

## Collateral Risk and Demographic Discrimination in Mortgage Market Equilibria

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**Abstract:** Observations of significant differences in loan terms between demographically distinct groups of borrowers are often interpreted as evidence of ethnic, racial or gender discrimination by lenders. We consider, in stark contrast to existing models of demographic discrimination, a model of mortgage lending in an economy having complete markets, common knowledge and arbitrage-free pricing. Market equilibria in this classical environment may exhibit discriminatory loan pricing by lenders even when borrowers, distinguished only by differences in an observable demographic trait, share identical measures of individual credit risk. Such pricing will be observed if these traits are directly correlated with certain features of the property securing a mortgage loan which, while omitted from standard statistical underwriting and regulatory review procedures, reduce the value of the collateral to the lender in case of foreclosure. When loans are secured by such properties and both lenders and borrowers act strategically, discrimination on this basis maximizes mortgage returns to lenders and will invariably be observed in mortgage market equilibria.

**Keywords:** Credit rationing; Discrimination; Efficient markets; Perfect equilibrium

**JEL Classifications:** G13, G15, O16

### 1. Introduction

Recent research into mortgage and consumer lending in the U.S. and elsewhere supports claims that some demographically distinct borrowers appear to encounter significantly different loan terms and higher rates of loan denials than do other borrowers to whom they appear identical in their measure of credit risk. Evidence of such lending discrimination, particularly in regard to residential mortgages, poses a significant anomaly for the classical assumption of efficiency in credit markets.<sup>1</sup>

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<sup>1</sup> A.B.A. (2012) provide a legal description of the criteria for lending discrimination in the U.S., while Nickerson, Nebhut and Courchane (2000), Calem and Longhofer (2002), Avery, Brevoort and Canner (2012), Federal Reserve Board (2015) and Ang, Flynn, Kearney, and Schatz (2017), among others, discuss data and econometric testing for such discrimination. LaCour-Little (1999) and Ross and Yinger (2002) provide comprehensive surveys of research into mortgage lending discrimination.

Two different explanations for this evidence exist, each depending on implicit or explicit situations of market failure. The first, popular among policymakers and the public, asserts that discrimination, in the form of disparate treatment of borrowers, is the result of lender preferences being either biased against certain borrowers owing to demographic traits that appear unrelated to credit risk or affected by behavioral limits on cognition which lead to suboptimal loan decisions. Such preferences, when accompanied by market concentration, could lead to systematic variation in loan terms and even to the rationing of credit through systematic denials of loan applications.<sup>2</sup>

Traditional economic explanations, in contrast, involve inefficiencies affecting loan underwriting, arising from either exogenous constraints on the accurate pricing of individual credit risk or from an asymmetric dispersion of information across lenders and borrowers. Both adverse selection and moral hazard, for example, can produce dispersion in loan terms and denial rates across distinct groups of borrowers who differ by unobservable probabilities of default. The competitive structure of most credit markets, the availability of credit histories and the widespread use of increasingly sophisticated statistical tests of credit risk posed by loan applicants, however, render the realism of these respective explanations suspect.

We offer a novel and fundamentally different explanation for observations of discrimination in mortgage markets and demonstrate that evidence of credit allocations consistent with lending discrimination can arise simply from the maximization of the returns to mortgage lending in conditionally efficient credit markets.

We consider a representative credit market embedded in a traditional continuous-time economy exhibiting complete markets, arbitrage-free valuation and strategic exercise by both borrowers and lenders of the options embedded in a standard secured mortgage loan. Exercise by each party will depend on certain characteristics of the type of property securing such a loan. If maximizing lenders perceive that the risk to the value of this collateral arising from these characteristics is correlated with classes of borrowers distinguished by observable demographic traits, then mortgage market equilibria will invariably exhibit systematic differences in the terms and volume of credit offered to member of different classes even when all borrowers appear identical by standard measures of credit risk.

The paper is organized as follows. We review the relevant previous research on equilibrium credit discrimination in the next section. Our model is described in Section 3. Analytical and numerical results are presented in Section 4. Section 5 offers a discussion of the implications of our model for both applied economic theory and public policy. Concluding remarks appear in the final section.

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<sup>2</sup> Neither argument is satisfactory. The first implies that management of financial institutions systematically fails to maximize the value of shareholder equity while the second, in principle, lacks any a priori restrictions on lender behavior, which consequently implies that regulation can enhance efficiency in any market equilibrium.

## 2. Literature and Context

The traditional approach to explaining idiosyncratic lending patterns across different borrowers relies on the pioneering work of Hodgman (1960), Jaffee and Modigliani (1969), Barro (1976), and others. These papers attributed a differential access to credit to an exogenous inability of lenders to accurately price credit risk among individual borrowers. While able to approximate empirical evidence of discrimination, these models could not generate endogenous equilibria exhibiting discrimination based on observable borrower characteristics.

