

Volatility Linkage of Nominal and Index-linked Bond Returns: A Multivariate BEKK-GARCH Approach

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Abstract: This paper investigates the behavior of volatility linkage between nominal and index-linked bond returns using a multivariate BEKK-GARCH approach. Based on daily return data for French bonds with different maturity dates and different reference indices, our analysis reveals two empirical regularities.

First, the relationship, in uncertainty, between nominal and index-linked bond returns is strongly significant and is mostly due to the liquidity risk.

Second, the co-persistence in volatility returns for the studied bonds is also confirmed.

This empirical study provides a useful method that may be employed by decision makers to quantitatively manage and reallocate their portfolios.

JEL Classifications: C58, G12, G32

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1. Introduction

This study investigates the behavior of volatility linkage between nominal and index-linked bonds. We study this linkage because they are very important for a variety of long term investment and risk management decisions and also for issuers. Firstly, Investors, for example, usually shift investments from conventional bonds to inflation index linked bonds when they expect an increase of inflation rates. Conversely, when inflation rates are expected to be quite stable, investors would decide to invest in conventional bonds to reduce liquidity risk. The inflation and liquidity risk reduction depends on the volatility co-movement between conventional and index linked bond returns. If the correlation between volatility changes across these bonds are high, then, index linked bond may not give the safe refuge against the inflation that investors are looking for. Moreover, if volatility changes across bond returns are highly correlated, investors can not eliminate the liquidity risk by replacing index linked bonds by conventional bonds.

Volatility co-movement can also be considered in setting regulatory policy, for example, in insurance companies actuary have to appreciate the nature of volatility co-movement to suitably measure capital adequacy and solvency regimes. It is known that bonds represented the majority of insurance industry investments, exceeding 75% of total cash and invested assets in the developed countries. Likewise, actuary should consider this volatility co-movement to predict the effect of an eventual policy changes.

In this paper we investigate the volatility co-movement between indexed and conventional bonds. Our contribution is twofold. First, we use a general autoregressive conditional heteroskedasticity model proposed by Baba, Engle, Kraft and Kroner, called BEKK-GARCH, to reveal nature of volatility dependence in bond market. The main merit of this model is that it resolves the problem of ensuring the positive definiteness of the covariance matrix by using a quadratic form for the conditional covariance equations.

Second, our study is focused on the French bond market. The choice of the French market may be argued by the fact that France is one of the largest issuers of inflation-linked bonds in the European area.

Moreover, French institutional investors, particularly insurers, banks and mutual funds, have been the main investors in inflation-linked bonds in Europe.

There is currently no empirical study on the volatility co-movement between French conventional and index-linked bonds. This paper fills the gap in the existing literature.

In addition to the original paper by Vasiliki et al. (2006), Kim et al. (2006), Brière et al. (2009), Conrad et al. (2010), and more recently Kanas (2014), and Benlagha (2014) with applications to the Euro area bond market, the literature on the volatility co-movement on bond markets is indeed scarce comparing to those on Asian or American markets.

The results reveal a strong association between conventional bond volatility and the French index-linked bond one. This strong relationship is also supported among conventional and European index-linked bond returns. We argue that this volatility linkage is due to the liquidity risk.

Besides, we display a presence of co-persistence in volatility. Thus, the price of a bond typically depends on the conditional covariance with a benchmark portfolio.

The rest of the paper is organised as follows. The next section briefly review the literature. Then, section 3 introduces the multivariate GARCH model to be estimated. Section 4 describes data and analyse some preliminary statistics for the studied time series. The estimation results are discussed in section 5. Finally, concluding remarks and directions for future research are suggested in section 6.

