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An Integrated Model for the Cost-Minimizing Funding of Corporate Activities over Time

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Abstract: To enhance the value of a firm, the firm's management must attempt to minimize the total discounted cost of financing over a planning horizon. Unfortunately, the variety of sources of funds and the constraints that may be imposed on accessing funds from any one source make this exercise a difficult task. The model presented and illustrated here accomplishes this task considering issuing new equity and new bonds, refunding the bonds, borrowing short-term from financial institutions, temporarily parking surplus funds in short-term securities, repurchasing its stock, and retaining part or all of a firm's earnings. The proportions of these sources of funds are determined subject to their associated costs and various constraints such as not exceeding a specific debt/equity ratio and following a stable dividend policy, among others.

Keywords: Cost-minimization; Discounted value; Financing costs; Financial constraints; Funding decisions; Funding requirements; Optimal financial policy; Optimization; Planning horizon; Sources of funds

JEL Classifications: C61, G32, G35

1. Introduction

A principal problem for managers of a firm is to provide funds over successive periods of time for its planned operations at the lowest possible cost of financing. This financing cost-minimization problem, however, remains largely unresolved because the management has a vast array of sources available to obtain funding whose costs differ and vary over time, thus compounding the financing problem (Baker and Wurgler, 2002, Hovakimian, Opler, and Titman, 2001, Huang and Ritter, 2009, Leary and Roberts, 2005, and Welsh, 2004).

Furthermore, managers also face a variety of constraints in using these sources of funds that may be either fixed or vary with time. Such constraints may include paying a desired dividend over successive periods of time, maintaining a permanently constant debt-to-equity ratio or one that varies over time, or borrowing for a short term at any time, or maintaining a specific ratio of short-term to long-term borrowings within a predefined limit. These constraints need to be satisfied in each time interval. In addition, there are consequences and costs associated with failing to meet these constraints.

Limited attempts have been made in the literature to handle the decision-making on one or another of these sources of funds. Concentrating on only one or two of the several sources available results in sub-optimal decisions in providing the firm with the funds it requires.

This paper presents a model designed to tackle the problem of financing current and projected activities of a firm at the lowest present value cost of financing. Factors associated with financing activities, such as new equity issues, bond issues, bond refunding, short-term borrowings,

investment of surplus funds, payment of desired dividends, stock repurchases, and their inter-relationships are simultaneously taken into consideration over time.

The second section of this article presents a review of the literature addressing the application of optimization techniques to finance. In the third section the basics of the model are developed. In the fourth section the model is illustrated. A numerical illustration and its solution are shown in the fifth section. The conclusions follow in section six.

2. Literature Review

Past attempts in the literature mostly concentrated on one or at most a few of the variables needed to be considered in handling the funding decisions. For instance, Morellec (2001) and Sibilkov (2009) address the problem of asset liquidity. Dhillon, Noe, and Ramirez (2001) and King and Mauer (2000) take the problem of bond calls into account. Bates, Kahle, and Stulz (2009), Lie (2000), Mikkelson and Partch (2003), and Opler, et al. (1999) discuss the problem of corporate cash holdings. In addition, Chen, et al. (2013) and Lee, et al. (2015) consider the problem of dividend policy. While important, concentrating on only one or two of the several variables that need to be considered is likely to result in sub-optimal financing decisions.

The managerial problem of deciding the optimal amount and timing of undertaking the financing operations has been developed as a problem in mathematical programming. Ashton and Atkins (1979), Carlton (1970), Chambers (1971), Hamilton and Moses (1973), Lee, Lee, and Lee (2009), Myers and Pogue (1974), and Weingartner (1966) provide mathematical programming procedures to optimally fund a firm over a given planning horizon that takes into consideration the different costs associated with each funding source and the restrictions imposed on the firm to ensure payment on its obligations.

Much of the early applications of mathematical programming were concentrated in the area of production and resource allocation. Among the first studies to apply mathematical programming to financial planning under conditions of certainty were the studies of Charnes, Cooper, and Miller (1959) and Robichek, Teichroew, and Jones (1965). Models combining the investment decisions with allocating funds from different sources of financing began to appear soon afterwards. For instance, Myers and Pogue (1974) developed a mixed-integer linear programming model that simultaneously considers the investment, financial, and dividend options facing the firm in optimizing financing. Such models were then used to develop multi-period financial policies (Brick, et al., 1983, and Mulvey and Vladimirou, 1992).

Unfortunately, there have been few practical applications of these procedures reported (Crum, Klingman, and Tavis, 1979, and Fox, 1971). We attempt to provide a model that minimizes the present value of the costs of a firm's financing and give an illustration of how this model may be applied. The main point of the model presented here is that it takes into account all of the variables that need to be considered simultaneously in a very comprehensive fashion so that the financing decision will be optimal.

3. The Model

The problem we attempt to resolve in this paper can be stated briefly: Given a finite length of time, the projected funding requirements, and the profits of the firm for each time interval, the problem is to determine the optimal time sequence and the amount of funding from each source that is available. The objective is to minimize the total discounted cost of meeting the funding requirements over the entire planning horizon. This is to be achieved while satisfying the managerial constraints on the debt/equity mix and the payout policy.

To carry out the analysis in a multi-period framework, let the planning horizon be divided into N discreet intervals as shown in the following diagram:

Time	t_0	t_1	t_2	t_3	$\cdots$	t_{i-1}	t_i	$\cdots$	t_{N-1}	t_N
Interval No.	1	2	3		$\cdots$		i	$\cdots$		N

where t_0 is the initial time. For any time interval let the decision be made at the beginning of that time interval. For instance, any decision for interval 2 is made at the time t_1. Also, let a state vector $\mathbf{Y}_i$ be defined as:

$$\mathbf{Y}_i = [Y_{1,i}, Y_{2,i}, Y_{3,i}]'$$

where $Y_{1,i}$ = the amount of stock issued in the interval i in dollars ($)

$Y_{2,i}$ = the amount of treasury stock bought in the interval i ($)

$Y_{3,i}$ = amount (positive or negative) of the earnings retained in the interval i ($).

