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Confidence Intervals and Hypothesis Tests for a Population Mean Using Ranked Set Sampling: An Auditing Application

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Abstract: Balanced ranked set sampling (RSS) provides a systematic way to use additional information readily available in a population to enable a researcher to select sample observations for measurement that yield a more complete picture of the entire population (and thereby greater precision in associated statistical analyses) than is possible by simple random sampling (SRS). This approach is particularly valuable when the readily available information for instituting RSS to select the sample units is easy to access but the actual measurement of the quantity of interest on the selected sample units is time consuming (and, hence, costly) or difficult to obtain. This can certainly be the case in many auditing applications, including inventory audits, receivable confirmations, etc. In this study we describe how to use balanced RSS to construct confidence intervals and test associated hypotheses about a population mean in an inventory valuation setting. Computer simulation is used to compare the accuracy and precision of these RSS confidence intervals (and associated hypothesis tests) with the corresponding confidence intervals and hypothesis tests under SRS. Mathematical formulas are also provided for the calculation of RSS confidence intervals for auditors working in the field. Results demonstrate that the confidence intervals generated under RSS are up to 35% narrower than those derived from SRS. The narrower confidence intervals imply more powerful hypothesis tests without increasing sample sizes. Accordingly, audit sampling cost is significantly lower when using RSS instead of SRS.

Keywords: Accuracy Interval Estimation; Ranked Set Sampling; Simple Random Sampling; Statistical Auditing; Statistical Sampling.

JEL Classifications: C12, C13, C83, and M42

Abbreviations: MUS = Monetary Unit Sampling; RSS = Ranked Set Sampling; SRS = Simple Random Sampling

1. Introduction

The conversion to digital documentation by modern businesses has changed the way auditors approach attesting to financial statements. The traditional paper trail of business records has been replaced with data entries on computer hard drives and servers. Overall, this is a positive development for auditing because sampling error has been reduced–especially when auditing internal control compliance–since auditors now have the luxury of evaluating population parameters rather than mere statistics. In such situations, computer programs can examine and tabulate the results of every transaction or event, and creating these summaries is nearly cost free when compared to traditional methods. Nevertheless, statistical sampling still plays an important role when auditors evaluate costly or difficult to observe evidence, such as the physical examination of inventories; property, plant, and equipment; accounts receivable; and accounts payable.

Modern audit sampling uses a variety of methods, including simple random sampling, stratified sampling, and monetary unit sampling (AICPA, 2008; Bailey, 1981; and Arens et al., 2012). The current popularity of monetary unit sampling may be due in large part to the AICPA endorsement given in Audit Guide: Audit Sampling (2008) rather than being based on any research demonstrating its superiority over other methods. Other sampling methods, however, still play an important role in current auditing, especially the time-honored classical methods based on simple random sampling (SRS).

There is, however, a theoretical paradox in the application of SRS to auditing financial statement account balances. An alternative sampling technique, known as ranked set sampling (RSS), is considerably more efficient in that it provides increased statistical precision for a given audit sample size. The paradox lies in the fact that modern auditing theory and practice does not address this superior audit sampling methodology (Arens et al., 2012; Messier et al., 2014; and Louwers et al., 2014). Based on computer simulations, Gemayel et al. (2012) showed that balanced RSS is always superior to SRS in terms of obtaining a sample that is more representative of a given accounting population. This more representative sample would enable auditors to reduce sample sizes without sacrificing audit confidence levels. Such a theoretical result is important, yet there are still unanswered questions regarding the implementation of RSS in auditing practice: the procedure for obtaining confidence intervals using RSS in audit settings, and the formula for obtaining an ex-ante sample size using the RSS technique.

This study describes the methodology for constructing confidence intervals and testing associated hypotheses when applying balanced RSS to the audit problem of testing financial statement account balances. Computer simulation is used to compare the accuracy and precision of these confidence intervals to those that would be attained under SRS. A mathematical formula is also provided for the calculation of RSS confidence intervals for auditors working in the field. Results demonstrate that RSS confidence intervals are up to 35% narrower than those derived from SRS. Accordingly, audit sample sizes (and audit costs) can be significantly reduced simply by switching from SRS to the RSS methodology.

2. Ranked Set Sampling

According to classical sampling theory, SRS provides external validity to experiments and studies because the random selection of sample units is the best way to obtain a sample that is truly representative of the population (Fisher, 1925). In the 1950s researchers discovered an alternative to SRS that significantly increased the representativeness of statistical samples. This new technique, RSS, would consistently provide greater levels of precision for a given sample size than SRS whenever there was a convenient concomitant (ranking) variable that could be used to help select the sample (McIntyre, 1952, 2005). The superiority of RSS over SRS is a result of the additional structure imposed on the data collection process by using the concomitant variable in selecting sample units. Patil (2002) points out that as long as the concomitant variable is reasonably correlated with the variable of interest, then samples selected via RSS will provide greater statistical precision than those afforded by SRS.

Early applications of RSS involved the US forestry service and their study of timber valuations on land. Visual estimation of the heights of trees were used as the concomitant variable, and RSS proved clearly superior to SRS in terms of minimizing required sample sizes in these forestry studies. Nevertheless, the value of the RSS methodology is related to the accuracy of the concomitant variable, and visual estimation of tree height could be prone to error based on observational circumstances and the talent of the observing party. This concomitant variable error is, however, almost nonexistent in financial statement audit applications because the concomitant variable (the book value of the account according to client records) is available to the auditor cost-free and is highly correlated with the audited (true) value. In fact, one could argue that auditing applications of RSS produce the most accurate concomitant variable rankings possible in practical statistical sampling settings. The clear superiority of RSS over SRS for statistical auditing has been established through computer simulations which show required audit sample sizes for a given precision level are much smaller when auditors use RSS (Gemayel et al., 2012).

Paradoxically, the auditing profession has never investigated RSS for use in statistical audit sampling. The authors theorize that part of the explanation lies in the previously described problems of inertia in the evolution of audit sampling and the official endorsement of monetary unit sampling (MUS) by the AICPA. Additionally, applying RSS to audit sampling requires the widespread availability of computers and software that were not available at the time when MUS was selected by the profession as the sampling method of choice.