2. Literature Review

In view of the important implications of the volatility co-movement among assets, many approaches have been implemented in the literature to address this issue, and the most commonly adopted method of dependence is the Granger causality test, Granger (1969, 1980), which focuses on the effect of past shocks of one asset on the current volatility of another. In this line of studies the commonly used method is the linear vector autoregressive regression (VAR) using estimated volatility proxies. For examples, Eun and Shim, (1989) uses a nine-market VAR model to study the stock market co-movement. Likewise, Soydemir (2000) investigates the volatility dependence among developed and emerging markets. Henry et al. (2007) uses Cheung and Ng's (1996) causality-in-mean and causality in variance tests to examine the stock market return and volatility spillovers among Asian countries and they found that stock markets may exhibit high degrees of co-movements.

Recently, Diebold and Yilmaz (2009) propose a new spillover index to analyse the co-movement and spillover effects across major stock markets over the world. The proposed Diebold-

Yilmaz spillover index is based on the variance decomposition associated with an N -variable vector autoregressive (VAR) model.

The second approach used to investigate the degree volatility dependence between assets is the parametric modelling of conditional variance. Engle et al. (1990), Hamao et al. (1990) and Booth et al. (1997) are among the first to investigate the volatility linkage between different markets using univariate generalized autoregressive conditional heteroscedastic GARCH model in tow stage . After that, the multivariate GARCH model becomes the commonly used parameter approach to investigate the volatility linkage. Karolyi (1995) used the multivariate GARCH model to investigate the international transmission of stock returns between the United States and Canada. Chiang et al. (2007) apply a dynamic conditional correlation model (DCC-GARCH) to nine Asian daily stock-return data to investigate the volatility and contagion effects between these stock markets. Billio and Caporin, (2010) introduce a simultaneous equation system with GARCH errors to model the contemporaneous relationships among Asian and American stock markets.

More recently, Bekiros (2013) adopt vector autoregressions and various multivariate GARCH representations to examine the dynamic linear and nonlinear causal linkages among the US, the EU and the BRIC markets using a sample covering the after-Euro period and including the financial crisis and the Euro-zone debt crisis.

3. Methodology

The BEKK GARCH is one of the most widely used models of conditional covariance and also conditional correlations, hence we use this approach to study the volatility co-movement between conventional and indexed bonds.

The basic framework for multivariate GARCH modelling was first introduced by Bollerslev, Engle and Wooldridge (1988), who extended the univariate GARCH model to include a vectorised conditional covariance matrix. Although the unrestricted model, known as the VECG-GARCH, is very general, it is practically infeasible to estimate for more than two variables because of the large number of parameters contained within it. Consequently, Bollerslev and Engle (1993), among others, have proposed a number of restrictions and specifications to reduce the number of parameters needed to be estimated. So, different specifications are proposed. Particularly, a model that can be viewed as a restricted version of the VEC model is the Baba-Engle- Kraft-Kroner (BEKK) discussed in Engle and Kroner (1995). Besides, nonlinear combination of univariate GARCH models is proposed in Engle (2002) and known as a constant conditional correlations (CCC-GARCH) model. Also, an extension to the CCC-GARCH model was introduced by Jeantheau (1998), called the Extended-CCC-GARCH (ECCGARCH) model, allowing a richer autocorrelation structure for the squared observed returns than the standard CCC-GARCH. More recently, Kawakatsu (2006) defined a multivariate exponential GARCH model in which parameter restrictions are used to ensure positive definiteness in the matrix which are not needed.

In order to keep the comparison between different M-GARCH model and their applications on financial assets we can refer to Nelson (1991), Hansson et al. (1998) or, more recently, Bauwens et al. (2006), Brooks (2006), Engle and Colacito (2006), Silvennoinen and Terasvirta (2008) and Mills and Markellos (2008).

Let's recall that the multivariate GARCH discussed by Bollerslev, Engle and Wooldridge (1988) is initially used to analyze the effect of news on various assets simultaneously. So, this model is interesting when considering multivariate models to describe the volatility of different time series jointly. In other words, this model tests if several assets moves together over time.

A multivariate GARCH model is presented as: $\varepsilon_t = z_t \sqrt{H_t}$ where $\varepsilon_t = (\varepsilon_{1t}, \dots, \varepsilon_{kt})'$ is a k -dimensional process and z_t is a k -dimensional *i.i.d* process with mean zero and covariance matrix equals to identity matrix I_k . To accomplish the modelling, the main task is a specification of parameters for the conditional covariance matrix H_t .