In a subsequent approach, Stiglitz and Weiss (1981) analyzed the endogenous decisions of lenders who, owing to the assumed presence of either adverse selection or moral hazard, could price risk to individual borrowers only on the basis of the average degree of credit risk of a group of borrowers. Loan terms, as a result, could vary across groups of borrowers and under extreme circumstances a group could experience credit rationing. Lang and Nakamura (1993), Campbell (2012) and others applied this idea to explain discrimination across distinct groups of borrowers. Models based on asymmetric information are unsuitable, however, in explaining discrimination since equilibrium allocations of credit in these models necessarily depends on the unobservable characteristics of borrowers germane to their individual credit risk. Like those models relying on an exogenous ability to price risk, the market equilibria in asymmetric information models are necessarily inefficient in allocating risk.

Our model and results are distinguished in two fundamental ways from this earlier research. First, we endogenously derive both the existence and magnitude of equilibrium lending discrimination using only the classical assumptions of financial asset-pricing models. Second, we show that, in our model, these assumptions generate equilibria consistent with a efficient allocation of credit risk.

The model used in this paper, more specifically, is devoid of any exogenous source of market failure and instead assumes an economy with complete capital markets, common knowledge of the actuarial risk of default on the part of both borrowers and lenders, and, as a result, arbitrage-free pricing of credit risk. Our model augments these assumptions by explicitly incorporating both the strategic aspects of mortgage loan negotiation and the presence of observable demographic traits which, while independent of the measure of credit risk in standard statistical underwriting methods, may differentiate borrowers in the common perception of risk held by lenders. This allows us to apply the familiar replication techniques of contingent claims analysis to accurately value, at origination and at any time up to and including the maturity date, the mortgage contract to a given lender and any borrower.

We can, consequently, determine whether systematic variation in loan terms across demographically distinct groups of borrowers requires the assumption of market inefficiency or whether it can be explained solely in terms of the strategic exercise of the embedded options each party holds in a standard mortgage loan. In turn, this allows us to conclude whether the extant assumption of either idiosyncratic lender preferences or market inefficiency is a necessary or simply sufficient condition for lending discrimination to be observed and whether discrimination can simply result from the economic incentives of lenders and borrowers inherent in actual mortgage lending.

Our paper is also related to research on the non-strategic exercise of the default and prepayment options possessed by mortgage borrowers. Kau, Keenan, Muller and Epperson (1995), Deng, Quigley and Van Order (2000) and others have used the risky-debt model of Merton (1994) to value both fixed and adjustable rate mortgage contracts in the absence of the options, such as

default or prepayment, embedded in standard mortgage contract. Our paper differs from these in analyzing the option to default in the strategic context of loan negotiation between lender and borrower. We consequently expand this traditional option-based literature on mortgage termination to include strategic options, deriving the endogenous response of the lender's offer of loan terms and supply of mortgage credit to the strategic timing of the decision of the borrower to exercise his default or prepayment options as well as to the characteristics of the property which secures his loan.

Finally, our approach to the design and valuation of mortgage contracts parallels that of the 'strategic debt service' literature, as exemplified by Anderson and Sundaresan (1996), and Acharya *et al.* (2006). Extending the Merton (1994) model to a game-theoretic context, these authors analyze the effect on debt pricing of strategic renegotiation and bankruptcy costs. Our model differs from those, however, in using collateral risk as the basis for loan valuation and in its focus on the degree of lending discrimination as an endogenous function of collateral risk.

### 3. The Model

#### 3.1 Assumptions

Using a framework similar to that of Jones and Nickerson (2002), we model the standard fixed-rate mortgage contract negotiated between a representative borrower and lender as a stochastic differential game in which the observed terms of that contract emerge from the conditions of a perfect Markov equilibrium in that game.<sup>3</sup> This contract is secured by the property being purchased and displays a constant coupon amortization structure over a finite term to maturity. This form of contract is common to the United States and elsewhere.<sup>4</sup> Efficiency in our model is conditional on the nature of this contract.

The initial market value of this property is common knowledge to both agents but that value evolves randomly and consequently has, at each moment, an uncertain future value. Owing to his equity claim, the borrower receives a measurable flow of rent or the equivalent value of housing services from the property. The options embedded in this contract include the choice of the borrower to default by failing to remit his scheduled payment, and the subsequent choice of the lender to foreclose upon the borrower, allowing him to resell the securing property. And we assume the property depreciates at an exogenous rate.<sup>5</sup> We assume the property depreciates at an exogenous

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<sup>3</sup> Bensoussan, Siu, Yam, and Yang (2014) analyze perfect Markov equilibria in stochastic differential games with properties similar to those of our model.