First, the cumulative equity funds available through stock sales over i intervals of time are:

$$\sum_{k=0}^{i} Y_{1,k} \text{ ,where } Y_{1,k} \geq 0 \text{ for all } k \text{ and } i = 1, \ldots, N \qquad \text{(la)}$$

$\boldsymbol{Y}_1 = [Y_{1,1}, Y_{1,2}, \ldots, Y_{1,N}]'$ is the vector of new equity issued over the N period planning horizon. $Y_{1,0}$ is the existing equity at the beginning of time interval 1. A positive value of $Y_{1,k}$ indicates that new equity is issued in the interval k and a zero value of $Y_{1,k}$ is indicative of no equity being issued in that time interval.

Second, a firm can buy back its own equity in the market. Let $Y_{2,i}$ be defined as the treasury stock acquired in the time interval i such that $\boldsymbol{Y}_2 = [Y_{2,1}, Y_{2,2}, Y_{2,3}, \cdots; Y_{2,N}]'$ is the vector of treasury stock acquisition over the N period planning horizon. If no treasury stock is acquired in any time interval k, $Y_{2,k}$ is then zero. Thus the cumulative treasury stock acquired over i intervals of time is given by the expression (lb).

$$\sum_{k=0}^{i} Y_{2,k} \text{ , where } Y_{2,k} \geq 0 \text{ for all } k \text{ and } i = 1, \ldots, N \qquad \text{(lb)}$$

It is not considered desirable, though probably permissible, that the cumulative treasury stock acquired at any time should exceed the cumulative retained earnings. Thus, the constraint on treasury stock acquisition is:

$$\phi \sum_{k=0}^{i} Y_{3,k} - \sum_{k=0}^{i} Y_{2,k} \geq 0 \quad \text{for all } i \qquad \text{(1c)}$$

where ϕ is a constant greater than zero and less than one. In general, firms use only a portion, ϕ at most, of their retained earnings for acquiring treasury stock.

Third, the cumulative funds available from retained earnings over i time intervals are:

$$\sum_{k=0}^{i} Y_{3,k} \text{ ,} \quad \text{for all } i = 1, \ldots, N. \qquad \text{(ld)}$$

where $\mathbf{Y_3} = [Y_{3,1}, Y_{3,2}, Y_{3,3}, \ldots, Y_{3,N}]'$ is the vector of retained earnings over the N period planning horizon. The retained earnings in time interval i may be expressed as:

$$Y_{3,i} = E_i - Z_i \tag{1e}$$

where E_i is the earnings for the time interval i and Z_i is the dividends paid during the interval. Thus the incremental retained earnings for that interval equals the difference between the earnings, E_i, and the dividends paid, Z_i.

Also, a dividend is paid in any time interval i only if the sum of retained earnings (in excess of treasury stock acquisition) through the time interval i and the earnings for that time interval, E_i, are positive. That is,

$$Z_i > 0 \qquad \text{if } \sum_{k=0}^{i}(Y_{3,k} - Y_{2,k}) + E_i > 0 \tag{lf}$$

The total equity funds available over any time interval i is the sum of the expressions (la) and (1d),

$$\text{i.e.} \qquad \sum_{k=0}^{i} Y_{1,k} + \sum_{k=0}^{i} Y_{3,k} \tag{1g}$$

For bond financing, the following logical assumptions are: (i) If in any time interval a bond is issued, only a single issue is made, and (ii) any time a bond is called, the entire issue is called. Now, let us define a state vector $\mathbf{X}_i$ as follows:

$$\mathbf{X}_i = [X_{1,i}, X_{2,i}, X_{3,i}, \cdots, X_{N_0,i}, X_{N_0+1,i}, \cdots, X_{N',i}]'$$

where $N' = N_0 + N$ and each element $X_{j,i}$ represents the number of bonds in the jth issue outstanding in the interval i. The first N_0 terms of the state vector $\boldsymbol{X}_i$ represent the N_0 bonds previously outstanding at time $t = t_0$. The last N terms of the state vector represent the bonds to be issued in the succeeding N time intervals, one issue per each interval. For example, $X_{N_0+1,i}$ corresponds to the bond issued in the first interval.

If no bond is issued in the first interval, then $X_{N_0+1,i}$ is zero for all i greater than or equal to 1. Also, if this particular bond is called in interval i-1, then $X_{N_0+1,i}$ also becomes zero. From this, it should be clear that in general:

$$X_0 = \left[X_{1,0}, X_{2,0}, X_{3,0}, \cdots, X_{N_0,0}, 0, 0, \cdots, 0\right]'$$

$$X_1 = \left[X_{1,1}, X_{2,1}, X_{3,1}, \cdots, X_{N_0,1}, X_{N_0+1,1}, 0, 0, \cdots, 0\right]'$$

$$\vdots$$

$$X_i = \left[X_{1,i}, X_{2,i}, X_{3,i}, \cdots, X_{N_0,i}, X_{N_0+1,i}, \cdots, X_{N_0+i,i}, 0, 0, \cdots, 0\right]'$$

$$\vdots$$

$$X_N = \left[X_{1,N}, X_{2,N}, X_{3,N}, \cdots, X_{N_0,N}, X_{N_0+1,N}, \cdots, X_{N_0+N,N}\right]'$$

The relationship between the state vectors $\boldsymbol{X}_i$ and $\boldsymbol{X}_{i\text{-}1}$ is as follows:

$X_{j,i} = X_{j,i-1}$ if a bond j is not issued, called, or matured in the interval i

$= 0$ if the bond is called in the interval i or $j > N_0+i$, the bond has matured or was never issued.

$= X_{j,i}$ if $j = N_0+i$ and a bond is issued in interval i. In this case $X_{j,i-1}= 0$.