Particularly, in this search, we describe and apply the BEKK specification. The BEKK model can be considered as an alternative representation of the multivariate GARCH (p, q) model.

In this specification the conditional covariance matrix H_t is given by

$$H_t = CC' + \sum_{i=1}^p \sum_{l=1}^L A_{li}' \varepsilon_{t-l} \varepsilon_{t-l}' A_{li} + \sum_{j=1}^q \sum_{l=1}^L B_{lj}' H_{t-l} B_{lj}$$

Where A_{li} and B_{lj} are (k, k) matrices and C is lower triangular parameter matrix, p and q are the lags used and l specifies the lags used. The fully parameterized model includes $(p+q)Lk^2 + k(k+1)/2$ parameters. The conditional covariance matrices are positive definite by construction, and the conditional variances are positive, regardless of the parameter signs. The main merit of the BEKK-GARCH model is that resolves the problem of ensuring the positive definiteness of the covariance matrix by using a quadratic form for the conditional covariance equations. But, the fully parameterized BEKK models are feasible only for small values of k , generally less than ten. The interpretation of the estimated parameters is somewhat difficult as they do not correspond to the impact of lagged values of $\varepsilon_t \varepsilon_t'$ and H_t as in the VEC model of Bollerslev et al. (1988). Particularly, the first order BEKK model ($p = q = l = 1$) for bivariate case, $k = 2$, can be written analytically as:

$$\begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 \end{bmatrix} = C'C + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1} \varepsilon_{2,t-1} \\ \varepsilon_{2,t-1} \varepsilon_{1,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \begin{bmatrix} \sigma_{1,t-1}^2 & \sigma_{1,t-1} \sigma_{2,t-1} \\ \sigma_{2,t-1} \sigma_{1,t-1} & \sigma_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

The covariance stationary of the BEKK model is discussed in Engle and Kronker (1995). Recently Boussama et al. (2011) analyzed stationary and ergodicity proprieties using Markov chain theory combined with algebraic geometry. The parameters of the Full BEKK model are estimated by maximizing the Gaussian function of the multivariate process.

4. Data and Preliminary Analysis

The data comprises daily returns for French bonds with different maturity dates and different reference indices:

- **OAT**: (In the French language we say: "Obligations Assimilable au Trésor", the appropriate terms in English could be the "Treasury bonds"). These bonds have become the method of choice for placing the French government's long-term debt. These fungible securities are issued with maturities of seven to 50 years, generally by auction according to an annual calendar published in

advance. In this paper we study two types of conventional bonds, the first with maturity of 2017 and the second with maturity of 2020.

- *OATi*: Indexed bonds are parts of the State's strategy of issuance policy diversification. These are fixed-rate bonds, the principal of which the time of maturity and coupons are paid annually. The *OATi* is protected against French inflation. The reference index for this bond is the consumer price index (CPI) excluding tobacco for all households residing in France.

- *OATei*: In October 2001, the French government issued an *OAT* indexed on the inflation indexed on the euro-zone price index (*OATei*).

In this study, data concerns *OATi* with maturity of 2017 and *OATei* with maturity of 2020. Data are retrieved from Bloomberg, it concerns 1413 days of return observations (283 financial weeks) between January 2003 and May 2012. For each bond, the continuously compounded return is estimated as $r_t = 100[\log(p_t) - \log(p_{t-1})]$, where p_t is the bond price at time t .

Before proceeding to present empirical results, let us discuss general issues which are implicit in almost any statistical analysis of asset returns.

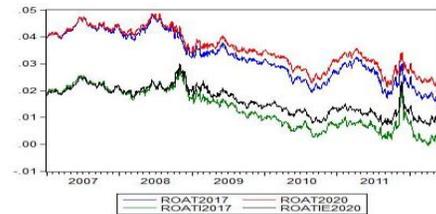


Figure 1. Daily returns of conventional and index-linked bonds (*OAT17*, *OAT20*, *OATi17*, and *OATei20*)

Figure 1 presents the time plot of the series, which fluctuates on a daily basis. The first impression is that the returns of conventional bonds (*OAT17* and *OAT20*) follow a similar movement. The same figure shows a presence of co-movement among returns of index-linked bonds.