<sup>4</sup> While we use a specific amortization structure here, the set of mortgage contracts feasible for our model is quite general and subsumes any fixed-rate mortgage with a finite maturity, arbitrary amortization structure and any realistic provisions for foreclosure and partial or full collateralization, inclusive of the property being financed. For example, although we treat the mortgage contract in our model as non-recourse, our results can easily be applied to partial or full recourse mortgage loans by reinterpreting the asset securing the loan to include other assets in the borrower's portfolio in addition to the property being financed.

<sup>5</sup> This flow of property services is, consequently, exactly analogous to the flow of dividends on a share of corporate stock.

rate and that the value of housing services, depreciation and any investment by the borrower in the property are common knowledge.

Two implications of this contract are particularly relevant to our analysis. First, the value of the past flow of housing services cannot be retrieved by the lender through foreclosure. Since this value was capitalized in the initial market value of the property, it is not available at the time of default. Both borrower and lender, consequently, are aware that an inverse relationship exists between the conditional value of the loan collateral at any time before the loan matures and the flow of housing services received by the borrower. Second, the timing of the borrower's option to default or prepay and the lender's option to foreclose after default will be chosen in their respective interests. A non-cooperative equilibrium requires that, should either decide to exercise them, these choices will be that party's optimal response to the choices made by the counterparty.

The optimal strategies of each party are determined through the valuation of their respective claims on the property securing the loan. Since the economy is assumed to have complete markets, we apply the standard arbitrage-free valuation method to derive a pair of linked partial differential equations and corresponding boundary conditions, which must be satisfied by the values of these contingent claims. Defining the state space of the game to be the support of all asset values and dates relevant to the decisions of the borrower and lender, we employ recursive methods to endogenously derive a numerical solution for the equilibrium of the game by finding those sequential paths in the state space which represent the "best-reply" strategies of the parties. Each choice of a set of parameters for the contract yields a distinct and unique equilibrium. Comparing these alternative equilibria measures the magnitude of lending discrimination in the form of loan terms and the volume of credit exchanged between lenders and different classes of borrowers.

### **3.2 Specification**

Consider a representative loan market in which lender and borrower negotiate a standard mortgage contract secured by the pledge of a representative residential property, and possibly other assets possessed by the borrower, of value  $a(t)$ . The lender chooses an initial balance to lend at date 0 for which the borrower is obligated to remit continuous coupon payments at a constant rate  $c$  until maturity  $T$  or the unpaid balance  $C(t)$  at any date  $t \in [0, T]$ . Both parties also have access to a riskless asset with maturity  $T$  and return  $r$ . The rate premium  $\gamma$  paid by the borrower is, consequently, determined by the constant coupon rate specified in the contract, relative to the riskless rate  $r$ .

The mortgage market operates in an economy satisfying the assumptions of the classical asset valuation environment. In particular, this economy has a complete filtered probability space  $[\Omega, \mathfrak{F}, P]$ , where  $\Omega$  represents the space of events in this economy,  $\mathfrak{F}$  represents the corresponding filtration, a set of sequential sigma algebras  $\{\mathfrak{F}_t\}_{0 \leq t \leq T}$  representing information available to traders at time  $t$ , and  $P$  represents the actuarial probability measure over property value defined on  $\Omega$ . Parties share common knowledge of each  $\mathfrak{F}_t$  and base all decisions at each date  $t, 0 \leq t \leq T$ , on this observation and  $t$ . The evolution in value of any other assets in this economy is adapted in this space. Since the markets for all assets, including property, are complete with respect to risk, property is valued in the absence of arbitrage.

All properties eligible as loan collateral are assumed to belong to a suitably defined set  $\Phi$ , and qualities distinguishing different properties are indexed by the vector  $\varphi \in \Phi^6$ . The value of the representative property at date  $t$ ,  $a(t)$ , evolves according to the diffusion

$$a(t) = \alpha(a, t)dt + \sigma a(t)dz(t) \quad (1)$$

where  $z(t)$  is a standard Brownian motion,  $\alpha(a, t)$  is the expected drift at all  $t$  and  $\sigma$  measures the conditional volatility of this value. The values of different properties are distinguished by different values  $\sigma_\varphi$  of this volatility.

Each borrower is assumed to display an exogenous degree of some demographic trait, observable by the lender but intrinsically independent of any measure of the borrower's risk of default. We index this trait over some bounded subset  $\Theta$  of the real line and denote the degree of this trait displayed by an arbitrary borrower by  $\theta \in \Theta$ .