Also, in each interval of time, the maturity characteristics of each bond outstanding in the firm's bond portfolio can be determined as follows. Let the maturity of a bond j in the interval i be $m_{j,i}$. Bond j is issued in the interval $(j - N_0) = k$. Since M_k is the maturity of the bond issued in the interval k, we can write:

$m_{j,i} = M_k - (i - k)$ for all $j > N_0$

$= m_{j,0} - i$ for all $j < N_0$

where $m_{j,0}$ is the maturity of the bond j outstanding at $t = t_0$.

The funds available through all the outstanding bonds in the interval i are:

$$V\sum_{j=1}^{N'} X_{j,i} \tag{2}$$

where V is the face value of one bond.

The total funds available from long-term sources of bonds and equity at any time interval i are defined as B_i. Then

$$B_i = \sum_{k=0}^{i} Y_{1,k} + \sum_{k=0}^{i} Y_{3,k} + V\sum_{j=1}^{N'} X_{j,i} \tag{3}$$

Let us define the cumulative requirements, D_i, in interval i as follows:

$$D_i = \sum_{k=1}^{i} R_k \tag{4}$$

where $\boldsymbol{R} = [R_1, R_2, R_3, ..., R_N]'$ is the vector of funding requirements over the N period planning horizon.

3.1 The costs of funds

1. *Flotation Costs*. If g_i and g_c are the variable and the fixed flotation costs, respectively, the total equity flotation cost G_i is:

$$G_i = (g_c + g_i Y_{1,i})\sigma_i \text{, where } \sigma_i = 1 \text{ if } Y_{1,i} > 0 \tag{5}$$
$$= 0 \text{ otherwise}$$

The bond flotation cost in general is lower than the equity flotation cost. Let α and β be the fixed and the variable flotation costs for bonds, respectively. Then the total bond flotation cost F_i in any given time interval is:

$$F_i = [\alpha + \beta(X_{N_0+i,i})]\delta_i \text{, where } \delta_i = 1 \text{ if } X_{N_0+i,i} > 0 \tag{6}$$
$$= 0 \text{ otherwise}$$

2. *Repurchasing Costs*. When a company repurchases its stock, there is a fixed cost, η, involved. Also, the company generally repurchases its stock at a premium, ω, to the market value of the equity repurchased. Thus the repurchase cost of equity is:

$$U_i = [\eta + \omega(Y_{2,i})]\theta_i \text{ , where } \theta_i = 1 \text{ if } Y_{2,i} > 0 \qquad (7)$$
$$= 0 \text{ otherwise}$$

3. *The Equity and Interest Costs*. Let s_i be the ($/$) cost of equity for the firm for the time interval i. Then the total equity cost for time interval i will be I_i where

$$I_i = \sum_{k=0}^{i} (Y_{1,k} + Y_{3,k}) s_i \text{ , where } i = 1, \ldots, N \qquad (8)$$

A coupon rate c_j has to be paid on the jth bond issue if it is outstanding at the time interval i. Thus the total interest cost on long-term bonds payable at the time interval i, C_i, becomes:

$$C_i = \sum_{j=1}^{N'} c_j X_{j,i} \qquad (9)$$

If any bond j is called in the interval i, then the corresponding $X_{j,i}$ becomes zero and no more interest is payable on this bond. The call premium H_i becomes payable in the time interval as the jth bond is called. Thus,

$$H_i = \sum_{j=1}^{N'} (X_{j,i-1}) e_{j,i} \quad \text{for all } X_{j,i} = 0 \text{ and } m_{j,i} \neq 0 \qquad (10)$$

where $e_{j,i}$ is the ($/bond) call premium rate for the jth bond issue. The call premium $e_{j,i}$ is known to be a declining function of the time to maturity and the coupon rate of the bond (Weingartner, 1967). A precise analytical expression for $e_{j,i}$ is needed, but for the purposes of this model a call premium schedule over time would suffice.

3.2 Bonus & penalty costs

Two types of penalty/bonus costs are taken into consideration:

(a) A penalty is incurred (bonus obtained) if the funds required (D_i) are greater than (less than) the funds available from all the long-term sources (B_i). Thus,

$$L_i = (D_i - B_i)\rho_i \text{ , where } \rho_i = b_i \text{ if } B_i > D_i$$
$$\rho_i = p_i \text{ if } D_i > B_i \qquad (11)$$

where L_i is a bonus if $B_i > D_i$ and a penalty if $D_i > B_i$.

The bonus rate is the short-term interest rate in the market less the transaction costs of lending. In general, the bonus rate (b_i) is less than the required rate of return on bonds (c_j). This will rule out the possibility of the firm indulging in excessive long-term borrowing only to lend short term. The penalty rate (p_i) is the cost of borrowing short-term. Even if the nominal short-term borrowing rate is less than the long-term rate, the effective rate on short-term loans is generally more because (i) banks are reluctant to let the firms borrow short-term for long-term needs and often impose constraints on the firms to prevent this, (ii) banks often require the borrowers to maintain compensating balances that further increases the effective rate on short-term loans, and (iii) it is not considered prudent for firms to depend heavily on short-term loans that may not become readily available should the money market conditions turn tight (Gupta, 1972, and Houston and Venkataraman, 1994). Furthermore, the promotion of tax benefits, the risk-induced accumulation of capital reserves, and the volatility of assets favor long-term debt over short-term debt (see Boyce and Kalotay, 1979, Brick and Ravid, 1991, Jun and Jen, 2003, and Wiggins, 1990). Therefore, a

penalty rate (p_i) is charged in case the long-term borrowings fall short of the desired level and is assumed to be greater than c_i, i.e. $p_i > c_i > b_i$.