Figure 2 displays series in logarithm differentiation, four plots show a presence of significant volatility clustering;

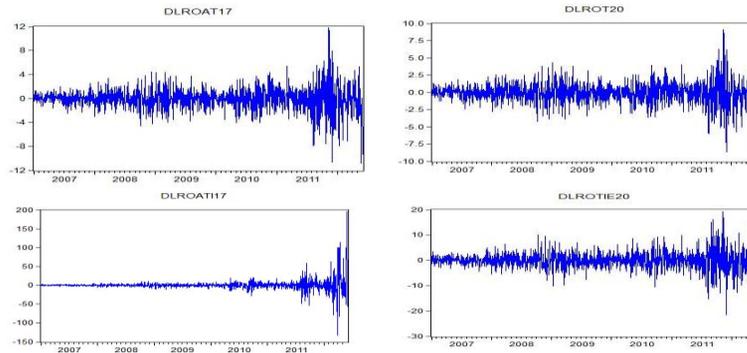


Figure 2. Series in logarithm differentiation for conventional and index-linked bonds returns (*OAT17*, *OAT20*, *OATi17*, and *OATei20*)

We show that large changes are followed by large changes while small changes are followed by small changes. The switching between high and low volatility states is the mechanism leading to volatility clustering. This volatility clustering feature indicates that bond returns are not independent across time.

There is different possible mechanisms which can be considered as origins of this volatility clustering.

Firstly, heterogeneity in agent's time horizons could be considered as a possible source for volatility clustering presented in figure (2). Thus, long term investors, as insurers and hedge funds, obviously focus on long-term behavior of prices, whereas traders intend to take advantage of short-term fluctuations. Granger (1980), suggested that long memory in economic time series can be due to the aggregation of a cross section of time series with different persistence levels. This suggestion was proposed by Andersen & Bollerslev (1997) as a possible justification for volatility clustering in terms of aggregation of different information flows.

Secondly, the volatility clustering can be due to the switching of economic agents between two behavioral patterns leading to large aggregate fluctuations. Thus, in the context of financial markets, these behavioral patterns can be perceived as trading rules and the resulting aggregate fluctuations as large movements in the market price.

In this perspective, the econometric challenge is to identify how the information is employed to forecast the mean and variance of the return, conditional on the past information.

5. Empirical Results

5.1 Volatility Behavior of Bond Returns

A prerequisite to applying the GARCH or EGARCH model is that the underlying bond's returns behave like a GARCH process. We therefore test for the presence of ARCH in the daily returns on the bond. A simple way to test for ARCH, proposed by Bollerslev (1986), is to use the standard Box-Jenkins method, usually applied for the identification of ARMA processes, but in our case applied to the squared series or squared residuals rather than to the series themselves. Table 1 shows that the estimated ARCH parameter is highly significant in four series. Moreover, the statistic TR^2 is also significant for all the different bond returns. These results suggest the presence of ARCH in the data.

Table 1. Estimation Results for ARCH

	<i>OAT17</i>	<i>OATi17</i>	<i>OAT20</i>	<i>OATei20</i>
ARCH(1)	0.089	89.75	0.112	0.0000
p-value	0.0012	0.0000	0.0000	0.0000
TR^2	10.53	25.96	17.16	174.94
p-value	0.0011	0.0000	0.0000	0.0000

These results obtained by the ARCH test supported by the descriptive statistics allow us to proceed by family GARCH models estimation.

Table 2 reports the GARCH and EGARCH estimated parameters. Looking for example, at the results for the *ROAT* and *ROATi* series, we conclude that the parameters which allow the volatility to evolve over time, α in GARCH type model are significant.

