x_B{}^a = 1$ if $h(x_C^0, x_B^0 : 1)$ is greater than $h(x_C^0, x_B^0 : 0)$ and choose no action $x_B{}^a = 0$ otherwise;

$$x_B{}^a = \begin{cases} 1 \;\; if \;\; h(x_C^0, x_B^0 : 1) > h(x_C^0, x_B^0 : 0), \\ 0 \;\; if \;\; h(x_C^0, x_B^0 : 1) \le h(x_C^0, x_B^0 : 0). \end{cases} \tag{31}$$

This decision rule is a function of (x_C^0, x_B^0) for taking $x_B{}^a = 1$ or $x_B{}^a = 0$ and it is rewritten as

$$x_B{}^a = 1 \;\; if \;\; (x_C^0, x_B^0) \in R \;\; and \;\; x_B{}^a = 0 \;\; if \;\; (x_C^0, x_B^0) \notin R$$
$$\text{with} \;\; R \equiv \{(x_C^0, x_B^0) : h(x_C^0, x_B^0; 1) / h(x_C^0, x_B^0; 0) > 1\}. \tag{32}$$

In Appendix it is shown that when the PVCIT is repeated by taking (x_C^0, x_B^0) repeatedly, an optimality of this decision rule in equation (32) holds through the generalized Neyman-Pearson Lemma in statistical hypothesis testing theory.

2.3.1 Decision for Action or no Action

Here again, we use the following Cobb-Douglas type utility which satisfies equations (28) and (29):

$$U_{B2} = \exp\left(-\alpha_B\left(\tilde{x}_B^1 - x_B^a\right)^2 - \left(1-\alpha_B\right)\left(x_C^n - x_B^a\right)^2\right) \tag{33}$$

where $0 \le \alpha_B \le 1$. This utility is completely similar to the one in equation (12) with x_B^* and x_C^* replaced by x_B^a and x_C^n respectively and hence by Proposition 2.1 the integral becomes

$$\bar{u}_{B2}(x_B^a, x_C^n; \alpha_B) = c_B D_B \exp\left[-\left\{ \frac{\alpha_B (x_B^a - \mu_B^1)^2}{2\alpha_B \left(\sigma_B^1\right)^2 + 1} + (1-\alpha_B)(x_B^a - x_C^n)^2 \right\} \right] \tag{34}$$

Note that though c_B depends on x_B^a, c_B >0.98 for any x_B^a. Also as in equation (31), for given (x_C^0 , x_B^0), B is assumed to choose action $x_B{}^a = 1$ if $\bar{u}_{B2}(1, x_C^n; \alpha_B)$ is greater than $\bar{u}_{B2}(0, x_C^n; \alpha_B)$ and to choose no action $x_B{}^a = 0$ otherwise

$$x_B{}^a = \begin{cases} 1 & if\ \bar{u}_{B2}(1, x_C^n; \alpha_B) > \bar{u}_{B2}(0, x_C^n; \alpha_B), \\ 0 & if\ \bar{u}_{B2}(1, x_C^n; \alpha_B) \le \bar{u}_{B2}(0, x_C^n; \alpha_B). \end{cases} \tag{35}$$

Theorem 2.2 The necessary and sufficient condition for $x_B{}^a = 1$ in equation (35) is that

$$\beta_B \left[\alpha_B \mu_B^1 + (1-\alpha_B) x_C^n \right] + (1-\beta_B)(1-\alpha_B) x_C^n > 0.5\beta_B - 0.5k \tag{36}$$

where $k = \log[c_B(1)/c_B(0)] \approx 0$ and $\beta_B = 1/\left(2\alpha_B \left(\sigma_B^1\right)^2 + 1\right)$.

Proof. Taking the ratio $\bar{u}_{B2}(1, x_C^n; \alpha_B) / \bar{u}_{B2}(0, x_C^n; \alpha_B)$ yields

$$\frac{c_B(1)}{c_B(0)} \exp\left[-\left\{ \frac{\alpha_B[(1-\mu_B^1)^2 - (\mu_B^1)^2]}{2\alpha_B \left(\sigma_B^1\right)^2 + 1} + (1-\alpha_B)[(1-x_C^n)^2 - (x_C^n)^2] \right\} \right] > 1,$$

from which $\alpha_B[1 - 2\mu_B^1] + (1-\alpha_B)[1 - 2x_C^n](2\alpha_B \left(\sigma_B^1\right)^2 + 1) < k(2\alpha_B \left(\sigma_B^1\right)^2 + 1)$.
Rearranging the terms gives the result.□

The left side of equation (36) for taking an action is regarded as the two step averages:

1) $v_B \equiv \alpha_B \mu_B^1 + (1-\alpha_B) x_C^n$; B's character-weighted average of B's posterior mean view and C's Nash assessment, which takes C's Nash assessment into account, and
2) $\beta_B v_B + (1-\beta_B) x_C^n$; second average of v_B and C's Nash assessment, where β_B will be greater than 0.97. When α_B = 0.5 and σ_B^1 = 0.5/3, β_B = 0.97. Hence practically this second average will not much affect B's decision.