Finally, we choose to model the economic source of discrimination as the perceived volatility  $\sigma_\varphi$  of a property. If the representative lender perceives a correlation between the observable degree to which a borrower exhibits the common demographic trait,  $\theta$ , and those characteristics of the specific property this borrower wishes to finance,  $\varphi$ , then under these circumstances, the lender acts as if the effective volatility in the inter-temporal value of this property as a function of the degree  $\theta$  of the borrower's trait. The lender's perception of the credit risk posed by this borrower will consequently be:

$$\sigma = \sigma(\tilde{\varphi}(\theta)) \quad (2)$$

We will use the notation  $\sigma$ , the volatility parameter in the value (1) in the generic asset, for notational simplicity in what follows but we will use this perceived risk or uncertainty on the part of the lender to solve for the presence and magnitude of lending discrimination below.

The initial value of this asset is common knowledge, as is all parameters in (1). Since these features are exogenous and cannot be influenced by either party to the loan, neither moral hazard nor adverse selection are present. While he services the loan, the borrower receives a continuous flow of housing services,  $\pi(a, t)$ , measurable across any use he makes of the property. Foreclosure, following a failure by the borrower to pay  $c$  at any date  $t$ , results in sale of the asset. The lender receives  $\max [a(t) - b(a, t), 0]$  from this sale, where  $b(a, t)$  is the liquidation cost incurred by the lender, and the borrower receives any residual funds exceeding the unpaid balance  $C(t)$ .

The respective spaces of strategies available to each party includes their choice of the timing of any exercise of the options inherent in the mortgage contract. The principal element of the space of the lender is, in the current paper, his specification of the initial amount of credit advanced,  $C(0)$ , and, contingent on default, the date of foreclosure.<sup>7</sup> The principal strategic elements chosen by the borrower are the timing of both his exercise of the option to default or to prepay the existing loan balance. The strategy of the lender and of the borrower are each selected to maximize, subject to the strategy of his counterparty, the value of his contingent claim on the asset collateralizing the

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<sup>6</sup> The index  $\varphi$  could be interpreted as a vector but for simplicity (without loss of generality), we will treat  $\varphi$  as a scalar. Analogously, the respective sets  $\Phi$  and  $\Theta$  are assumed to be discrete but each could equally well be interpreted as a continuum.

<sup>7</sup> Clearly this space could, in a more complex version of the model, be augmented to include several additional options, including the right to call the loan.

loan. We denote by  $L(a,t)$  the resulting value to the lender and by  $B(a,t)$  the value of each party's claim under the mortgage contract.

Standard arbitrage pricing determines the respective functions  $L(a,t)$  and  $B(a,t)$  for all possible combinations  $(a, t)$ . Solutions for these functions, under each choice of parameters, represent the respective values of the debt and equity claims on the asset in a perfect Markovian equilibrium. These solutions satisfy a pair of partial differential equations, linked by the best-reply strategies selected by each party and by the respective boundary conditions for each equation. This pair of equations is

$$rL = \left(\frac{1}{2}\right)(a\sigma)^2 L_{aa} + (ra - \pi)L_a + c + L_t \quad (3)$$

$$rB = \left(\frac{1}{2}\right)(a\sigma)^2 B_{aa} + (ra - \pi)B_a + c + B_t \quad (4)$$

with the corresponding boundary conditions

$$L(\check{a}, t) = \max\{0, \check{a} - \eta(\check{a}, t)\} \quad (5)$$

$$\pi(\check{a}, t) - c + E_t B^*(\check{a}, t) = 0 \quad (6)$$

and

$$L^*(\hat{a}, t) = C(t) \quad (7)$$

$$\pi(\hat{a}, t) - c + E_t B^*(\hat{a}, t) = 0 \quad (8)$$

The term  $E_t(\cdot)$  is the expectations operator under the unique equivalent martingale measure induced by our assumption of complete markets and  $B^*(a, t) = (e^r dt)B(a + da, t + dt)$  and  $L^*(a, t) = (e^r dt)L(a + da, t + dt)$  are the respective risk-adjusted values of the claims of the borrower and lender, discounted at the riskless interest rate. Denoting by  $\bar{B}(a, t)$  and  $\bar{L}(a, t)$  the respective values of the parties' claims if the loan terminates through the exercise of an option by either party, the terms  $\check{a}$  and  $\hat{a}$  are the respective asset values triggering default and prepayment by the borrower. At these values the functions  $B(a, t)$  and  $L(a, t)$  satisfy the value-matching and smooth-pasting criteria. The value-matching condition requires the borrower's value function to be continuous at the respective asset value inducing him to default at date  $t, \check{a}$ , or to prepay at date  $t, \hat{a}$ , as defined respectively by

$$B(\check{a}, t) = \bar{B}(\check{a}, t) \quad (9)$$

$$B(\hat{a}, t) = \bar{B}(\hat{a}, t) \quad (10)$$

while the smooth-pasting condition requires the first derivatives of  $B(\cdot)$  and  $\bar{B}(\cdot)$  to be continuous at these same points. The lender's value function is required to satisfy analogous criteria at those distinct points where he would exercise his option to foreclose or, if a particular specification of the game allows this option, to call.