Results show, also, that the GARCH processes are strictly stationary ($\alpha + \beta = 1$) for the French and European index-linked bond returns with maturity of 2017. While, the GARCH is covariance stationary ($\alpha + \beta < 1$) for the conventional bond with maturity of 2020.

Table 2. Statistics of parameter estimates for GARCH and EGARCH models

	GARCH				EGARCH			
	<i>OAT17</i>	<i>OAT20</i>	<i>OATi17</i>	<i>OATie20</i>	<i>OAT17</i>	<i>OAT20</i>	<i>OATi17</i>	<i>OATie20</i>
ω	0.0054	0.0073	0.0057	0.0153	-0.066	-0.0705	-0.1140	-0.0670
Std(ω)*	0.0033	0.0038	0.0150	0.0105	0.0100	0.0108	0.0110	0.0100
α	0.0480	0.0530	0.0810	0.0427	0.0913	0.0959	0.1520	0.0950
Std(α)	0.0067	0.0076	0.0094	0.0067	0.0130	0.0146	0.0160	0.0140
β	0.9520	0.9460	0.9300	0.959	0.9960	0.9946	1.0000	0.9970
Std(β)	0.0061	0.0075	0.0080	0.0063	0.0150	0.0021	0.0009	0.0015
θ	-	-	-	-	-0.0277	-0.0307	-0.0053	-0.0230
Std(θ)	-	-	-	-	0.0087	0.0095	0.0090	0.0100

* **Std:** standard deviation

Besides, we show that, with all time series, the stationary condition of the EGARCH model is well supported since the estimated parameter β is less than the unit ($\beta < 1$).

Furthermore, the leverage effect term θ for different type of bond returns is negative and statistically different from zero at level of 1%. This result indicates the existence of the leverage effect in bonds' returns during the sample period. Thus, positive shocks or good news on the bond market generate less volatility, in the subsequent period, than negative shocks or bad news.

The parameter β measures, also, the persistence in conditional volatility regardless of any event happening in the market. In this modeling β is relatively small, then volatility takes a short time to die out following a crisis in the market.

Finally, the comparison between the two conditionally heteroscedastic specifications, using various information criteria, shows that EGARCH model performs very well against GARCH for different bond returns.

Given the existence of a negative leverage effect, we can draw important implications for asset allocation. Obviously, in the short term, investors may do better by holding bonds in their portfolio when the high volatility prevails. If the investor ignores volatility levels, they would hold too much bond for too long period of time and would not be appropriate to shift to another secure investment when the high volatility hits.

Also, the result of the persistence in conditional volatility have important implications for long term investors, as insurers and hedge funds. Since the volatility of bond returns takes a short time to die out following a crisis in the market, investors prefer to invest in bonds rather than other assets to face possible crisis risk on the long horizon.

5.2 The BEKK GARCH Model Results

The BEKK GARCH is applied on the stationary series to investigate the conditional covariance and also conditional correlations. Hence, we use this approach to study the volatility co-movement between conventional and indexed bonds. Table 4 reports parameter estimates for the five BEKK GARCH (1, 1) models. We have to take into consideration that the first model is applied to describe the relationship in uncertainty between conventional bond returns with different maturity. While the second, the third and the fourth are used to test the co-movement of volatility among conventional and index-linked bonds with different maturities. The last model tries to

identify the relationship in volatility among index-linked bond returns with maturities, respectively to 2017 and 2020.

Table 3. Parameter estimates for the BEKK GARCH(1,1)