Table 1. RSS Numerical Example, Set Size Equal 3

Nine Simple Random Sampling Values		
680.91	130.38	329.21
178.52	478.43	717.36
794.18	629.45	213.33
Above Values Ranked by Rows		
130.38	329.21	680.91
178.52	**478.43**	717.36
213.33	629.45	**794.18**
Diagonal Values are in RSS Sample		
130.38	**478.43**	**794.18**

The structure for RSS is well described in a number of sources (see, for example, Gemayel et al., 2012; Wolfe, 2012). The RSS process can be intuitively represented with the following simplified numerical example. Suppose an auditor is selecting a sample from the accounts receivable subsidiary ledger for audit confirmation. The first step would be to select a set size for purposes of drawing the sample. Typically, set sizes are between three and ten items for most RSS applications; however, this example will use a set size of three because it provides an easily understandable visualization of the RSS process. Having selected a set size of three, the auditor would draw 3^2 or nine accounts at random from the list of accounts in the accounts receivable subsidiary ledger (if the set size were five, then 5^2 or 25 accounts would have been drawn, and so forth). The nine book values selected would be placed at random in a 3x3 matrix (if the set size were five, then a 5x5 matrix would be used). Next, each row of the matrix is sorted in increasing order; the three accounts whose book values appear on the diagonal are selected to be in the auditor's sample. This example is illustrated in Table 1. The auditor would then flag these three items (130.38, 478.43, and 794.18) for confirmation. The off-diagonal accounts would be discarded.

The above example represents a single RSS cycle for the selection of three accounts to be audited. The auditor would repeat this process until the desired sample size is achieved. If the set size is equal to k, and the number of cycles is m, then the final sample size can be represented as n = m*k accounts. Thus, in the case with a set size equal to three, the final sample size would be some multiple of three depending on the desired level of precision. In actual auditing practice set sizes larger than three would be used because they provide more information about the characteristics of the population. The RSS sampling process would be tedious if performed manually; however, computer software applications can effortlessly extract the sample in a matter of seconds because the ranking variable (client book value) is readily available and the selection process itself follows a simple algorithm.

Intuitively speaking, RSS is superior to SRS because the ranking process *forces* the selection of a more representative sample of the population than is achieved with SRS. The structural advantage of RSS over SRS can be understood by recognizing that RSS uses readily available ranking information from $m*k^2$ accounts, of which only m*k accounts are actually audited. Thus, RSS is using considerably more information than SRS in selecting a representative sample. The strength of this advantage will increase as the accuracy of the client book values as a ranking variable for actual values increases. Conversely, in a worst-case scenario where there are widespread discrepancies between audited and book values, the precision level of RSS would decline but never fall below the level of precision achieved with SRS. In the vast majority of actual auditing cases, the rank correlation between audited and book values will be high. Accordingly, RSS will usually achieve superior precision over SRS, and will never yield inferior results to SRS.

3. RSS for Verifying Account Balances

RSS is best applied to auditing problems when assessing the true value of an account balance is time-consuming. Accounts such as inventory; accounts receivable; property, plant, and equipment; and accounts payable usually fall into this category. The auditor will draw a sample from these accounts through their respective subsidiary ledgers, and then proceed with on-site inspections, recalculations, confirmations, and other auditing procedures.

Table 2. Descriptive Statistics for the Audited Values of the Inventory Data

Mean	375.57
Median	376.30
Standard Deviation	112.69
Minimum	17.97
Maximum	771.60
Total	1,877,836.72
Count	5,000

This study applies RSS to a population of 5,000 different inventory items derived from the financial records of a retail clothing store. (The full data set of 5,000 book values and audited values for these financial records are available upon request from the authors.) Each of these 5,000 items represents the inventory value of a particular retail item. The non-normality of accounting populations is well documented in the fraud literature, and accounting numbers that have not been artificially restricted or manipulated will tend to follow Benford's Law (Nigrini, 2011). Accordingly, the data set was checked for conformity to Benford's Law using Nigrini's Mean

Absolute Deviation test and the chi-square test for distribution conformity; both tests show strong conformity to Benford's Law. Thus, this data set would be a reasonable representation of what an auditor would expect to see in an actual inventory valuation setting. We choose inventory for our study because it is one of the more commonly manipulated accounts in financial statement frauds (Albrecht et al., 2013; Wells, 2011). Table 2 provides some descriptive statistics for the population.

The total shown in Table 2 is assumed to be the audited or true value of the total inventory account. Therefore, the subsidiary numbers that comprise this sum are also audited values. In reality, these numbers would be unknown unless every item in the population were audited. Using the audited values as a base, a fraudulent data set was created by randomly overstating individual inventory values to simulate a typical inventory fraud. In the 5,000 accounts, the number of overstated accounts is 500, or 10% of the accounts. The amount of overstatement for each fraudulent account was generated using random numbers, with an average overstatement factor of twice the true (audited) value. The fraudulent data set represents modest inventory overstatements, consistent with a company looking to boost its earnings by a material amount, yet without attracting inordinate attention. Table 3 summarizes the fraudulent data set considered in this paper.

Table 3. Features of the Fraudulent Data Set

Percentage of Fraudulent Accounts	Number of Fraudulent Accounts	Total Inventory Book Value	Total Inventory Audited Value	Percentage Overstatement
10%	500	2,053,578.34	1,877,836.72	9.4

4. Confidence Intervals and Hypothesis Tests for the Population Mean Using Data from a RSS

In a previous paper, Gemayel et al. (2012) described how to use RSS data to estimate the mean of an auditing population of interest. They used extensive simulations to demonstrate how RSS can lead to dramatic improvement in precision over SRS for a given sample size or to substantial reduction over SRS in sample size needed to achieve a desired precision level. To obtain those results, the simulation process was replicated 5,000 times to provide an accurate estimate of the standard deviation of the RSS sample mean $\overline{X}_{RSS}$ to evaluate its precision relative to that of the SRS sample mean $\overline{X}_{SRS}$. In an auditing application, however, we cannot use simulation for this purpose, since we have available only a single RSS set of data. In such an application, we are primarily interested in using the observed RSS data set to construct confidence intervals and conduct hypothesis tests about population parameters of interest, with the goal of reaching a statistical conclusion as to whether or not the population contains fraudulent entries and, if so, some statistical indication as to both the percentage of fraudulent entries and the magnitude of the fraud.

In Appendix A, we provide the statistical details for obtaining an approximate $100(1-\alpha)\%$ confidence interval for the mean μ of a population. The expression for this approximate confidence interval is presented in equation (9) in Appendix A. For the data setting discussed in Section 3 we are interested in two population quantities, namely, the total amount of fraud in the book value entries and the percentage of book value entries that are fraudulent. Let μ_F denote the average amount of fraud per account in the financial records of the retail clothing store (so that $5{,}000\mu_F$ represents the total amount of fraud in the financial records) and let p_F denote the percentage of fraudulent accounts in the financial records.

In this section we illustrate how to use a single RSS from this population to obtain approximate 95% confidence intervals for both μ_F and p_F. Details for the collection of the RSS and the necessary calculations to obtain these two confidence intervals are provided in Appendix B. We show here only the principal steps leading to the two confidence intervals.