Corollary 2.2 If $\min(\mu_B^1, x_C^n) > 0.5$, equation (36) is satisfied and hence action $x_B{}^a = 1$ is taken.

Proof. By its form in equation (36), it suffices to show the case $\mu_B^1 = x_C^n > 0.5$, which is clear. □

As has been discussed in Sections 1 and 2, this decision will affect the level of C's satisfaction in each one time PVCIT and so the relationship between B and C in the long run. The level of C's satisfaction in each PVCIT depends on (x_C^0 , x_B^0) and (α_C, α_B) but when the PVCIT is repeated or equivalently when (x_C^0 , x_B^0) is repeated, the average level of his satisfaction depends on the combination (α_C, α_B) of their character.

3. MPACC and Optimal Combinations of Self-loyal Character under Repetitions

In the MPACC where the above PVCIT is repeated routinely, the mean effectiveness of the PVCIT will be assessed by management *A*. We make a simulation analysis on it from a viewpoint of the effectiveness of organizational hierarchy and find optimal combinations of their character. It is

assumed that the empowerment hierarchy gives a decision-making power to *B*, and so by using it, *B* is required to contribute to value creation as a team. Therefore he will need to maintain an effective PVCIT by holding a harmonious cooperative relationship with his subordinates. The management *A* will check the effectiveness of PVCIT through *B*'s reports or interviews and control it by finding optimal combinations (α_C, α_B) of self-loyalty character between *B* and *C* for increasing firm-value creation.

In the PVCIT repetitions, taking into account how C randomly meets new bf-information in business-front, it is likely that the mean views { x_C^0 } on the success of valuable implementation, which C forms repeatedly, are approximately normal, while for simplicity we assume that x_B^0 is uniform distribution on [0, 1]; $x_B^0 \sim U(0,1)$, though there will be some other alternatives including beta distribution.

Here the management A is assumed to have the following utility function to judge the compatibility of character combinations for checking the effectiveness of the PVCIT.

$$U_A = U_A(z_B, z_C : \alpha_A) \quad \text{on } [0,1]\times[0,1]\times[0,1] \tag{37}$$

where U_A is continuous, positive and differentiable in its interior points

$$z_B = \left|x_B^n - x_B^a\right|, \quad z_C = \left|x_C^n - x_B^a\right| \tag{38}$$

$$\partial U_A(z_1, z_2)/\partial z_1 < 0 \quad \text{and} \quad \partial U_A(z_1, z_2)/\partial z_2 < 0$$

Here U_A is a function of variables z_B and z_C, which respectively measure the gap between B's action and his own Nash statement and the gap between B's action and C's Nash statement. It is noted that U_A itself attains the maximum value when the both variables are 0, which is the almost impossible case $x_B^n = x_C^n = x_B^a$ as $x_B{}^a = 1$ or 0 since most likely $0 < x_B^n, x_C^n < 1$. But if $x_B^n = x_C^n$ and both x_B^n and x_C^n are close to 0 or 1, then both B and C will tend to be satisfied and C will be cooperative and increase their ability to carry out the implementation plan when $x_B^n = x_C^n$ is close to 1. In the utility equation (37) it is assumed that the larger the parameter α_A is, the greater the weight for z_B is. In fact, α_A represents A's relative weight to z_B over z_C and it is a control parameter for A to choose. If α_A =1, A assesses only the consistency of B's action x_B^a with B's Nash statement x_B^n.