(b) The firm may have a desired level of dividends it would like to pay in each interval ($Z_{des,i}$). Most firms want to follow a stable dividend policy. Changes in dividend payouts are a means by which the managers of a corporation signal its future prospects to the investment community. Shareholders have less information about the prospects for the corporation than do the managers. As such, an announced increase in the dividends is likely to be viewed positively by shareholders, and a dividend decrease will be viewed negatively by financial market participants. The management of a corporation will increase dividends only if it thinks future earnings will be sufficient to sustain the higher dividend level. Conversely, the management will attempt to avoid reductions in the dividend payments to prevent a negative equity market reaction (Bhattacharya, 1979, Miller and Rock, 1985, Ross, 1977, and Ross, 1978). Thus a penalty should be charged in the design of the model if management cannot maintain the level of dividends.

We introduce a penalty cost indicating the weight management attaches to maintaining the desired level of dividends should financing cost considerations dictate the payment of dividends lower than $Z_{des,i}$. However, the penalty should not be charged if the cumulative retained earnings plus the earnings in the current period are not enough to pay the desired level of dividends. Also, the penalty should not be charged if the management decides to pay more dividends than $Z_{des,i}$. Thus,

$$P_i = (Z_{des,i} - Z_i)\, p_{di}\,, \qquad \text{if} \quad Z_{des,i} > Z_i \text{ and } \sum_{k=0}^{i}(Y_{3,k} - Y_{2,k}) + E_i > Z_{des,i}$$
$$= 0\,, \qquad \text{otherwise} \qquad (12)$$

The penalty rate p_{di} should be greater than s_i. Otherwise the model will tend not to pay any dividends.

In addition to bonus/penalty costs, additional constraints on working out a cost-minimizing financial policy for the firm may be imposed by the management. One such constraint, as suggested by Hovakimian, Opler, and Titman (2001), could be that the debt/equity ratio should not exceed a predetermined maximum level, say RDE_{max}, and always should be greater than a predetermined minimum level, say RDE_{min}. The fact is that managements do try to maintain a specific debt/equity ratio and try to keep that ratio within a specific range. Determination of the optimal debt/equity ratio is beyond the scope of the paper. Nevertheless, the model accepts a range of debt/equity ratios desired by the management in providing for the requisite funds at the minimum cost.

The RDE_{max} and RDE_{min} may differ from firm to firm or industry to industry that reflect industry norms, management risk orientation, and management thoughts of what is appropriate for the firm. In our notation, such a constraint will take the form of:

$$RDE_{\min} \le \left[\{D_i - \sum_{k=0}^{i}(Y_{1,k} + Y_{3,k})\} / \sum_{k=0}^{i}(Y_{1,k} + Y_{3,k}) \right] \le RDE_{\max} \qquad (13)$$

Other logical constraints can also be specified by the management. For example, as Heine and Harbus (2002) and Hovakimian, Opler and Titman (2002) point out, the short-term borrowing ability of the firm at any time interval may be severely limited because of volatile earnings, restricted credit, or excessive short-term borrowing relative to long-term borrowing.

3.3 The objective function

Having written down all the cost factors for the interval i, we can now write the total cost T_i for the interval i as follows:

$$\begin{aligned} T_i &= G_i + I_i + U_i + P_i + F_i + C_i + H_i + L_i \\ &= \sigma_i(g_c + g_i Y_{1,i}) + \sum_{k=0}^{i}(Y_{1,k} + Y_{3,k})s_i + \theta_i(\eta + \omega Y_{2,i}) + \mu_i\{Z_{des,i} - (E_i - Y_{3,i})\}\cdot p_{di} \\ &+ \delta_i(\alpha + \beta X_{N_0+i,i}) + \sum_{j=1}^{N'} c_j X_{j,i} + \sum_{j=1}^{N'}(X_{j,i-1})\cdot e_{j,i}\cdot \varepsilon_{j,i} + \rho_i(D_i - B_i) \end{aligned} \tag{14}$$

The indicator parameter for the penalty cost, P_i, is:

$\mu_i = 1$ if $\sum_{k=0}^{i}(Y_{3,k} - Y_{2,k}) + E_i > Z_{des,i}$

$= 0$ otherwise.

Also, $\mu_i = 0$ if $(E_i - Y_{3,i}) > Z_{des,i}$

In addition, the indicator parameter for the call premiums on bonds paid, H_i, is:

$\varepsilon_{j,i} = 1$ if $X_{j,i} = 0$

$= 0$ otherwise

3.4 Discounting of total cost, *T*, in the interval *i* to its present value

If we define d_c as an appropriate discounting rate, then the discounted total cost, T'_i, is:

$$T_i' = T_i/(1+d_c)^i \tag{15}$$

If the discount rate, d_c, is non-stationary over time, then

$$T_i' = T_i / \Pi_{k=1}^{i}(1+d_k) \tag{16}$$

Our objective is to minimize the discounted total cost over the planning horizon, i.e.

$$\text{Minimize } \sum_{i=1}^{N} T_i' \tag{17}$$

subject to the condition given by equation (14) and the following constraints:

1. $X_{j,i} \geq 0$ for all j and i, also it is an integer number.
2. $Y_{1,i} \geq 0$ for all i
3. At any time i, the ratio

[Total bonds + short-term borrowing] / [stock (net of treasury stock) + Retained earnings] should be greater than some RDE_{min} and smaller than RDE_{max}.

$$RDE_{\min} \leq \left[\{D_i - \sum_{k=0}^{i}(Y_{1,k} + Y_{3,k})\} / \sum_{k=0}^{i}(Y_{1,k} + Y_{3,k})\right] \leq RDE_{\max}$$

4. Illustration

To understand how the optimization of the model described in the preceding section can be carried out and what the output of the model would be, consider a simple illustration with only four time intervals. The time scale with corresponding state vectors is shown below.

t_0	t_1	t_2	t_3	t_4
$\mathbf{Y}_0$	$\mathbf{Y}_1$	$\mathbf{Y}_2$	$\mathbf{Y}_3$	$\mathbf{Y}_4$
$\mathbf{X}_0$	$\mathbf{X}_1$	$\mathbf{X}_2$	$\mathbf{X}_3$	$\mathbf{X}_4$

Assume the following for the illustration:

1. Initially, at $t = 0$, there is some stock existing, but there is no retained earnings and no treasury stock.
2. Also, at $t = 0$, there is only one bond outstanding, i.e. $N_0 = 1$. This bond cannot be called within the first three intervals and it has a coupon rate c_0.
3. For simplicity, assume that c_i, the coupon rate for the bonds issued in the interval i, is known as a function of r_i, the market interest rate in the interval i. Also, when the bond is issued, it is issued at a fixed maturity, M_0. It is also assumed that the stock cost rate s_k is known for all the intervals.
4. The call premium schedule is such that the bond issued cannot be called for the first four intervals.