As noted in Appendix B, we applied the R command RSS(*k*, *m*, *x*) in the R package NSM3 developed by Schneider (2014) for the third edition of Hollander, et al. (2014) to the previously discussed population of 5,000 inventory items from the financial records of a retail clothing store. For illustrative purposes, we choose a set size of k = 10 and number of cycles m = 40, so that the total number of audited values in our RSS is $n = mk = 400$. (Since the population of inventory values contains only 5,000 entries, we used sampling with replacement between each selection to generate our mk^2 = 4,000 sets to obtain the 400 observations in the RSS.) The two quantities of interest here are whether or not an account is fraudulent ($I_F = 1$ if the book value is larger than the audited value for the account, = 0, otherwise) and the amount of fraud in the account (given by $Z_F = BV - AV$, the difference between the book value BV and the audited value AV). A detailed listing of this information for the RSS observations, cross-tabulated by the cycle number and the judgment rank order, is provided in Appendix B.

We see from Appendix B that the k = 10 individual judgment rank sample means for the amount of fraud in this RSS are

$$\bar{Z}_{F[1]}=0, \bar{Z}_{F[2]}=0, \bar{Z}_{F[3]}=0, \bar{Z}_{F[4]}=0, \bar{Z}_{F[5]}=0,$$

$$\bar{Z}_{F[6]}=0, \bar{Z}_{F[7]}=6.179, \bar{Z}_{F[8]}=30.324, \bar{Z}_{F[9]}=69.2055, \bar{Z}_{F[10]}=260.1955$$

The corresponding k = 10 judgment rank sample proportions for the presence of fraud in this RSS are

$$\hat{p}_{F[1]}=0,\ \hat{p}_{F[2]}=0,\ \hat{p}_{F[3]}=0,\ \hat{p}_{F[4]}=0,\ \hat{p}_{F[5]}=0,$$

$$\hat{p}_{F[6]}=0, \hat{p}_{F[7]}=.025, \hat{p}_{F[8]}=.10, \hat{p}_{F[9]}=.225, \hat{p}_{F[10]}=.70$$

Thus, from equation (1) in Appendix A, the RSS estimate of the average amount of fraud in the population of inventory values is given by

$$\hat{m}_F = \bar{Z}_{F,RSS} = \frac{1}{10}\sum_{r=1}^{10}\bar{Z}_{F[r]} = \$36.5904\,,$$

so that the RSS estimate of the total amount of fraud in the population is $5{,}000\,\hat{m}_F$ = 5,000($36.5904) = $182,952. In addition, using the fraud indicator data in equation (1), the RSS estimate of the percentage, p_F, of fraudulent accounts in the population of inventory values is given by

$$\hat{p}_F = \frac{1}{10}\sum_{r=1}^{10}\hat{p}_{F[r]} = .105 \ \text{(i.e., 10.5\%)}.$$

Using the sample estimates $S^2_{F[1]}, \mathbf{K}, S^2_{F[10]}$ for the variances of the k individual judgment order statistics (details in Appendix B), we estimate the variance of $\hat{m}_F = \bar{Z}_{F,RSS}$ to be

$$\widehat{Var}[\bar{Z}_{F,RSS}] = \frac{1}{40(10)^2}\sum_{r=1}^{10}S^2_{F[r]} = \frac{1}{4000}(64711.95) = 16.178$$

Similarly, using the sample estimates $T^2_{F[1]}, \ldots, T^2_{F[10]}$ for the variances of the k individual judgment sample percentages (again details in Appendix B), the estimate of the variance of $\hat{p}_F$ is given by

$$\hat{Var}[\hat{p}_F] = \frac{1}{40(10)^2}\sum_{r=1}^{10} T^2_{F[r]} = \frac{1}{4000}(.51154) = .0001279.$$

We are now in a position to use equation (9) in Appendix A to obtain the desired 95% confidence intervals for m_F and p_F. With α = .05, the upper .025 percentile for the standard normal distribution is $z_{.025} = 1.96$ and an approximate 95% confidence interval for the average amount of fraud, m_F, in the population of inventory values is

$$\bar{Z}_{RSS} \pm z_{a/2}\sqrt{\frac{1}{mk^2}\cdot\sum_{r=1}^{k} S^2_{F[r]}} = 36.5904 \pm 1.96\sqrt{16.178} = 36.5904 \pm 7.8835 = (28.7069, 44.4739),$$

so that an approximate 95% confidence interval for the total amount of fraud in the population of inventory values is (5,000($28.7069), 5,000($44.4739)) = ($143,534.50, $222,369.50). We note that in this example the true amount of fraud in the population, namely, $175,741.62, correctly belongs to this approximate 95% confidence interval and the associated approximate α = .05 hypothesis test would correctly reject the null hypothesis of no fraud in the accounts, since 0 does not belong to the confidence interval.

Similarly, an approximate 95% confidence interval for the percentage of fraudulent accounts, p_F, in the population of inventory values, is

$$\hat{p}_F \pm z_{a/2}\sqrt{\frac{1}{mk^2}\cdot\sum_{r=1}^{k} T^2_{[r]}} = .105 \pm 1.96\sqrt{.0001279} = .105 \pm .02217 = (.08283, .12717),$$

so that we are 95% confident that between 8.283% and 12.717% of the accounts in the population are fraudulent. As with the total fraud, the true percentage of fraudulent accounts in the population, namely, 10%, correctly belongs to this approximate 95% confidence interval and the associated approximate α = .05 hypothesis test would correctly reject the null hypothesis of no fraudulent accounts, since 0% does not belong to the confidence interval.

5. Assessing the Advantage of RSS over SRS

In Section 4 we presented methodology for obtaining approximate confidence intervals and hypothesis tests about population means and proportions using RSS data. In this section we use simulation to illustrate the advantage of using this RSS approach as opposed to standard procedures based on SRS data with the same sample size. We compare both the lengths of the confidence intervals and the simulated powers of the associated tests. For our study, we used a total sample size of n = 400 for both the SRS and RSS procedures and the four different set sizes k = 5, 10, 20, and 25 for collecting the RSS observations. (Again we used sampling with replacement between each selection to generate our RSS sets to obtain the 400 observations in each of our samples. The R code used in these simulations is available upon request from the authors.) This simulation process was repeated 5,000 times and the RSS and SRS approximate 95% confidence intervals for m_F, the average amount of fraud per account in the financial records of the retail clothing store, and p_F, the percentage of fraudulent accounts in the financial records were obtained for each sample. (Again, remember that 5,000 m_F represents the total amount of fraud in the records.) Figure 1 below plots the averages of the lower endpoints and the upper endpoints of these 5,000 confidence intervals for

m_F separately for each of the set sizes k = 1, 5, 10, 20, and 25. Figure 2 plots the same averages for the associated 5,000 confidence intervals for p_F. Note that SRS corresponds to set size k = 1.