Since the finite maturity of the loan precludes an analytical solution, we characterize market equilibrium through numerical solutions for the valuation equations and boundary conditions (3) –

(8), which are derived on the basis of a standard recursive finite difference procedure.<sup>8</sup> This requires representation of the respective strategies of the lender and borrower in terms of subsets of the underlying state space of our game. This state space is defined by all specific pairs of exogenous values  $a$  of the asset and corresponding dates  $t$  relevant to the respective strategy choices by the lender and borrower. If the set of all asset values is denoted by  $A$  and the compact set of all dates relevant to the loan contract by  $T$ , then the state space of the game is  $A \times T$ , which is the support of the continuum of all possible states  $(a, t)$ . Any strategies chosen by the lender and borrower are nested within  $A \times T$ .<sup>9</sup>

## 4. Results

When lenders perceive a relation between the degree of one or more demographic traits displayed by borrowers with the salient characteristics of the property securing the loan, the equilibrium strategies and values to each party of the mortgage contract will be conditional on the degree to which a given borrower displays one or more of the observable demographic traits assumed. Since the number of demographic classes in most databases is limited, we present our results in terms of two such classes, implying that  $\Theta$  has only two elements.<sup>10</sup> Also only for purposes of simplicity, we assume that the respective members of each such class wish to finance a property with volatility  $\sigma$ , the quality distinguishing properties in (1)-(2), common to the property of every other member of their class but distinct from those financed by members of the other class. These assumptions are simultaneously reflected in equation (2) for each of the two values of  $\theta$ .

The presence and magnitude of equilibrium discrimination can consequently be deduced directly through the comparing the equilibrium loan terms for each of the two classes of borrowers. Selection of alternative sets of values of the parameters determining the stochastic evolution of the representative property for each class and those representing the institutional features of the market allow us to compare the strategies of borrowers and lenders and their respective values of equity and debt in the equilibrium arising for each class. Such comparisons allow us to measure, in numerical terms, the relative difference between loan terms and the volume of credit exchanged in each equilibrium.

The existence of a correlation between classes of loan borrowers distinguished by traits unrelated to credit risk and the parameters of the diffusion process describing the evolution of property value generate different loan terms and balances offered to the members of each such class. Since the features most frequently cited in empirical evidence of lending discrimination, such as in analyses of HMDA data in the U.S. are comparisons of the amount of credit obtained at loan origination and selected loan terms, we focus the presentation of our results on these two properties of credit market equilibrium. We use the initial loan balance, which is also the current value of the loan to the lender  $C(0)$ , to measure the amount of credit and the rate premium  $\pi$  to represent the

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<sup>8</sup> A standard reference to the implementation of the finite-difference method we use is Trottenberg, Oosterlee, and Schuller (2000).

<sup>9</sup> We also specify six gridpoints, or equivalently five 'periods,' over the state space  $A \times T$  for our calculations of the numerical solutions to equations (2) – (7).

<sup>10</sup> This is done without loss of generality. The binary values of  $\Theta$  can be interpreted to designate 'majority' and 'minority' borrowers, 'white' and 'black' borrowers, 'male' and 'female' borrowers, and so on, depending upon the nature of the data to which the model is applied.

terms of the loan and compare these values across the equilibria corresponding to alternative sets of parameters describing the exogenous features of the credit market.

The exogenous features of the market we consider include the instantaneous mean  $\alpha(a,t)$  and, conditional on the borrower's type,  $\theta$ , the volatility  $\sigma(\theta)$  exhibited over time by the collateral asset; the net flow of value  $\pi(a,t)$  accruing to equity in the asset; the cost  $b(a,t)$  incurred by the lender in liquidating the asset in the event of foreclosure; and any cost  $f(C(t))$  incurred by the borrower should he prepay the loan. We assume, for simplicity in the interpretation of the results below, that the net revenue flow from the asset  $\pi(a, t)$ , liquidation costs  $b(a, t)$  and prepayment costs  $f(C(t))$  are all independent of time and homogeneous in their arguments. These allow us to represent the initial loan balance as a percentage of the initial value of the asset, obviating the need to interpret our results as conditional on the absolute size of the loan. We interpret as annual the per-period values for the riskless rate and rate premium, the asset volatility and the maturity of the loan as a convenience for the reader.