N', the dimension of the state vector for this problem, is $N_0 + N = 1 + 4 = 5$. The state vectors **X** and **Y** in general are:

$$\mathbf{Y}_0 = \left[y_{1,0}, y_{2,0}, y_{3,0} \right]' \qquad \mathbf{X}_0 = \left[x_{1,0}, 0, 0, 0, 0 \right]'$$

$$\mathbf{Y}_1 = \left[y_{1,1}, y_{2,1}, y_{3,1} \right]' \qquad \mathbf{X}_1 = \left[x_{1,1}, x_{2,1}, 0, 0, 0 \right]'$$

$$\mathbf{Y}_2 = \left[y_{1,2}, y_{2,2}, y_{3,2} \right]' \qquad \mathbf{X}_2 = \left[x_{1,2}, x_{2,2}, x_{3,2}, 0, 0 \right]'$$

$$\mathbf{Y}_3 = \left[y_{1,3}, y_{2,3}, y_{3,3} \right]' \qquad \mathbf{X}_3 = \left[x_{1,3}, x_{2,3}, x_{3,3}, x_{4,3}, 0 \right]'$$

$$\mathbf{Y}_4 = \left[y_{1,4}, y_{2,4}, y_{3,4} \right]' \qquad \mathbf{X}_4 = \left[x_{1,4}, x_{2,4}, x_{3,4}, x_{4,4}, x_{5,4} \right]'$$

where the elements of the state vectors **X** and **Y** have the following meanings:

$y_{1,0}$ = stock existing in the beginning.

$x_{1,0}$ = the number of bonds outstanding in the beginning.

$y_{1,1}$, $y_{1,2}$, $y_{1,3}$, and $y_{1,4}$ are the stocks issued in intervals 1, 2, 3, and 4, respectively.

$y_{2,1}$, $y_{2,2}$, $y_{2,3}$, and $y_{2,4}$ are the treasury stocks acquired in intervals 1, 2, 3, and 4, respectively.

$y_{3,1}$, $y_{3,2}$, $y_{3,3}$, and $y_{3,4}$ are the retained earnings in intervals 1, 2, 3, and 4, respectively.

$x_{2,1}$, $x_{3,2}$, $x_{4,3}$, and $x_{5,4}$ are the bonds issued in intervals 1, 2, 3, and 4, respectively.

Our objective is to determine all the state vectors $\mathbf{Y}_i$s and $\mathbf{X}_i$s that will minimize the total cost.

For the illustration, the state vectors simplify as follows:

$$\mathbf{Y}_0 = \left[y_{1,0}, 0, 0 \right]' \qquad \mathbf{X}_0 = \left[x_{1,0}, 0, 0, 0, 0 \right]'$$

$$\mathbf{Y}_1 = \left[y_{1,1}, 0, 0 \right]' \qquad \mathbf{X}_1 = \left[x_{1,0}, x_{2,1}, 0, 0, 0 \right]'$$

$$\mathbf{Y}_2 = \left[y_{1,2}, y_{2,2}, y_{3,2} \right]' \qquad \mathbf{X}_2 = \left[x_{1,0}, x_{2,1}, x_{3,2}, 0, 0 \right]'$$

$$\mathbf{Y}_3 = \left[y_{1,3}, y_{2,3}, y_{3,3} \right]' \qquad \mathbf{X}_3 = \left[x_{1,0}, x_{2,1}, x_{3,2}, x_{4,3}, 0 \right]'$$

$$\mathbf{Y}_4 = \left[y_{1,4}, y_{2,4}, y_{3,4} \right]' \qquad \mathbf{X}_4 = \left[x_{1,4}, x_{2,1}, x_{3,2}, x_{4,3}, x_{5,4} \right]'$$

In terms of the elements of the state vectors $\mathbf{X}_i$s and $\mathbf{Y}_i$s, the objective function is evaluated as below.

$$T_1^{'} = [\sigma_1(g_c + g_1 Y_{1,1}) + s_1(Y_{1,0} + Y_{1,1}) + \mu_1(Z_{des1} - Z_1)\,\rho_{d1}$$
$$+ \delta_1(\alpha + \beta X_{2,1}) + (c_0 X_{1,0} + c_1 X_{2,1}) + \rho_1(D_1 - B_1)]/(1 + d_{c1})$$

$$T_2^{'} = [\sigma_2(g_c + g_2 Y_{1,2}) + s_2(Y_{1,0} + Y_{1,1} + Y_{1,2} - Y_{2,2}) + \mu_2(Z_{des2} - Z_2)\rho_{d2}$$
$$+ \delta_2(\alpha + \beta X_{3,2}) + (c_0 X_{1,0} + c_1 X_{3,2}) + \rho_2(D_2 - B_2)]/(1 + d_{c1})(1 + d_{c2})$$

$$T_3^{'} = [\sigma_3(g_c + g_3 Y_{1,3}) + s_3(Y_{1,0} + Y_{1,1} + Y_{1,3} - Y_{2,2} - Y_{2,3}) + \mu_3(Z_{des3} - Z_3)\rho_{d3}$$
$$+ \delta_3(\alpha + \beta X_{4,3}) + (c_0 X_{1,0} + c_1 X_{2,1} + c_2 X_{3,2} + c_3 X_{4,3})$$
$$+ \rho_3(D_3 - B_3)]/(1 + d_{c1})(1 + d_{c2})(1 + d_{c3})$$