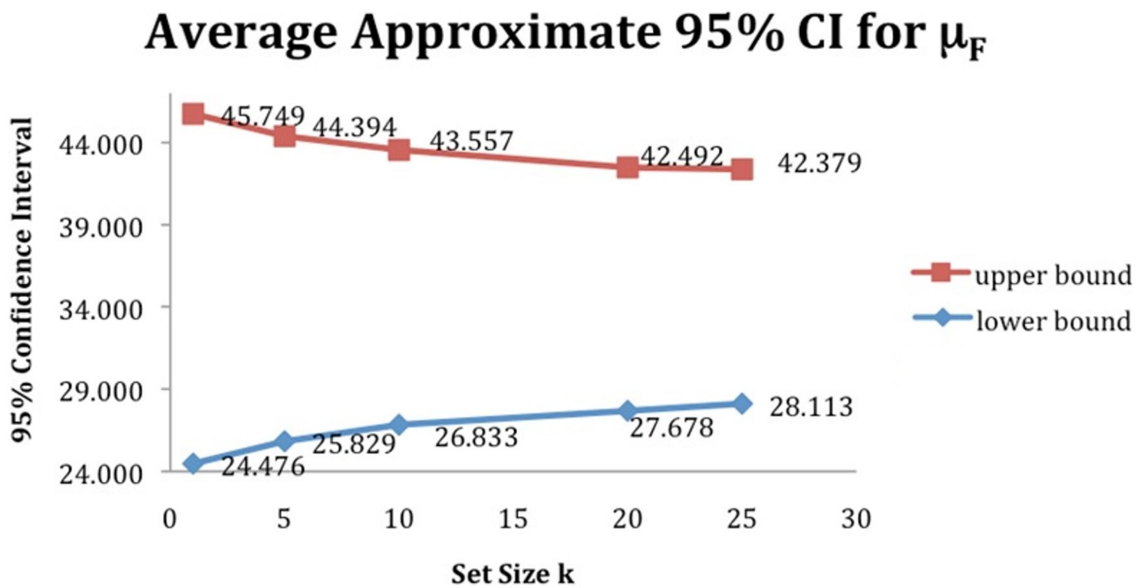

Figure 1. Average Approximate 95% Confidence Intervals
For Average Amount of Fraud

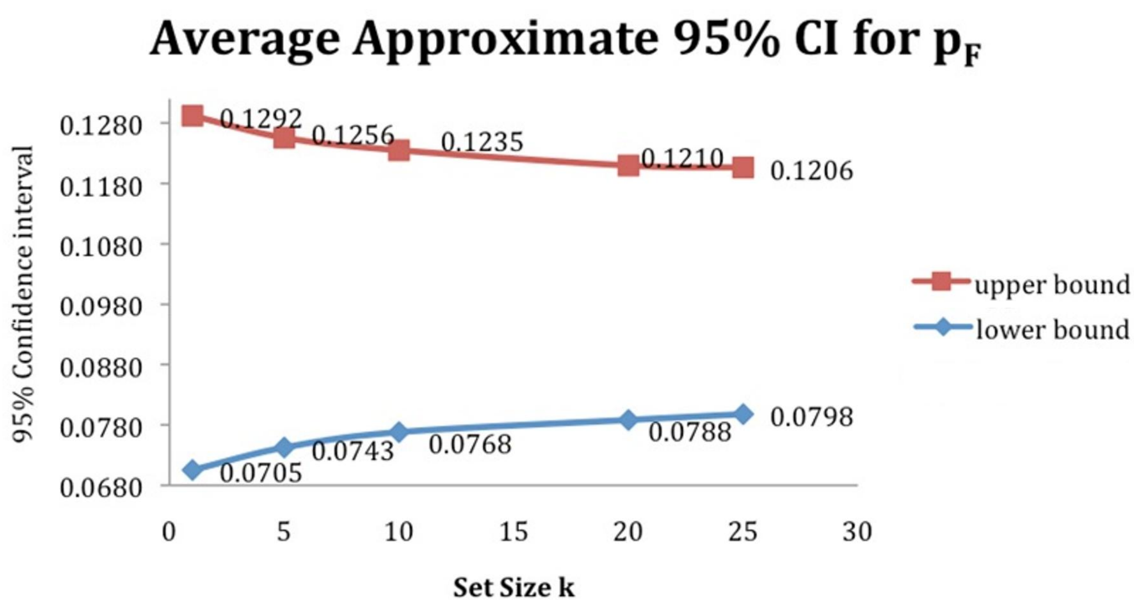

Figure 2. Average Approximate 95% Confidence Intervals
For Percentage of Fraudulent Accounts

The purpose of Figures 1 and 2 is to demonstrate that the lengths of the RSS confidence intervals for m_F and p_F are, in both cases, decreasing functions of the set size k for a given total sample size of n = 400 and all of the RSS intervals are shorter than their comparable SRS confidence intervals (k = 1). Note, however, that the greatest reduction in length has already occurred by set size k = 20 and we attain only diminishing marginal returns from increasing the set size further.

To get a more refined picture of how much better (i.e., shorter) the RSS confidence intervals are than their SRS counterparts, we also plot the ratios of the average lengths of the approximate 95% RSS confidence intervals to the lengths of the corresponding SRS counterparts. These results are pictured in Figures 3 and 4 for the confidence intervals for m_F and p_F, respectively.

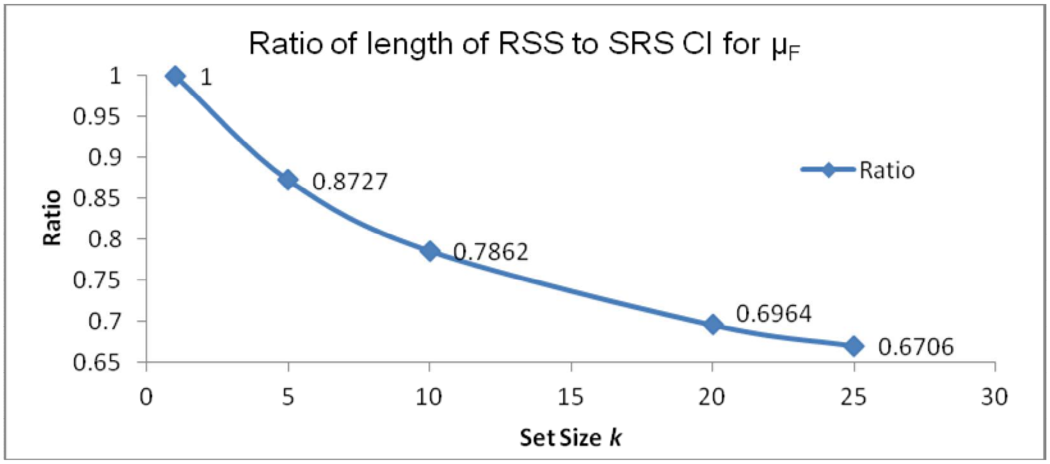

Figure 3. Plot of Ratios of Average Lengths of Simulated RSS and SRS 95% Confidence Intervals for Average Amount of Fraud