Variable	<i>OAT17_OAT20</i>	<i>OAT17_OATei20</i>	<i>OAT20_OATi17</i>	<i>OAT20_OATei20</i>	<i>OATi17_OATei20</i>
C_{11}	0.153***	0.098***	0.128***	0.123***	0.096***
Std(C_{11})	(0.0021)	(0.0009)	(0.013)	(0.0017)	(0.0031)
C_{12}	0.143***	0.193***	0.178***	0.249***	0.134***
Std(C_{12})	(0.002)	(0.0015)	(0.080)	(0.018)	(0.0031)
C_{oo}	-1.13e ⁻⁰⁵	-7.99×10 ⁻⁰⁵	-0.00065	0.0347***	0.00056***
Std(C_{oo})	(1.45×10 ⁻⁰⁵)	(0.0014)	(0.342)	(0.014)	(1.10×10 ⁻⁰⁶)
α_{11}	0.423	0.337***	0.227***	0.337***	0.561***
Std(α_{11})	(0.258)	(0.0018)	(0.00059)	(0.0017)	(0.0014)
α_{21}	0.084	-0.060***	-0.57231***	-0.090***	0.0261***
Std(α_{21})	(0.188)	(0.0074)	(0.0066)	(0.0146)	(0.00019)
α_{12}	-0.177	-0.053***	-0.0036***	-0.068***	-0.317***
Std(α_{12})	(0.322)	(0.00084)	(1.46×10 ⁻⁰⁵)	(0.0016)	(0.0017)
α_{22}	0.147	0.22***	0.506***	0.233***	0.188***
Std(α_{22})	(0.237)	(0.0018)	(0.00087)	(0.0044)	(0.00035)
β_{11}	0.960***	0.934***	0.965***	0.924***	0.870***
Std(β_{11})	(0.019)	(0.00016)	(0.00025)	(0.00044)	(0.00023)
β_{21}	0.011	-0.0196***	0.140***	-0.035***	-0.010***
Std(β_{21})	(0.014)	(0.0007)	(0.0022)	(0.006)	(6.16×10 ⁻⁰⁵)
β_{12}	0.0013	0.021***	0.0025***	0.025	0.107***
Std(β_{12})	(0.025)	(6.4×10 ⁻⁰⁵)	(2.04×10 ⁻⁰⁶)	(0.00025)	(0.00024)
β_{22}	0.953***	0.987***	0.902***	0.989***	0.989***
Std(β_{22})	(0.019)	(0.00015)	(0.00011)	(0.00042)	(0.00024)

* **Std:** standard deviation

As shown in the table 3 above, in this bivariate modeling, α_{12} , α_{21} , β_{21} and β_{12} are statistically insignificant rejecting any relationship in volatility among the conventional bonds with different maturities.

The statistical significance of α_{12} , α_{21} , β_{21} and β_{12} for the second, the third and the fourth models shows a strong association between conventional bond volatility and the French index-linked bond one. This strong relationship is also verified among conventional and European index-linked bond returns.

Consequently, we support a significant association, in uncertainty, between conventional and index-linked bonds returns.

Moreover, in the majority of cases, the parameters indicating the correlation in volatility among conventional and index-linked bonds returns are negative. So, the volatility correlation between these two types of bond is negative.

This result have important implication since the diversification in different types of bond will be profitable for investors.

For a better understanding of our results, we have to discuss the sources of risks for the index-linked bond.

While, investors in nominal bonds are manly faced to the volatility of inflation levels, those investing in index-linked bonds could face two types of risk;

First, investors in index-linked bonds are faced to the liquidity risk. The liquidity can be defined as a difference between the fundamental value of a security and the price at which the security is actually transacted at. Then a bond “A” is more liquid than a bond “B” when this gap for the first is smaller than the second. The main problem is that liquidity is correlated to trading, and the downside risk associated with liquidity is that very little trading exists in a particular security that an investor wishes to buy or sell. Indeed, the liquidity of a market or an asset is measured by both transactions and amounts outstanding.

By using the two previous measures, different studies show that the index-linked bonds are much less liquid than the nominal bonds. This result is confirmed for the US Treasury Inflation-Protected Securities (TIPS), UK inflation-linked Gilts and the French index-linked bonds (*OATi*).

Second, investors in index-linked bonds are faced to the inflation risk. It is commonly known that an inflation-indexed bond protects both investors and issuers from the uncertainty of inflation over the life of the bond. Unfortunately it is noticed that a perfect indexing and full protection from inflation is not possible also for the inflation-indexed bond. Since the practical necessity of lags in indexation would have a noticeable effect on the benefits from returns on index-linked bonds. Furthermore, it is shown that taxation could reintroduce inflation risk to indexed bonds. Consequently, the index-linked bonds are not free from inflation because of the indexation method and taxation.