Note that U_A can be evaluated deterministically when (x_B^n, x_C^n, x_B^a) is given. But since the above whole process is routinely repeated on daily basis and since as in equation (15), each of x_B^n and x_C^n is a function of (x_C^0, x_B^0) as well as (α_C, α_B) and so are z_B and z_C, the management assesses the effectiveness of the PVCIT based on the expected value of U_A with respect to the joint distribution $G_C(\cdot)\, G_B(\cdot)$ of (x_C^0, x_B^0):

$$\bar{U}_A = E(U_A) = E[U_A(z_B(x_C^0, x_B^0), z_C(x_C^0, x_B^0) : \alpha_A)] = \bar{u}_A^*(\alpha_A : \alpha_B, \alpha_C) \tag{39}$$

Here in repetitions the distribution function $G_C(\cdot)$ of C's mean view x_C^0 is assumed to be truncated normal distribution; $x_C^0 \sim N^t(0.5, (0.5/3)^2)$, while for simplicity the distribution function $G_B(\cdot)$ of his mean view x_B^0 it is assumed to be uniform distribution on [0, 1]; $x_B^0 \sim U(0,1)$.

Since x_B^n and x_C^n are nonlinear functions of (x_C^0, x_B^0), we maximize $\bar{U}_A$ with respect to α_A via simulation by generating (x_C^0, x_B^0) from $G_C(\cdot)\, G_B(\cdot)$. The number of sample is 4,000. It

may be interpreted as follows. In his business front, C is supposed to meet or find 3 to 5 pieces of information for possible business implementation in a day and works for 250 days in a year. Then, in the time horizon of 3 to 4 years he will meet about 4,000 pieces of bf-information.

It should be noted that the expected utility $\bar{U}_A$ is a function of (α_C, α_B) and α_A. The management A is assumed to maximize this expected utility with respect to α_A and to assess the effectiveness of the PVCIT and find optimal combinations of self-loyalty character (α_C, α_B) that should be consistent with management policy and empowerment hierarchy.

Here *A* is assumed to have the Cobb-Douglas type utility;

$$U_A = \exp[-\alpha_A (x_B^n - x_B^a)^2 - (1-\alpha_A)(x_C^n - x_B^a)^2], \tag{40}$$

which satisfies equations (37) and (38). Here U_A is a function of variables $z_B = \left| x_B^n - x_B^a \right|$ and $z_{B(C)} = \left| x_C^n - x_B^a \right|$, which respectively measure the gap between B's action and his own Nash assessment and the gap between B's action and C's Nash assessment. In equation (40), the larger α_A is, the greater the weight for z_B is, where α_A represents A's relative weight to z_B over $z_{B(C)}$ and it is a control parameter for A to choose. If α_A =1, A assesses only the consistency of B's action x_B^a with B's Nash value x_B^n. Note that U_A can be evaluated deterministically when (x_B^n, x_C^n, x_B^a) is given for each PVCIT.

If B and C are completely complementary in character, i.e., $\alpha_B + \alpha_C = 1$, then $x_B^n = x_C^n$ always holds whatever (x_C^0, x_B^0) may be. In this case the utility is independent of α_A;

$$U_A = \exp[-(x_B^n - x_B^a)^2] = \exp[-(x_C^n - x_B^a)^2].$$

Management A concerns the consistency of B's decision with his Nash assessment, and if α_A =1 is chosen as a result of the maximization of the expected utility, the PVCIT under repetitions will be accepted, which will be the case with A's satisfaction relative to the consistency of the empowerment hierarchy of the organization in his view. On the other hand, when the case with α_A =0 holds as a result of the maximization, the PVCIT with α_A =0 will not be acceptable for A, and B might be eventually demoted or replaced.

As a screening or threshold level for reporting in equation (4), we assume

$$d = 0.5 + 0.13\alpha_C \tag{41}$$

Here 0.5 corresponds to α_C =0, meaning that even a subordinate of the weakest self-loyal character does not report unless an initial mean view x_C^0 is greater than or equal to 0.5. On the other hand, the slope 0.13 corresponds to the fact that the probability that x_C^0 exceeds $d = 0.5 + 0.13 \times 1 = 0.63$ under the truncated normal distribution in equation (2) is approximately 0.25. In other words, even a subordinate of the strongest self-loyal character reports 25% of bf-information he meets. In the simulation we consider the combinations of self-loyal character:

$$\{(\alpha_C, \alpha_B); \alpha_C, \alpha_B = 0.1, 0.2, 0.3, \ldots, 0.9\},$$

and for each (α_C, α_B) A numerically maximizes $\bar{U}_A$ with respect to α_A. Table 1 gives the results. The top row and left column respectively represent $\alpha_B = 0.1, 0.2, \cdots, 0.9$ and $\alpha_C = 0.1, 0.2, \cdots, 0.9$. Further, in each cell of (α_C, α_B), the upper number is the maximizer α_A that maximizes $\bar{U}_A$ and the lower number represents the maximized value of $\bar{U}_A$. In the

table, the values of $\bar{U}_A$ are italicized when they are more than 0.9.