**Table 1.** Loan terms: Effects of liquidation costs

Rate Premium	Case I ( $b = 10\%$ )	Case II ( $b = 30\%$ )
$\gamma$	$C(0) / \%$	$C(0) / \%$
0.02	71.70	57.60
0.04	82.70	65.00
0.06	89.00	68.00
0.08	93.00	70.70
0.10	95.50	71.80
0.12	97.20	71.90
0.14	98.30	71.90

**Notes:** Table 1 illustrates the influence of liquidation costs on equilibrium lending terms. Case I shows that, for relatively low costs ( $b=10\%$ ) of liquidating the asset at default, the initial loan balance is 71.7% at a rate premium of 2%. This balance increases monotonically with this rate until it reaches 98.3% of the asset value at a 14% rate. Case II illustrates, however, that higher liquidation costs reduce both the amount of credit available at each rate premium as well as rate at which that amount increases at each successively higher rate. Liquidation costs of 30% reduce the initial loan balance to only 57.6% at a two percent premium while the increases in the balance for equal rate increases steadily decline until, at a rate premium of 10%, the balance reaches only 71.90%. True credit rationing occurs after this point, with subsequent rate increases producing no increase in credit. Higher rates provide no increase in value for the lender because liquidation costs imply that the increased risk of default they induce outweighs any increase in the value of higher coupon payments.

Our results appear in terms of equilibrium values of the initial loan balance  $C(0)$  and corresponding no-arbitrage rate premium  $\gamma$ , for alternative values of a chosen exogenous variable, holding constant all other parameters at benchmark values.<sup>11</sup> Since empirical evidence presented in

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<sup>11</sup> These benchmark values are a riskless annualized interest rate of  $r = 0.03$ , an annualized proportional volatility (standard deviation) in the value of the collateral asset of  $\sigma = 0.2$ ; an instantaneous flow of value to equity  $\pi$  of ten basis points, a maturity  $T$  of five periods, a one basis point flow of coupon payments  $c$ ; and both liquidation  $b(a,t)$  and refinancing  $f(C(t))$  costs of zero.

Seitzer(2008), van Order and Zorn(2000) and elsewhere suggests that the relative cost of liquidating a foreclosed property significantly influences differences in loan terms offered to different demographic groups, we illustrate the effects of property price volatility conditional on such liquidation costs.

We consider first the influence of liquidation costs on the equilibrium amounts and terms of credit and show that, above a certain threshold rate premium, the lender will rationally ration credit over and above the maximum initial loan balance he has chosen. Table 1 illustrates this by depicting how loan balances  $C(0)$  vary with successive two percentage point increases in the corresponding equilibrium rate premium  $\gamma$ , as the costs  $b$  of asset liquidation in foreclosure increase from a ‘low value’ (10%) to a ‘high’ value (30%). Higher balances correspond to higher rates in each case, but, as expected, higher liquidation costs reduce the initial loan balance at each rate premium and, averaged over all rates, initial balances differ by eight percentage points. As liquidation costs become sufficiently high, rate increases beyond a threshold point (8% in this example) elicit virtually no increases in initial balances. Credit, after this threshold, is strictly rationed.

Table 2 illustrates the direct influence of price volatility on credit available at each rate. When volatility is relatively ‘low’ ( $\sigma = 15\%$ ), for each increase in rates balances increase, but at a sharply decreasing rate. Doubling the volatility reduces the average amount of credit across the rate premiums by 10.2 percentage points.

**Table 2.** Loan terms: Effects of asset price volatility

Rate Premium	Case I ( $\sigma = 15\%$ )	Case II ( $\sigma = 30\%$ )
$\gamma$	$C(0) / \%$	$C(0) / \%$
0.02	78.10	58.10
0.04	88.70	73.70
0.06	93.60	82.10
0.08	96.20	87.50
0.10	97.70	91.10
0.12	98.70	93.70
0.14	99.20	94.40

**Note:** Analogously, Table2 illustrates the influence of annual volatility in asset value on equilibrium lending terms. Case I shows that, for relatively low volatility ( $\sigma = 15\%$ ), the initial loan balance is 78.10% at a rate premium of 2%. Credit available to the borrower increases monotonically with this premium until it reaches 99.2% of the asset value at a 14% rate. This increase in credit, however, exhibits a sharply decreasing rate of increase, from approximately a 14% growth in initial balance as the premium rises from a value of 2% to 4%, to only 1% growth as the premium rises from a value of 12% to 14%. Higher volatility ( $\sigma = 30\%$ ), as expected, reduces the amount of credit available, at each corresponding value of the rate premium, to an average balance of 83% relative to an average of 93.4% at the lower volatility.