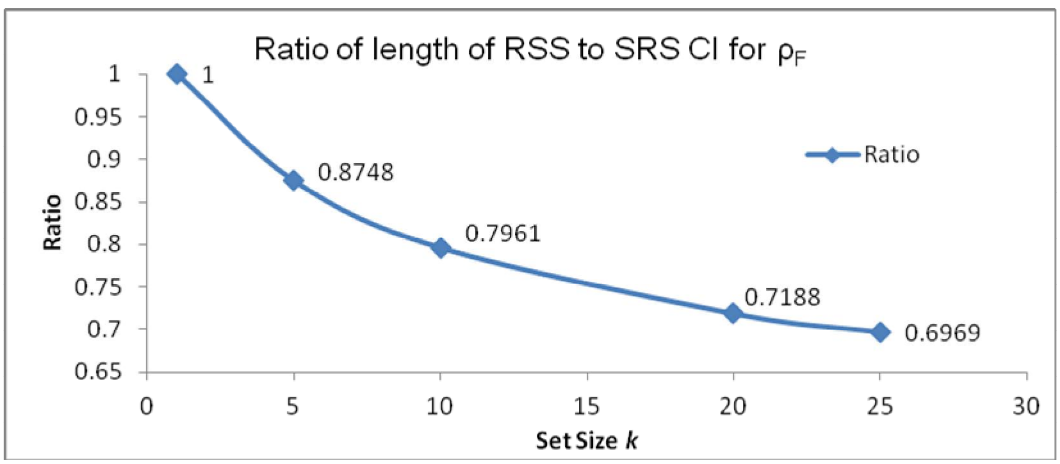

Figure 4. Plot of Ratios of Average Lengths of Simulated RSS and SRS 95% Confidence Intervals for Percentage of Fraudulent Accounts

Figures 3 and 4 demonstrate again the merits of RSS in producing narrower confidence intervals (as expected given the results we discussed in Section 4 and Appendix A), as well as the diminishing marginal returns from continuing to increase set size. For example, with overall sample size n = 400, an approximate 95% confidence interval for the population mean m_F using RSS with set size k = 20 is, on average, less than three-fourths as wide as the approximate 95% confidence interval for m_F using SRS. Increasing the set size to k = 25 yields only a small additional reduction in the length of the confidence interval. Similar comments apply to the approximate 95% confidence intervals for p_F based on RSS and SRS.

We also compare the empirical powers of the SRS and RSS hypothesis tests associated with their respective confidence intervals. We carried out 5,000 simulations each for all appropriate combinations of overall sample sizes *n* = 20, 50, 100, and 400 and the same set sizes *k* = 1, 5, 10, 20, and 25. (Once again, remember that SRS corresponds to set size *k* = 1.) For each of the 5,000 simulations, we recorded whether the associated confidence intervals for m_F and p_F contained the null values of $m_F = 0$, corresponding to no fraud in the recorded book values, or $p_F = 0$, also corresponding to no fraud in the recorded book values, respectively. In Tables 4 and 5 we report the numbers of our simulated confidence intervals that led to rejection of $m_F = 0$ (did not contain 0) or $p_F = 0$ (did not contain 0), respectively, for the four RSS configurations with *k* = 5, 10, 20, and 25 and the SRS setting (*k* = 1). (Thus, dividing the entries in Tables 4 and 5 by the number of simulations, 5,000, provides us with empirical power estimates for the associated hypothesis tests.)

Table 4. Number of Rejections (Empirical Power) of H_0: $\mu_F = 0$
for 5000 Simulations from the Fraudulent Data Set

N \ *k*	**1**	**5**	**10**	**20**	**25**
20	696 (.1392)	1169 (.2338)	1985 (.397)	NA	NA
50	3675 (.735)	3884 (.7768)	4245 (.849)	NA	4600 (.920)
100	4951 (.9902)	4979 (.9958)	4995 (.999)	4999 (.9998)	5000 (1)
400	5000 (1)	5000 (1)	5000 (1)	5000 (1)	5000 (1)

Table 5. Number of Rejections (Empirical Power) of H_0: $p_F = 0$
for 5000 Simulations from the Fraudulent Data Set

N \ *k*	**1**	**5**	**10**	**20**	**25**
20	696 (.1392)	1517 (.3034)	1998 (.3996)	NA	NA
50	3719 (.7438)	4225 (.845)	4252 (.8504)	NA	4686 (.9392)
100	4962 (.9924)	4980 (.996)	4995 (.999)	4999 (.9998)	5000 (1)
400	5000 (1)	5000 (1)	5000 (1)	5000 (1)	5000 (1)

As expected (since the SRS confidence intervals are, on average, longer than the corresponding RSS confidence intervals), the hypothesis tests of $H_0 : m_F = 0$ and $H_0 : p_F = 0$ associated with the RSS intervals are always at least as powerful as the corresponding tests associated with the SRS intervals. The empirical power for the RSS procedure is also an increasing function of the set size *k*, in agreement with the average lengths of the associated confidence intervals. Of course, all of the RSS procedures and the SRS procedures do extremely well for the larger sample sizes (*n* = 100 and *n* = 400), but there is a distinct advantage for the RSS procedures for the smaller sample sizes (*n* = 20 and *n* = 50).

6. Implications and Recommendations for Auditing

We have demonstrated both theoretically (in Appendix A) and empirically (in Section 5) that RSS can lead to much more precise estimates of either a population mean or a population proportion than a SRS of the same size. RSS also produces significantly narrower confidence intervals and more powerful hypothesis tests for both parameters. The degree of improvement possible through RSS depends, of course, on the reliability of the judgment rankings that lead to the

RSS data. In the fraudulent inventory account data example in this paper we used the easily obtained book values to perform these judgment rankings and select which accounts to audit for the true inventory values. Since the recorded book value is highly correlated with the true inventory value, these judgment rankings were quite accurate and led to the collection of a RSS that contained more information about the fraudulent nature of the accounts than could be obtained from a SRS without the aid of this additional information. How much improvement can be realized from using RSS instead of SRS in other settings depends very much on how reliable a mechanism is available for obtaining the relevant judgment rankings. The bottom line, however, is that inferences about population means or proportions based on RSS can never do any worse than inferences based on a SRS of the same size. This enables the use of RSS procedures with smaller sample sizes than would be necessary using SRS procedures to obtain the same effectiveness (precision of estimates, length of confidence intervals, power of tests) when making inferences about a population mean or proportion.

The error rates introduced into the inventory data population used in this study represent realistic attempts to materially overstate the value of inventory in a financial statement fraud. These types of fraudulent manipulations are commonplace whenever corporate management is under pressure to boost the bottom line (Albrecht et al., 2013). Policy implications would include the need to be alert to the possibility of inventory overstatements (or understatements for an income tax evasion scheme) whenever there are material weaknesses in internal controls related to the revenue/inventory cycle.