Table 1. The maximized utility $\bar{U}_A$ with respect to α_A for each (α_C, α_B)

	α_B 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
α_C 0.1	0 0.875	0 0.868	0 0.869	0 0.874	0 0.880	0 0.888	0 0.896	***0*** ***0.905***	***1*** ***0.913***
0.2	0 0.883	0 0.876	0 0.875	0 0.878	0 0.883	0 0.889	0 0.896	***1*** ***0.904***	***1*** ***0.913***
0.3	0 0.881	0 0.876	0 0.876	0 0.878	0 0.882	0 0.887	1 0.893	***1*** ***0.902***	***1*** ***0.912***
0.4	0 0.877	0 0.874	0 0.874	0 0.876	0 0.879	1 0.883	1 0.892	***1*** ***0.901***	***1*** ***0.911***
0.5	0 0.872	0 0.871	0 0.871	0 0.872	1 0.875	1 0.881	1 0.889	1 0.899	***1*** ***0.910***
0.6	1 0.870	1 0.870	0 0.870	1 0.870	1 0.873	1 0.879	1 0.887	1 0.898	***1*** ***0.909***
0.7	1 0.874	1 0.874	1 0.873	0 0.872	1 0.872	1 0.876	1 0.884	1 0.895	***1*** ***0.908***
0.8	1 0.888	1 0.886	0 0.885	0 0.881	0 0.875	1 0.874	1 0.881	1 0.892	***1*** ***0.905***
0.9	***1*** ***0.903***	***0*** ***0.903***	***0*** ***0.903***	***0*** ***0.900***	0 0.889	1 0.875	1 0.879	1 0.889	***1*** ***0.903***

In Figure 2, the 3-D graph of $(\alpha_C, \alpha_B, \alpha_A)$ in Table 2 gives the overall structure of the mean utility of the management A.

From this table, it follows that under the assumption of the threshold in equation (41):

1) All the maximizers of α_A turn out to be 0 or 1. This implies that for assessing the effectiveness of the PVCIT, management A does not choose α_A in 0< α_A < 1 to make a balance between the two person's gaps, but takes a decisive evaluation for the combinations of (α_C, α_B). If α_A =0, A will not think that the PVCIT is effective as it stands, relative to their empowerment hierarchy, even though A's expected utility is high.

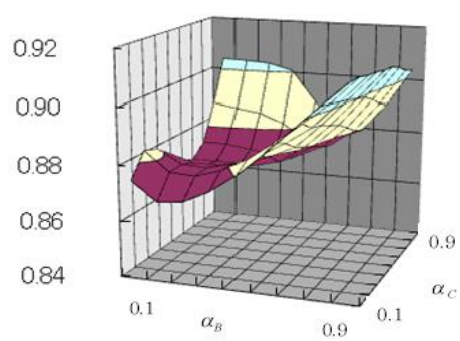

Fig. 2 The 3-D graph of $\bar{U}_A$ in $(\alpha_C, \alpha_B, \alpha_A)$

2) Consider the case that their self-loyal character is completely complementary with $\alpha_B + \alpha_C = 1$. This case corresponds to the combinations in the northeast-to-southwest (shortened as ne-sw) diagonal cells in the Table. Then naturally all the α_A's are 1 and the maximized $\bar{U}_A$ attains the largest value ***0.913*** at α_B =0.9, from which it decreases to the minimum 0.870 along the ne-sw diagonal cells as α_B decreases to 0.4, and then increases to ***0.903*** as α_B further decreases to 0.1. In other words, even though both C and B are satisfied with their character

relations for any case of (α_C, α_B) with $\alpha_B + \alpha_C = 1$, in the view of management the combination of $(\alpha_C, \alpha_B) = (0.1, 0.9)$ is the best as in the Table. Though the second best combination in the diagonal cells is $(\alpha_C, \alpha_B) = (0.2, 0.8)$ with $\bar{U}_A$=***0.904***, the third one is interestingly $(\alpha_C, \alpha_B) = (0.9, 0.1)$ with $\bar{U}_A$=***0.903*** and α_A=1, which is the only one case where A puts α_A=1 with satisfaction as much as $\bar{U}_A$>0.9 in the combinations of $\alpha_C > \alpha_B$. But in this case A might demote B as an incapable boss.