$$T_4^{'} = [\sigma_4(g_c + g_4 Y_{1,4}) + s_4(Y_{1,0} + Y_{1,1} + Y_{1,2} + Y_{1,3} + Y_{1,4} - Y_{2,2} - Y_{2,3} - Y_{2,4})$$
$$+ \mu_4(Z_{des4} - Z_4)\rho_{d4} + \delta_4(\alpha + \beta X_{5,4}) + (c_0 X_{1,4} + c_1 X_{2,1} + c_2 X_{3,2} + c_3 X_{4,3} + c_4 X_{5,4})$$
$$+ e_{1,4} X_{1,0} + \rho_4(D_4 - B_4)]/(1 + d_{c1})(1 + d_{c2})(1 + d_{c3})(1 + d_{c4})$$

$$T^{'} = T_1^{'} + T_2^{'} + T_3^{'} + T_4^{'} \tag{19}$$

where $B_1 = \mathrm{V}(X_{1,0} + X_{2,1}) + (Y_{1,0} + Y_{1,1})$

$B_2 = \mathrm{V}(X_{1,0} + X_{2,1} + X_{3,2}) + (Y_{1,0} + Y_{1,1} + Y_{1,2}) + Y_{3,2} - Y_{2,2}$

$B_3 = \mathrm{V}(X_{1,0} + X_{2,1} + X_{3,2} + X_{4,3}) + (Y_{1,0} + Y_{1,1} + Y_{1,2} + Y_{1,3})$
$\quad - (Y_{2,2} + Y_{2,3}) + (Y_{3,2} + Y_{3,3})$

$B_4 = \mathrm{V}(X_{1,4} + X_{2,1} + X_{3,2} + X_{4,3} + X_{5,4}) + (Y_{1,0} + Y_{1,1} + Y_{1,2} + Y_{1,3} + Y_{1,4})$
$\quad - (Y_{2,2} + Y_{2,3} + Y_{2,4}) + Y_{3,2} + Y_{3,3} + Y_{3,4}$

$Z_2 = E_1 - Y_{3,2}$, $\quad Z_3 = E_2 - Y_{3,3}$, $\quad Z_4 = E_3 - Y_{3,4}$, and

$$\mu_i = 0 \qquad \text{if } Z_i > Z_{des,i} \text{ or } \left[\sum_{k=0}^{i} (y_{3,k} - y_{2,k}) + E_i \right] < Z_{des,i}$$
$$= 1 \qquad \text{otherwise}$$

$X_{1,4} = 0$ or $X_{1,0}$ (either the initial number of bonds issued remain outstanding in the final interval or the call provision is exercised)

$e_{1,4} = 0$ if $X_{1,4} = X_{1,0}$, or

$= e_{1,4}$ if $X_{1,4} = 0$

We want to minimize T' given by Equation (19) by finding the optimum values of $Y_{1,1}$, $Y_{1,2}$, $Y_{1,3}$, $Y_{1,4}$, $Y_{2,2}$, $Y_{2,3}$, $Y_{2,4}$, $Y_{3,2}$, $Y_{3,3}$, $Y_{3,4}$, $X_{1,0}$, $X_{2,1}$, $X_{3,2}$, $X_{4,3}$, and $X_{5,4}$, subject to the following constraints:

1. $X_{j,i} \geq 0$ for $i = 1, \ldots, 4$ Also, it is an integer value and $j = 2, \ldots, 5$
2. $RDE_{MIN} \leq \left\{ D_i - \sum_{k=0}^{i} (Y_{1,k} + Y_{3,k}) \right\} \Big/ \left\{ \sum_{k=0}^{i} (Y_{1,k} + Y_{3,k}) \right\} \leq RDE_{MAX}$ for all $i = 1, \ldots, 4$

Given:

1. $X_{1,0}$, $Y_{1,0}$ (Initial bonds and stock)
2. α, β, $e_{1,4}$ (Fixed and variable bond flotation cost; Call premium)
3. c_0, c_1, c_2, c_3, c_4 (Coupon rates)
4. D_1, D_2, D_3, D_4 (Funding requirements)
5. E_1, E_2, E_3, E_4 (Expected earnings)
6. g_1, g_2, g_3, g_4, g_c (Stock floatation costs, and constant cost)
7. s_1, s_2, s_3, s_4 (Cost of equity)
8. ρ_1, ρ_2, ρ_3, ρ_4 (Penalty rates)
9. b_1, b_2, b_3, b_4 (Bonus rates)
10. p_{d1}, p_{d2}, p_{d3}, p_{d4} (Penalty rate for dividend)
11. d_{c1}, d_{c2}, d_{c3}, d_{c4} (Discount rates)
12. RDE_{MIN} (Minimum acceptable debt/equity ratio)
13. V = 1000 (Face value on the bonds issued)

5. Numerical Illustration

The funding requirements for any firm are determined by its planned capital investment, which in turn is determined by its growth rate. Moreover, the degree of technological innovation in an industry also influences the funds required for new capital investment (Maksimovic and Pichler, 2001). In general, the funds required are the result of the expected changes in working capital and fixed capital requirements.

For our illustration, let the vector of the expected funds required be given as follows:

$$\overline{\mathbf{D}} = [180{,}000,\ 204{,}000,\ 223{,}600,\ 242{,}660,\ 246{,}500]'$$

The first element in the funding requirement vector, $\overline{\mathbf{D}}$, corresponds to the funding requirement at $t = t_0$, which is satisfied by \$100 million in equity plus \$80 million in bonds currently available. Assume these bonds carry a coupon rate of \$60 per bond (or 6 percent) and there are no outstanding short-term borrowings or investments at this time. We further assume that

retained earnings at inception are zero. The discount rate, *d*, will be a constant 9.5 percent for each time interval. Also, let the opportunity cost of equity be 11 percent and the penalty rate for not paying the desired dividend, as explained earlier, be 16 percent. We have set this penalty rate to be higher than the cost of equity or the management will not tend to pay the desired dividend (Allen and Michaely, 1995).