We note that it would also be of interest to compare RSS directly with MUS. Such a comparison, however, would need to include a detailed discussion of the theoretical underpinnings for MUS to provide the proper context for the comparison, and the scope and magnitude of such an investigation warrants a completely separate manuscript.

Appendix A: Statistical Development

In a previous paper, Gemayel et al. (2012) gave a heuristic argument for the advantage of RSS over SRS, by noting that RSS uses easily-obtained ranking information from mk^2 units to obtain mk measurements, whereas SRS is only concerned with those mk units that happen to be included in the sample, and makes no use of ranking information whatsoever. We now provide a more rigorous argument for this advantage.

The first step in understanding the potential of RSS is to visualize the different judgment ranks as their own hypothetical populations. We can think of each of these hypothetical populations as having its own mean and variance. The RSS sample mean $\bar{X}_{RSS}$ is then simply the average of the k individual judgment rank sample means, which are given by $\bar{X}_{[r]} = \frac{1}{m}\sum_{i=1}^{m} X_{[r]i}$, for r = 1, …, k, where $X_{[r]i}$ is the i^{th} observation on the r^{th} judgment rank; that is,

$$\bar{X}_{RSS} = \frac{1}{mk}\sum_{r=1}^{k}\sum_{i=1}^{m} X_{[r]i} = \frac{1}{k}\sum_{r=1}^{k}\bar{X}_{[r]} \qquad \text{(A1)}$$

Now, let $m_{[r]}$ and $s^2_{[r]}$ denote the mean and variance of the distribution for the r^{th} judgment rank, r = 1, …, k. Expressions for $m_{[r]}$ and $s^2_{[r]}$ were initially obtained by Dell and Clutter (1972) and further developed, for example, in Mode, Conquest, and Marker (1999) in the context of sampling costs in ecological research. Although our contribution in this paper lies in the practical application of RSS to real-world auditing situations, we choose to include the short derivations of both $m_{[r]}$ and $s^{2}_{[r]}$ here as well for completeness of the discussion. Since the *mk* RSS observations are mutually independent, it follows from standard results for sums of independent random variables that

$$E[\overline{X}_{RSS}]=E\left[\frac{1}{k}\sum_{r=1}^{k}\overline{X}_{[r]}\right]=\frac{1}{k}\sum_{r=1}^{k}E[\overline{X}_{[r]}]=\frac{1}{k}\sum_{r=1}^{k}m_{[r]} \tag{A2}$$

and

$$Var[\overline{X}_{RSS}]=Var\left[\frac{1}{k}\sum_{r=1}^{k}\overline{X}_{[r]}\right]=\frac{1}{k^{2}}\sum_{r=1}^{k}Var[\overline{X}_{[r]}]=\frac{1}{mk^{2}}\sum_{r=1}^{k}s^{2}_{[r]} \tag{A3}$$

It can be shown that the average of the individual judgment rank population means is equal to the overall population mean (that is, $\frac{1}{k}\sum_{r=1}^{k}m_{[r]}=m$), so that $\overline{X}_{RSS}$ is an unbiased estimator for the population mean μ. This relationship does not, however, carry over to the variances. Instead, it can be shown that the overall population variance s^{2} can be decomposed as follows:

$$s^{2}=\frac{1}{k}\sum_{r=1}^{k}s^{2}_{[r]}+\frac{1}{k}\sum_{r=1}^{k}\left(m_{[r]}-m\right)^{2} \tag{A4}$$

The first term on the right-hand side of (A4) represents within-rank variability, while the second term measures the discrepancy between the means of the judgment ranks and the overall population mean m, thereby accounting for between-rank variability.

Let $\overline{X}_{SRS}$ denote the mean of a (generic) SRS of the same size $n=mk$ as our RSS. The variance of this estimator is given by the well-known formula $Var\,\overline{X}_{SRS}=\frac{s^{2}}{n}=\frac{s^{2}}{mk}$. Combining this expression with the results in (A3) and (A4) yields

$$Var[\overline{X}_{SRS}]=\frac{s^{2}}{mk}=\frac{1}{mk^{2}}\sum_{r=1}^{k}s^{2}_{[r]}+\frac{1}{mk^{2}}\sum_{r=1}^{k}\left(m_{[r]}-m\right)^{2}=Var[\overline{X}_{RSS}]+\frac{1}{mk^{2}}\sum_{r=1}^{k}\left(m_{[r]}-m\right)^{2}. \tag{A5}$$

Since $\frac{1}{mk^{2}}\sum_{r=1}^{k}\left(m_{[r]}-m\right)^{2}$ is a non-negative quantity, it is immediately clear from (A5) that $Var[\overline{X}_{SRS}]\geq Var[\overline{X}_{RSS}]$. Thus the RSS mean is at least as precise as its SRS counterpart for any given sample size, because the variance of the RSS mean involves only within-rank variability and does not include between-rank variability.

Consider now the sample counterparts to the mean and variance, $m_{[r]}$ and $s^{2}_{[r]}$, respectively, of the *r*th judgment rank population. A natural and unbiased estimator for the mean $m_{[r]}$ is simply the sample average of the *m* measurements taken from that rank, namely,

$$\hat{m}_{[r]}=\overline{X}_{[r]}=\frac{1}{m}\sum_{i=1}^{m}X_{[r]\,i} \tag{A6}$$

Similarly, a natural unbiased estimator of the variance $s^{2}_{[r]}$ is provided by the sample variance of those same *m* measurements, namely,

$$\hat{s}^2_{[r]} = S^2_{[r]} = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{[r]i} - \overline{X}_{[r]}\right)^2 \quad \text{(A7)}$$

We have already noted that the RSS estimator for the overall population mean is given by $\hat{m}_{RSS} = \overline{X}_{RSS}$ and from (A3) and (A7) it follows that an estimator for the variance of $\hat{m}_{RSS} = \overline{X}_{RSS}$ is given by

$$\hat{Var}[\overline{X}_{RSS}] = \frac{1}{mk^2}\sum_{r=1}^{k}\hat{s}^2_{[r]} = \frac{1}{mk^2}\sum_{r=1}^{k}S^2_{[r]}. \quad \text{(A8)}$$

Using these two estimators, in conjunction with the result that the distribution of $\hat{m}_{RSS} = \overline{X}_{RSS}$ can be well-approximated by a normal distribution when the number of cycles, *m*, is large, leads to the conclusion that an approximate 100(1-α)% confidence interval for the population mean μ is given by the expression

$$\overline{X}_{RSS} \pm z_{a/2}\sqrt{\frac{1}{mk^2}\cdot\sum_{r=1}^{k}S^2_{[r]}} \quad \text{(A9)}$$

where $z_{a/2}$ is the $(\alpha/2)^{th}$ upper percentile of the standard normal distribution.