3) In the region R_1=$\{(\alpha_C, \alpha_B) : \alpha_C + \alpha_B < 1,\ \alpha_C \le 0.5\}$ where both B and C are of weaker self-loyal character, α_A's are all zero so long as $\alpha_C \le 0.5$. Hence the combinations in the region R_1 will not be desirable for management A empowering B.

4) In R_2={ $(\alpha_C, \alpha_B) : \alpha_C + \alpha_B < 1$ and $\alpha_C \ge 0.6$ }, α_A=1 holds except for (α_C, α_B) = (0.6,0.3). However, this region is the cases where B's weaker self-loyalty is complemented by C's stronger self-loyalty and these cases are least favorable and should be replaced by (α_C, α_B)=(0.9,0.1) in 2).

5) However, in R_3={ $(\alpha_C, \alpha_B) : \alpha_C + \alpha_B > 1$, $\alpha_C \ge 0.6,\ \alpha_B < 0.6$ }, management's decisions are mixed. Except for the case $(\alpha_C, \alpha_B) = (0.9, 0.1)$ with $\bar{U}_A$=***0.903***, A rejects the cases with $\alpha_C = 0.9$ even though those cases satisfy $\bar{U}_A$>0.888. The other cases in R_3 are either α_A=0 or $\bar{U}_A$<0.873 with α_A=1, which management will not be satisfied.

6) In R_4=$\{(\alpha_C, \alpha_B) : \alpha_C + \alpha_B > 1, \alpha_B \ge 0.6\}$, A puts α_A=1 for all the cells in R_4, implying that the combinations in this region will be acceptable, though not all the cells in R_4 are of $\bar{U}_A$>0.9 and the cases with larger $\alpha_B \ge 0.6$ are preferred. Among others, the combinations with $\alpha_B = 0.9$ are mostly preferred by the management. The values of $\bar{U}_A$'s in the region $\{(\alpha_C, \alpha_B) : \alpha_B = 0.9\}$ are decreasing as α_C increases and so long as $\alpha_C \le 0.8$, the satisfaction levels of $\bar{U}_A$'s are greater than the level of the special case (α_C, α_B)=(0.9, 0.1). Though the best case is the case (α_C, α_B)=(0.1, 0.9), the other cases including the case of α_C=0.9 are also preferred when $\alpha_B = 0.9$. When $\alpha_B = 0.8$, the cases α_C=0.2, 0.3 and 0.4 are secondly preferred though the values $\bar{U}_A$'s are smaller than the cases with $\alpha_B = 0.9$ and $\alpha_C \le 0.8$.

As a conclusion, A positively assesses the effectiveness of the PVCIT when B is of the strongest self-loyal character no matter how self-loyal C may be. An important implication of this conclusion will be that so long as the boss B is strongly self-loyal, he will be able to have an effective PVCIT for any subordinate. It will be desired for B to have at least as large as 0.7 in the self-loyalty character. The worse cases are the combinations in the region $\{(\alpha_C, \alpha_B) : \alpha_C \le 0.5,\ \alpha_B \le 0.4\}$ in which the both C and B have weak self-loyal character.

Each PVCIT per bf-information needs some time in its completion and so too much information is costly unless the volume of bf-information is controlled or screened. This cost problem should be treated elsewhere.

4. Conclusion with Some Remarks

In this paper, in view of superior-subordinate relationship, we formulated a general mathematical model to analyze about the effect of combinations of self-loyalty character of direct superior B and subordinate C on the process of their transforming bf-information into a firm value by implementing it. Here the self-loyalty is the character that makes a person tend to stick to his own value base so long as issue is an important matter of his pride on intellectuality, capability and experiences that is related to his business individuality and identity. Because of this nature of the character, the PVCIT inevitably includes a character-dependent game between B and C about their views or assessments on the information.

For simplicity and analytical tractability we specify character-dependent utility functions, on which the game develops, by Cobb-Douglas type function, a complete description of the PVCIT for given mean views was made with a framework of Nash assessments on which they agree and with B's making an optimal decision on an action or no action.

We also formulated the MPACC that A evaluates the effectiveness of the average PVCIT under repetitions and finds optimal combinations of their self-loyal character. In other words, the effectiveness of the PVCIT itself is an important management issue for value creation.