These two types of risk can be considered as sources of volatility in index-linked bonds returns. Moreover, the existence of the previous types of risk for the conventional and index-linked bonds can be considered as a source of correlation in volatility returns of these bonds.

With reference to the previous results, we suggest that the main source of relationship in volatility is due to the existence of liquidity risk. This can be argued by the fact that correlation in volatility of nominal bonds is insignificant and thus the inflation risk can't be considered as a factor explaining this correlation.

Even more interesting is to observe the co-persistence in variance for each multivariate GARCH models.

As mentioned by Bollerslev and Engel (1993), while many financial time series appear to be persistent in variance, certain linear combinations of the variables may have no persistence, so that the variables exhibit a common long run component.

By definition, the multivariate stochastic process $\{y_t\}$ is said to be co-persistent in variance if there exist a vector $\gamma \in \mathfrak{R}^N$ such that $\{vec2(\lambda)\}_i \neq 0$ and $\limsup_{t \rightarrow \infty} \left| \left\{ H_t^*(s) \right\}_i \right| \neq 0$. Expressly, the

co-persistence in variance can be determined by calculating the largest eigenvalue of $A \otimes A + B \otimes B$. The estimated largest eigenvalue are reported in table 4.

Table 4. The estimated largest eigenvalues

	<i>OAT17_OAT20</i>	<i>OAT17_OATei20</i>	<i>OAT20_OATi17</i>	<i>OAT20_OATei20</i>	<i>OATi17_OATei20</i>
Largest eigenvalue	3.6637×10^9	4.0634×10^7	3.8693×10^8	3.1512×10^7	1.5997×10^8

Results show that the estimated eigenvalue for the five models are non zero which indicate a co-persistence in variance. Thus, whereas the effect of a shock in terms of the optimal forecasts for each of the conditional variance and the covariance of indexed and nominal bonds persist or die out very slowly, the long-run forecasts are tied together.

The presence of the volatility co-persistence between conventional and indexed bond returns have direct implication on the investor's decisions. Indeed, by calculating the co-persistence area, investors can eliminate persistence by reallocating their portfolios. It is noticed that the computation of the co-persistence area is somewhat difficult, however, some algorithms are useful, see Lee (2012).

Finally, from a financial perspective, the presence of co-persistence is very important in the understanding of bond pricing relationship.

Hence, the price of a bond typically depends on the conditional covariance with a benchmark portfolio, so that the pricing of long-term contracts may differ radically from that one period contracts if shocks to the conditional variance or covariance with the benchmark portfolio persist. At the same time, the pricing of linear combinations of the bonds may not be very sensitive to current information if the involved bonds show co-persistence in variance.

6. Conclusion

In this paper, we examine the volatility linkage between conventional and index-linked bonds returns by modeling their dependence using a multivariate GARCH. We found a significant and strong volatility linkage among the studied bond returns. We suggested that this linkage is mainly due to the existence of liquidity risk in bond markets.

We, also found a presence of co-persistence of the volatility returns for conventional and index-linked bonds. This co-persistence is of great importance for investors because it helps them to reallocate their portfolios.

This empirical study shows, also, that the BEKK-GARCH model is an appropriate tool to study the volatility linkage and the co-persistence among bonds returns. However, modelling the volatility linkage using BEKK-GARCH is certainly only one of many plausible multivariate models. Mainly, this multivariate GARCH does not model the dynamic conditional correlation among bond returns. A DCC-GARCH specification allowing for dynamic correlation investigation will be discussed in further work.

Moreover, multivariate GARCH models did not care about the structure of the volatility dependence of returns. A copula approach may be suitable to investigate discuss this problem.

Finally, from an economic point of view, it would be interesting to investigate the compensation between liquidity and inflation risk premium for nominal and index-linked bonds.

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