Next, assume that the fixed and the variable components of the equity flotation costs are $80,000 and 2.5 percent of the equity issued. Further assume that the fixed and variable components of the bond flotation costs are $45,000 and $15 per bond, respectively. When a bond is issued, it is generally not refundable for some time period after its issuance during what is called the call protection period. For this illustration the call protection period is four years without affecting the generality of the model.

The decision to repurchase stock is, among other things, a function of whether the stock is overvalued or undervalued in the market at any time interval and the expected future funding requirements. However, even in a short time period after the issuance of stock, the management may decide to repurchase stock as long as the benefits of stock repurchase outweigh its associated costs (Grullon and Ikenberry, 2000). When the management decides to repurchase the corporation's stock, it generally has to pay a certain premium over the prevailing market price. Assume here that the fixed and variable costs of stock repurchase are $60,000 and 5 percent of repurchase value, respectively. The other cost vectors are given as follows:

$\bar{\mathbf{c}} = [60, 74, 63, 76, 80]'$ (the coupon rates on bonds)

$\bar{\mathbf{p}} = [74, 82, 71, 87, 92]'$ (the penalty rates and cost of short-term borrowing)

$\bar{\mathbf{b}} = [51, 60, 48, 63, 68]'$ (the rates on short-term investments)

Note that at this stage we do not know what the values of $y_{1,i}$, $y_{2,i}$, $y_{3,i}$ and $x_{j,i}$ in the above state vectors. That is, the optimal financing policy for the firm is not evident. Optimal policy decisions require a simultaneous analysis of the trade-offs between the costs of different sources of financing, the funds required, the funds from retained earnings, and the penalty costs of not meeting the various managerial constraints.

Thus, given the above information, our objective is to determine the values for $y_{1,i}$, $y_{2,i}$, $y_{3,i}$, and $x_{j,i}$ for each time interval that minimizes the total discounted costs over the entire planning horizon. This problem can be solved in two ways. One can draw a tree diagram that delineates each possible path from one time period to next until the end of the planning horizon and then calculate the discounted total cost associated with each path. In this example, however, more than 5,000 paths are possible, which makes the tree diagram analysis impractical.[1] Another possibility to solving such an optimization problem is to use a method such as the method of conjugate gradients by Fletcher and Reeves (1964).[2] Briefly stated, the algorithm is as follows:

[1] For the first interval there are seven (7) possible choices to meet the funding requirement: 100% bonds, 100% stock, 100% borrowing, and four-way combinations of these three. Notice that the way these three options are combined to raise the required funds is not even counted here. For the intervals 2 to 4 there are nine (9) different ways to go to the next time period, the seven possible choices mentioned above plus whether stock would be repurchased or not. Therefore, total possible paths are 5,103 (i.e. $7x9^3$). See Gupta and Lee (2006) for an illustration of this method with a simpler case.

[2] The conjugate gradient method is an algorithm for finding the nearest local minimum function of *n* variables assuming that the gradient function can be computed. See Bulirsch and Stoer, (1991) for a discussion of the method.

(1) Arbitrarily choose a vector $\overline{\mathbf{X}}_0$ which serves as an initial guess for the vector $\overline{\mathbf{X}}$ that optimizes the function.

(2) Find $\overline{g_0} = g(\overline{\mathbf{X}_0})$, the gradient vector of the objective function at $\overline{\mathbf{X}_0}$.
Let $\overline{P_0} = -\overline{g_0}$ be the initial search direction.

(3) Let $\overline{\mathbf{X}}_{i+1}$ = the position of the minimum of the objective function *f*(x) on the line through $\overline{x}_i$ in the initial direction of $\overline{p}_i$.

(4) If $\left\|\overline{\mathbf{X}}_{i+1} - \overline{\mathbf{X}_i}\right\|$ is less than ε, a small number, terminate the process. The optimum is $\overline{\mathbf{X}}_{i+1}$.
If $\left\|\overline{\mathbf{X}}_{i+1} - \overline{\mathbf{X}}_i\right\| >$ ε, go to the next step.

(5) Find $\overline{g}_{i+1} = \overline{g}(\overline{\mathbf{X}}_{i+1})$, the gradient vector at $\overline{\mathbf{X}}_{i+1}$. Calculate $\beta_i = \dfrac{g_{i+1}^{(T)}(g_{i+1})}{g_i^{(T)}(g_i)}$ where *T* represents the transpose of the vector.

(6) Now $\overline{p}_{i+1} = -\overline{g}_{i+1} + \beta \overline{p}_i$ is the new search direction. Go to step (3) and repeat to step (6).

Note that step (3) is a simple one-dimensional optimization problem that can be solved by such methods as the Golden section method or quadratic interpolation, etc. (Pozrikidis, 1999; Belegunda and Chandrupatla, 2011; Fox, 1971; and Wilde and Beightler, 1967).

Results produced by a simple computer program following the above algorithm yielded the following optimal state vectors:

$\mathbf{Y}_0$ = [100,000, 0, 0]′	$\mathbf{X}_0$ = [80,000, 0, 0, 0, 0]′
$\mathbf{Y}_1$ = [100,000, 0, 4,000]′	$\mathbf{X}_1$ = [80,000, 0, 0, 0, 0]′
$\mathbf{Y}_2$ = [100,000, 0, 4,600]′	$\mathbf{X}_2$ = [80,000, 0, 35,000, 0, 0]′
$\mathbf{Y}_3$ = [100,000, 0, 6,060]′	$\mathbf{X}_3$ = [80,000, 0, 35,000, 0, 0]′
$\mathbf{Y}_4$ = [100,000, 0, 6,366]′	$\mathbf{X}_4$ = [80,000, 0, 35,000, 0, 0]′

5.1 Optimal policy decisions

The optimal state vectors also determine the optimal capital financing policy to be followed by the firm in each time period. That is, the policy that will minimize total discounted costs. Table 1 summarizes the optimal policy decisions.