In our Cobb-Douglas utility, when B is very strongly self-loyal, some combinations of the character were found to be effective, no matter how self-loyal C may be. However, if C is also strongly self-loyal, C may not cooperate effectively in the PVCIT with moral hazard when their Nash assessments are largely different, although A accepts the PVCIT.

There remain some important problems even if our framework for analysis is accepted. First of all, we selected the self-loyalty among others as human character that affects the effectiveness of the PVCIT. A fact of the matter will be a complexity of the problem and it is necessary to consider the complexity of the human character in various viewpoints. Second, looking into the internal structure, there will be other specifications of the utility functions in the PVCIT and the MPACC, which may lead to different results. We used the linear character-dependent information-screening function in equation (41), which may be replaced by some other nonlinear specifications. In addition, the truncated normal distributions and uniform distributions that we respectively assumed for C's and B's mean views on the success of implementing bf-information may be differently specified as, for example, beta distribution and there some dependency of the mean views may be introduced. Further the specifications in the formation of the posterior views will be changed.

Appendix: Optimality of decision rule in equation (32) under Repetitions of the PVCIT

When the above PVCIT with sub-processes (1), (2) and (3) is repeated for *N* times, C forms *N* prior subjective distributions based on the mean values $\{x_C^0(i): i=1,\cdots,N\}$, though not all the information is reported to his boss. In reality *N* is random though we fix it for convenience. For this repeated business practices, we will assume that the set of $x_C^0(i)'s$ is a random sample from a specific distribution function G_C, since C randomly encounters bf-information;

$$x_C^0 \sim G_C(z) = \int_0^z g_C(v)\,dv \text{ with pdf } g_C(v) \tag{42}$$

Correspondingly, B forms prior distributions based on the mean values $\{x_B^0(i_k): i_k=1,\cdots,N_B\}$, where N_B is the number of pieces of information that B receives from C. Similarly $x_B^0(i_k)$'s are assumed to be random samples from (42) with C replaced by B. We have assumed the independence of x_C^0 and x_B^0.

Since each of the Nash statements x_B^n and x_C^n is a function of (x_C^0, x_B^0) as well as (α_C, α_B), the expected utility in (34) is a function of random variables (x_C^0, x_B^0) in view of repeated samples from

the pdf $g_C(\cdot)\, g_B(\cdot)$. Using the expected utility in (34), we can define the joint density of (x_C^0 , x_B^0);

$$h_{pdf}(x_C^0, x_B^0; x_B^a) = c(x_B^a) h(x_C^0, x_B^0; x_B^a) g_C(x_C^0) g_B(x_B^0) \tag{43}$$

where $c(x_B^a)$ is the normalization constant so that the integral over [0,1]×[0,1] becomes 1. To show an optimality of the decision rule in (35), let $\phi(x_C^0, x_B^0)$ denote the probability of taking action $x_B{}^a = 1$ for given (x_C^0, x_B^0), where $0 \le \phi(x_C^0, x_B^0) \le 1$, and $1 - \phi(x_C^0, x_B^0)$ is the probability of taking $x_B{}^a = 0$ for given (x_C^0, x_B^0). Our problem is to maximize the expected probability $E[\phi(x_C^0, x_B^0) \mid 1]$ under action $x_B{}^a = 1$ with respect to decision rule ϕ with

$$E[\phi(x_C^0, x_B^0) \mid x_B^a] = \int_0^1 \int_0^1 \phi(x_C^0, x_B^0) h_{pdf}(x_C^0, x_B^0; x_B^a) dx_C^0 dx_B^0 \tag{44}$$

provided $E[\phi(x_C^0, x_B^0) \mid 0] = c$ for each given *c* in [0,1]. Then by the Neyman-Pearson Lemma (see, e.g., Ferguson (1967, page 291)), $\phi(x_C^0, x_B^0)$ that maximizes $E[\phi(x_C^0, x_B^0) \mid 1]$ is a decision of the form;

$$\phi^*(x_C^0, x_B^0) = \begin{cases} 1 \ if \ (x_C^0, x_B^0) \in R, \\ 0 \ otherwise, \end{cases} \tag{45}$$

where $R \equiv \{(x_C^0, x_B^0) : h(x_C^0, x_B^0; 1) / h(x_C^0, x_B^0; 0) > k'\}$ with $k' = kc(0) / c(1)$.

Then when $(x_C^0, x_B^0) \in R$, action $x_B{}^a = 1$ is taken, and otherwise $x_B{}^a = 0$ is taken. This is the decision rule with k'=1 in equation (32), implying its optimality in the framework of repetitions.

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