We now illustrate, in the last set of our selected results, the effects on the availability of credit from simultaneous variation in the volatility of asset value and costs of liquidating that asset at foreclosure. Table 3 measures these effects through four parametric combinations corresponding to those in Tables 1 and 2. In the first case, loan terms are shown for ‘low’ ( $b=10\%$ ) and ‘high’

( $b=30\%$ ) liquidation costs, conditional on the ‘low’ value (15%) of price volatility. The second case repeats this same increase in liquidation costs, but now at the ‘high’ volatility value (30%).

Two aspects of our results are especially significant. First, the data in Tables 1-2 clearly illustrate, as expected, that loan balances at any given rate are considerably lower for borrowers when lenders incur higher liquidation costs at constant price volatility or when price volatility increases at constant costs of liquidation. Table 3 illustrates, in addition, that the adverse impact on credit caused by a given increase in liquidation costs or volatility is significantly worsened by a respective increase in volatility or liquidation costs. Consider, for example, that the approximately 6 percentage point decline in average balances caused by a given rise in liquidation costs from 10% to 30% at an annual volatility of 15% increases to an approximately 22 percentage point decline in average balances for the same increase in liquidation costs at an annual volatility of 30%, a difference of 16 percentage points.

**Table 3.** Loan terms: Combined effects of liquidation costs and asset price volatility

Volatility ( $\sigma = 15\%$ )			Volatility ( $\sigma = 30\%$ )		
	$b = 0.10$	$b = 0.30$		$b = 0.10$	$b = 0.30$
$\gamma$	C(0)	C(0)	$\gamma$	C(0)	C(0)
0.0200	0.7690	0.7050	0.0200	0.5230	0.4990
0.0400	0.8400	0.7660	0.0400	0.6610	0.5600
0.0600	0.8700	0.7910	0.0600	0.7400	0.6030
0.0800	0.8840	0.7980	0.0800	0.7890	0.6340
0.1000	0.8900	0.8040	0.1000	0.8220	0.6540
0.1200	0.8940	0.8090	0.1200	0.8440	0.6480
0.1400	0.8970	0.8130	0.1400	0.8590	0.6420

**Notes:** The effects on equilibrium loan terms of simultaneous variation in the volatility of asset value and costs of liquidating that asset at foreclosure are illustrated in Table 3. Loan terms, again represented by the rate premium and corresponding initial loan balance, are shown for four combinations of parameter values. In the first case, loan terms are shown for the same ‘low’ ( $b=10\%$ ) and ‘high’ ( $b=30\%$ ) liquidations costs as considered in Table 1, conditional on the annual price volatility of 15% used for Case I in Table 2. The second case shows these same loan terms for the same two values of liquidation cost, but now conditional on the annual price volatility of 30% used for Case II in Table 2.

The second and most striking feature of our results is that not only can the ‘rationing’ of credit above a certain balance occur to different borrowers, based on the characteristics of the assets used to collateralize their respective loans, but, for possible, if extreme, parameter values in the market, increases in the rate premiums that rationed borrowers are willing to pay can actually reduce the loan balance they receive. Table 3 demonstrates that, when liquidation costs and price volatility are both high, each successive increase of two percentage points in rates above 10%, actually causes credit to decline by 60 basis points.

## 5. Discussion

Set in a classical economy devoid of market failures and idiosyncratic preferences, our results demonstrate that lending discrimination on a demographic basis can arise when rational lenders, conditional on their common beliefs, accurately risk-price credit to maximize returns on mortgage lending. Such discrimination takes the form of loan terms that feature costs of borrowing that increase as the risk to the value of the properties being financed by one demographically distinct class of borrowers increases relative to others. More specifically, owing to the strategic exercise by both borrowers and lenders of the options embedded in a typical mortgage loan, such discrimination will invariably occur when lenders perceive a direct relation between those borrowers distinguished only by an observable demographic trait and the relative collateral risk posed to lenders by the properties securing the loans of these borrowers, despite identical measures of credit risk exhibited by all borrowers.

To put our results in perspective, consider an example in which two borrowers, A & B, share identical measures of individual credit risk but are distinguished by different degrees of some observable demographic trait. Both borrowers wish to partially finance the common purchase price, \$400,000.00, of their respective properties. The representative lender believes that the annualized price volatility in the property securing a loan to B is significantly higher ( $\sigma = 30\%$ ) than that ( $\sigma = 15\%$ ) analogous property securing A's loan. Using the results in Table 1, the initial loan balance offered to A is, for any rate premium, considerably larger than that offered to B, with the difference ranging between \$16,000.00 to \$80,000. Equivalently, borrower B would be charged, in the absence of regulatory constraints, a rate premium two to four percentage points higher than that paid by A for approximately identical mortgages. Using the results in Table 2, if the lender alternatively believes that the unit costs of liquidating B's collateral is higher (30%) than those of A (10%), the difference between the initial mortgage balance offered to A ranges from \$56,400.00 to \$94,400.00 as the rate premium rises from 2% to 10%. Finally, if the lender believes both price volatility and liquidation costs for B's property are correspondingly higher than for A's, Table 3 indicates that the difference between the initial mortgage balances respectively offered to A and B range from \$108,000.00 to \$96,800.00 and, since the lender denies any application by B for credit over \$261,600.00, there is no rate difference that would allow B to obtain the same amount of mortgage credit as A.