We also note that the improvement in precision from using the RSS estimator $\overline{X}_{RSS}$ instead of the SRS estimator $\overline{X}_{SRS}$ is represented by the difference in their variances, since both estimators are unbiased. We see from (A5) that this improvement in precision is given by

$$\text{I} = Var[\overline{X}_{SRS}] - Var[\overline{X}_{RSS}] = \frac{1}{mk^2}\sum_{r=1}^{k}(m_{[r]} - m)^2, \quad \text{(A10)}$$

which we previously referred to as the between-rank variability. It follows that

$$\hat{I} = \frac{1}{mk^2}\sum_{r=1}^{k}(\overline{X}_{[r]} - \overline{X}_{RSS})^2 \quad \text{(A11)}$$

is a natural estimator for the amount of improvement from using RSS data instead of SRS data to estimate the population mean μ.

Appendix B: Detailed Example of Confidence Intervals and Hypothesis Tests for a RSS

To provide a detailed example of the process for obtaining a confidence interval for μ_F, the average amount of fraud per account in the financial records of the retail clothing store, and p_F, the percentage of fraudulent accounts in the financial records, we applied the R command RSS(*k*, *m*, *x*) in the R package NSM3 developed by Schneider (2014) for the third edition of Hollander et al. (2014) to the previously discussed population of 5,000 inventory items. For illustrative purposes, we choose a set size of *k* = 10 and number of cycles *m* = 40, so that the total number of audited values in our RSS is *n* = *mk* = 400. We present the 400 RSS observations (the book values BV and the audited values AV) in Table 6 at the end of this paper, categorized by both the cycle number and the associated judgment rank order.

From the AV and BV values in Table 6, we obtain the two pieces of information Z_F = BV – AV = amount of fraud in the account and the fraud indicator I_F = 1 or 0 if there is or is not fraud, respectively, in the account. Averaging across the cycles for the fraud data in Table 6, we see that the *k* = 10 individual judgment rank sample means for the amounts of fraud in this RSS are

$$\bar{Z}_{F[1]} = 0,\ \bar{Z}_{F[2]} = 0,\ \bar{Z}_{F[3]} = 0,\ \bar{Z}_{F[4]} = 0,\ \bar{Z}_{F[5]} = 0,$$
$$\bar{Z}_{F[6]} = 0,\ \bar{Z}_{F[7]} = 6.179,\ \bar{Z}_{F[8]} = 30.324,\ \bar{Z}_{F[9]} = 69.2055,\ \bar{Z}_{F[10]} = 260.1955,$$

and the corresponding k = 10 judgment rank sample percentages for the presence of fraud in the RSS are

$$\hat{p}_{F[1]} = 0,\ \hat{p}_{F[2]} = 0,\ \hat{p}_{F[3]} = 0,\ \hat{p}_{F[4]} = 0,\ \hat{p}_{F[5]} = 0,$$
$$\hat{p}_{F[6]} = 0,\ \hat{p}_{F[7]} = .025,\ \hat{p}_{F[8]} = .100,\ \hat{p}_{F[9]} = .225,\ \hat{p}_{F[10]} = .70.$$

Using the expression $\hat{s}^2_{[r]} = S^2_{F[r]} = \frac{1}{39}\sum_{i=1}^{40}\left(Z_{F[r]\,i} - \bar{Z}_{F[r]}\right)^2$ from equation (A7), the sample estimates $S^2_{F[1]}, \mathbf{K}, S^2_{F[10]}$ of the individual judgment rank variances $s^2_{[r]}$ for the amounts of fraud in the various judgment ranks are then

$$S^2_{F[1]} = 0,\ S^2_{F[2]} = 0,\ S^2_{F[3]} = 0,\ S^2_{F[4]} = 0,\ S^2_{F[5]} = 0,$$
$$S^2_{F[6]} = 0,\ S^2_{F[7]} = 1527.202,\ S^2_{F[8]} = 9700.704,\ S^2_{F[9]} = 17841.5,\ S^2_{F[10]} = 35642.54.$$

Using the indicator functions for the presence or absence of fraud in conjunction with equation (A7) again, the sample estimates $T^2_{F[1]}, \mathbf{K}, T^2_{F[10]}$ of the individual judgment rank variances $s^2_{[r]}$ for the percentage of fraud in various judgment ranks are then

$$T^2_{F[1]} = 0,\ T^2_{F[2]} = 0,\ T^2_{F[3]} = 0,\ T^2_{F[4]} = 0,\ T^2_{F[5]} = 0,$$
$$T^2_{F[6]} = 0,\ T^2_{F[7]} = .025,\ T^2_{F[8]} = .09231,\ T^2_{F[9]} = .17885,\ T^2_{F[10]} = .21538.$$

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Table 6. RSS of n = 400 Observations (Book Value = BV and Audited Value = AV) from the Inventory Population Using Set Size k = 10 and Cycle Size m = 40

		Judgment Rank Order									
Cycle		Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8	Rank 9	Rank 10
1	BV	261.38	294.74	350.15	362.12	321.51	303.35	478.59	463.76	595.84	826.48
	AV	261.38	294.74	350.15	362.12	321.51	303.35	478.59	463.76	199.74	396.38
2	BV	268.49	215.73	268.2	283.34	421.5	444.97	469.19	485.01	625.25	926.83
	AV	268.49	215.73	268.2	283.34	421.5	444.97	222.03	485.01	365.69	498.6
3	BV	154.6	320.8	317.27	361.25	373.07	475.36	357.11	549.74	490.03	788.97
	AV	154.6	320.8	317.27	361.25	373.07	475.36	357.11	314.6	490.03	335.73
4	BV	133.65	327.41	417.88	372.21	356.37	478.63	470.51	580.43	729.29	606.84
	AV	133.65	327.41	417.88	372.21	356.37	478.63	470.51	580.43	467.69	606.84
5	BV	183.3	329.11	265.44	378.28	337.6	384.07	551.44	490.61	476.63	620.24
	AV	183.3	329.11	265.44	378.28	337.6	384.07	551.44	490.61	476.63	620.24
6	BV	169.84	316.23	349.38	377.85	447.16	487.63	501.38	423.31	510.38	662.73
	AV	169.84	316.23	349.38	377.85	447.16	487.63	501.38	423.31	510.38	458.95
7	BV	65.86	340.92	334.17	292.85	396.03	411.75	398.77	440.75	661.91	475.32
	AV	65.86	340.92	334.17	292.85	396.03	411.75	398.77	440.75	661.91	475.32
8	BV	113.06	235.72	215.89	357.63	389.26	372.79	406.89	459.43	587.73	900.11
	AV	113.06	235.72	215.89	357.63	389.26	372.79	406.89	459.43	587.73	415.56
9	BV	156.45	241.7	381.91	321.75	315.21	433.93	359.2	473.32	477.02	836.68
	AV	156.45	241.7	381.91	321.75	315.21	433.93	359.2	473.32	477.02	388.13