According to the optimal state vectors, the sequence of optimal decisions to be undertaken by the firm over each interval of the four-year planning horizon is as follows:

1. The firm increases its cumulative retained earnings to $4 million from zero and meets its dividend payout target of $14 million. Neither any bond nor equity is issued. However, the firm is short of required funds by $20 million. It makes up the difference by optimally borrowing short-term in the amount of $20 million.

2. The firm issues bonds of $35 million (or 35,000 bonds), enough to pay off the outstanding short-term loan of $20 million from the first interval. The $15 million remaining from the bond issue after liquidating the short-term loan is added to the retained earnings of $4.6 million for

the period to provide the required funding of $223.6 million once stockholders are paid the desired dividend of $15.4 million.

3. The firm meets its funding requirements of $242.66 million by borrowing $13 million short-term and optimally retaining $6.06 million of its earnings after meeting its dividend payment obligations of $16.94 million.

4. The firm's funding requirements of $246.5 million are more than adequately met by the increase in retained earnings of $6.366 million after dividend payment requirements. The remainder is used to reduce the short-term borrowing from $13 million to $10.474 million.

Table 1. Decision Matrix for Optimal Policy Decisions (in $000)

	(1)	(2)	(3)	(4)	(5)	(6)
Time Interval	Dividend Payout	Cumulative Retained Earnings	Fund from Equity	Fund from Short-term Borrowing	Funds from Bonds	Total Required Funds (2)+(3)+(4)+(5)
1	14,000	4,000	100,000	20,000	80,000	204,000
2	15,400	8,600	100,000	-	115,000	223,600
3	16,940	14,660	100,000	13,000	115,000	242,660
4	18,634	21,026	100,000	10,474	115,000	246,500

5.2 Optimal cost information

The optimal state vectors and the policy decision discussed above also determine the optimal sequence of financial costs incurred in each time interval. Table 2 gives the cost breakdown for each time interval associated with the optimal financing decision in Table 1.

Table 2. Cost Associated with the Optimal Policy Decisions (in $000)

	(1)	(2)	(3)	(4)	(5)	(6)
Time Interval	Equity Cost	Interest on Borrowing	Coupon Cost on Bond	Bonds Floatation Cost	Total Cost (1)+(2)+(3)+(4)	Total Discounted Cost $(5)/(1+d)^i$
1	11,000.00	1,480.00	4,800.00		17,280.00	15,780.82
2	11,440.00		7,390.00	570.00	19,400.00	16,179.81
3	11,946.00	1,131.00	7,390.00		20,467.00	15,588.77
4	12,612.60	963.61	7,390.00		22,352.21	14,583.56
						62,132.96

Discussion of the discounted total cost by time interval is as follows:

1. The firm pays $4.8 million in interest on $80 million of bonds at 6 percent (or $60 per bond) and $1.48 million for short-term borrowings of $20 million at 7.4 percent. There is also $11 million in the cost of equity that is based on the equity outstanding. The total discounted cost for this interval is $15.8 million.

2. During this time period, the firm incurs a cost of equity of over $11.4 million. The short-term borrowings of the first interval are paid off, so there is no interest to be paid here. However, the firm optimally issues new bonds at a coupon rate of 7.4 percent. As such, the firm incurs $0.57

million of bond flotation costs and pays almost $7.4 million in bond interest. Thus the total discounted cost for this period is $16.2 million.

3. The coupon payments on long-term bonds and the cost of equity are $7.39 million and $11.95 million, respectively. Interest on short-term borrowing is $1.13 million on a loan of $13 million. The total discounted cost for this period is $15.59 million.

4. In the last interval, the interest on short-term borrowing is lowered to $0.963 million as some of the short-term loans are liquidated in this period. The coupon payments on long-term bonds and the cost of equity are $7.4 million and $12.6 million, respectively. The total discounted cost for this period is $14.58 million.

In this example the discounted total costs over the four-year planning horizon are about $62.10 million.

5.3 Model revision

The decision vector for each time interval derived here is optimal. However, economic conditions change and the costs associated with various sources of funds may also change (John, 1993, and Panno, 2003). In addition, a firm is an ongoing entity and its funding requirements may change over time. Management may change or modify some or all of the constraints with respect to the debt/equity mix, the desired level of dividends payable to shareholders, or the most desired mix of short-term to long-term financing (Baskin, 1987, and Lang, Ofek, and Stulz, 1996).

The decision vectors derived here should minimize the total discounted cost of providing the necessary funds to the firm. These decisions simultaneously take into account all the information available and their inter-relationships over successive periods of time. As the information available changes, the model could be rerun and new decision vectors derived. These decision vectors again would be optimal until new information becomes available.

6. Conclusions

This paper has presented a model that deals with the problem of financing the current and projected activities of a firm at the lowest present value cost. The projected activities are influenced by the growth rate the firm wants to maintain and the types of projects the firm wants to undertake. The model considers all the dimensions of the funding problem and all the sources available to the firm, such as new equity issues, bond issues, bond refunding, short-term borrowings, surplus funds investment, desired dividends payments, stock repurchases, and earnings retention. Simultaneously, the model takes into account all their inter- and intra-period relationships.

In addition, the model accounts for both the possible managerial constraints, such as the debt-to-equity ratio to be maintained, the dollar limits on short-term borrowing and a predetermined ratio of short-term debt to long-term debt and the desired level of dividends to be paid over successive periods of time. By taking these considerations into account, the model matches the amount of funds supplied with the amount of funds required. In so doing, the model accomplishes the matching of financing and investment needs that Mauer and Triantis (1994) said was of fundamental importance and helps to control the agency cost of the managerial overinvestment incentive (Jung, Kim, and Stulz, 1996).

Overall, the output of the model produces a sequence of optimal financing decisions to be undertaken by the firm over time to meet its funding requirements at a minimum total discounted cost. The model is easily adaptable to changes in both the costs of the different financing sources and the constraints imposed on a firm's management on those sources that may occur with time.

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