10	BV	232.77	269.87	326.5	337.61	443.17	363.81	395.31	497.11	441.91	762.44
	AV	232.77	269.87	326.5	337.61	443.17	363.81	395.31	497.11	441.91	267.41
11	BV	215.72	233.56	392.62	322.88	388.22	458.05	417.6	467.87	426.1	712.03
	AV	215.72	233.56	392.62	322.88	388.22	458.05	417.6	467.87	426.1	255.7
12	BV	204.64	325.7	339.73	269.78	364.85	484.65	391.89	358.03	803.01	553.67
	AV	204.64	325.7	339.73	269.78	364.85	484.65	391.89	358.03	538.56	553.67
13	BV	308.72	237.91	369.63	341.99	408.23	442.25	378.39	514.31	587.16	841
	AV	308.72	237.91	369.63	341.99	408.23	442.25	378.39	514.31	587.16	511.17
14	BV	242.75	296.23	286.68	399.74	376.56	375.22	433.62	508.17	471.97	755.96
	AV	242.75	296.23	286.68	399.74	376.56	375.22	433.62	508.17	471.97	264.49
15	BV	259.25	225.19	279.38	289.21	425.38	370.11	513.55	473.17	594.44	573.81
	AV	259.25	225.19	279.38	289.21	425.38	370.11	513.55	473.17	274.65	573.81
16	BV	73.86	199.93	378.84	252.62	460	332.34	381.03	532.99	575.28	582.98
	AV	73.86	199.93	378.84	252.62	460	332.34	381.03	532.99	575.28	582.98
17	BV	188.89	200.24	298.9	351.87	306.04	440.27	459.16	450.08	478.2	519.74
	AV	188.89	200.24	298.9	351.87	306.04	440.27	459.16	450.08	478.2	519.74
18	BV	217.34	268.71	320.67	312.8	398.06	356.15	406.52	415.24	517.64	688.98
	AV	217.34	268.71	320.67	312.8	398.06	356.15	406.52	415.24	517.64	427.04
19	BV	168.89	305.59	244.28	379.73	369.07	370.61	470.69	496.23	619.03	527.16
	AV	168.89	305.59	244.28	379.73	369.07	370.61	470.69	496.23	310.02	527.16
20	BV	143.24	199.1	305.56	404.72	395.61	447.4	466.27	378.5	503.69	533.35
	AV	143.24	199.1	305.56	404.72	395.61	447.4	466.27	378.5	503.69	533.35
21	BV	172.87	236.29	235.98	285.78	378.52	424.59	496.56	474.37	472.93	588.33
	AV	172.87	236.29	235.98	285.78	378.52	424.59	496.56	474.37	472.93	373.66
22	BV	157.75	321.05	213.83	397.11	426.25	419.53	516.59	482.16	431.5	774.83
	AV	157.75	321.05	213.83	397.11	426.25	419.53	516.59	482.16	431.5	324.92
23	BV	171.4	315.81	355.78	423.55	433.63	407.34	387.47	476.08	476.48	751.32
	AV	171.4	315.81	355.78	423.55	433.63	407.34	387.47	476.08	476.48	401.93
24	BV	54.03	325.69	318.2	305.13	339.33	409.22	445.07	488.64	543.29	803.62
	AV	54.03	325.69	318.2	305.13	339.33	409.22	445.07	488.64	543.29	451.75
25	BV	209.51	294.84	394.46	396.12	342.56	425.26	527.32	458.63	499.6	764.41
	AV	209.51	294.84	394.46	396.12	342.56	425.26	527.32	256	499.6	329.4

26	BV	280.17	271.19	327	369.68	435.71	335.23	389.07	432.55	523.51	508.49
	AV	280.17	271.19	327	369.68	435.71	335.23	389.07	432.55	523.51	256.15
27	BV	127.24	277.69	330.87	353.8	371.85	389.2	374.02	526.66	531.9	949.66
	AV	127.24	277.69	330.87	353.8	371.85	389.2	374.02	526.66	531.9	668.12
28	BV	75.55	259.04	310.27	369.79	400.7	361.81	418.5	446.44	486.63	874.56
	AV	75.55	259.04	310.27	369.79	400.7	361.81	418.5	446.44	486.63	631.96
29	BV	155.42	264.85	283.61	371.81	365.35	421.25	419.04	436.29	593.12	578.36
	AV	155.42	264.85	283.61	371.81	365.35	421.25	419.04	436.29	593.12	297.29
30	BV	233.07	128.6	312.11	321.56	331.1	314.95	460.3	544.27	692.28	683.38
	AV	233.07	128.6	312.11	321.56	331.1	314.95	460.3	544.27	277.99	315.37
31	BV	188.21	185.11	430.52	357.67	309.27	344.23	430.24	722.03	540.43	528.03
	AV	188.21	185.11	430.52	357.67	309.27	344.23	430.24	238.78	540.43	528.03
32	BV	239.39	284.15	287.44	330.04	367.44	389.84	367.53	413.56	644.49	808.57
	AV	239.39	284.15	287.44	330.04	367.44	389.84	367.53	413.56	434.9	444.26
33	BV	110.79	304.68	262.56	280.83	274.57	481.45	468.16	490.01	498.2	531.43
	AV	110.79	304.68	262.56	280.83	274.57	481.45	468.16	490.01	498.2	531.43
34	BV	249.98	267.44	304.21	280.52	461.14	419.94	419.85	471.64	472.44	788.05
	AV	249.98	267.44	304.21	280.52	461.14	419.94	419.85	471.64	472.44	410.61
35	BV	164.48	307.24	383.09	288.49	307.29	418.83	389.17	487.62	515.03	586.87
	AV	164.48	307.24	383.09	288.49	307.29	418.83	389.17	487.62	515.03	295.96
36	BV	270.67	281.8	400.96	327.86	391.64	452.99	532.18	432.27	494.22	705.77
	AV	270.67	281.8	400.96	327.86	391.64	452.99	532.18	432.27	494.22	396.54
37	BV	227.92	274.6	400.23	367.36	294.38	384.43	480.11	502.33	471.5	1002.75
	AV	227.92	274.6	400.23	367.36	294.38	384.43	480.11	502.33	471.5	503.84
38	BV	181.33	235.21	414.57	338.36	414.25	335.85	444.22	476.36	668.75	551.27
	AV	181.33	235.21	414.57	338.36	414.25	335.85	444.22	476.36	334.92	551.27
39	BV	181.98	298.92	320.93	247.11	419.96	398.24	453.21	491.64	582.09	840.86
	AV	181.98	298.92	320.93	247.11	419.96	398.24	453.21	199.7	582.09	448.74
40	BV	232.78	330.21	262.4	290.93	396	391.16	491.47	475.29	613.73	1042.64
	AV	232.78	330.21	262.4	290.93	396	391.16	491.47	475.29	613.73	577.2