Equilibrium discrimination, in its most general form, arises in our model for two reasons. First, lenders and borrowers both behave strategically in order to maximize the value accruing to them from their position in the loan contract. This strategic behavior includes decisions by each party whether and at what time to exercise the options, embedded in a standard fixed-rate mortgage contract he controls. The non-cooperative equilibrium features each party to the contract choosing an exercise strategy to maximize the value of the contract to him, conditional on the strategy chosen by his counterparty. This equilibrium may result in both parties being disadvantaged relative to a hypothetical equilibrium in which the players could cooperate by committing to choosing only those strategies which jointly maximize the aggregate value of the contract, but neither party can regard his counterparty's commitment to be credible.

Second, both parties are aware that the inter-temporal flow of housing services to the borrower has economic value. As equity holder, the borrower receives this flow as long as he services his loan. Since, however, the expected value of this flow was capitalized, in our model, at the time of origination, the lender cannot retrieve the value of past service flows in the event of default. The

collateral value of the borrower's property is diminished, to the detriment of the lender, to the extent that this flow is higher or the likelihood of its loss through default is greater.

These two features guarantee that those borrowers, even with a standard measure of credit risk identical to other borrowers, who finance properties which pose relatively greater collateral risk to the lender will bear correspondingly higher costs of borrowing in equilibrium. The case in which discrimination is observed to occur on purely demographic grounds can now be seen to arise immediately when lenders in this market possess a common belief in a correlation between one or more demographic traits distinguishing a borrower and any characteristics of the property that borrower wishes to finance that pose, relative to the properties of other borrowers, a higher risk of loss to the lender in the event of default. The loan terms offered to that class of borrowers who pose higher risk will worsen as this risk increases, relative to those offered to other borrowers.<sup>12</sup> The property characteristics that will increase credit risk include greater degrees of price volatility, higher foreclosure and liquidation costs, and rates of depreciation and property services. When lenders perceive that these characteristics of the representative property pledged as collateral by this disadvantaged class of borrowers imply a degree of risk beyond an endogenous threshold, then our results show that value-maximizing lenders will rationally deny these borrowers access to the amount of mortgage credit they demand. If this risk is sufficiently higher than this threshold, the initial balance lenders offer will actually decline as the rate acceptable to borrowers increases.

The most extensive documentation of lending discrimination on the basis of differences in racial or ethnic characteristics occurs in the American market for residential mortgage loans. Consider an example in which the neighborhoods of a city are segregated on the basis of one or more features of a minority group and a majority group.<sup>13</sup> If lenders believe that depreciation rates are higher in a given minority neighborhood than in a majority neighborhood, perhaps owing to property crime arising from the relative scarcity of police services in the former neighborhood, or are less certain about the appreciation of property prices or foreclosure and liquidation costs in the former neighborhood, perhaps owing to relatively fewer or less accurate property appraisals then lenders will perceive that the collateral value of the representative property will be less in that neighborhood than in a majority neighborhood.<sup>14</sup> As a consequence, even if they share similar measures of credit risk with their majority counterparts, prospective homebuyers or current property owners in the minority neighborhood will only be able to obtain mortgages (and similar loans) that have significantly less attractive terms.

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<sup>12</sup> Observe, from equations (7) -(10), that, since the composition function (2) describing housing price volatility is continuous, decreasing values of this correlation commensurately reduce the costs of a disparity in loan terms received by demographically distinct borrowers.

<sup>13</sup> This segregation could have occurred from a preference by the members of one or both groups for various locational qualities, such as geographical proximity to other members of their class relative to the other class, or be caused from sources entirely exogenous to the mortgage market, such as the differential treatment of these two groups in regard to the provision by local government of public or private services in their neighborhoods, or from a variety of other reasons beyond the influence of lenders.

<sup>14</sup> Empirical studies of the relationships between location, neighborhood qualities and lending discrimination include Dawkins (2005), Feinberg and Nickerson (2002), Deng, Ross and Wachter (2003), Li, Hossain and Ross (2010) and Leonard, Jha and Zhang (2015).

## 6. Concluding Remarks

The objective of this paper is to provide the first consistent and rigorous explanation for the extensive evidence of demographic discrimination in mortgage and other credit markets which eschews the assumptions of adverse lender preferences or informational market failures used in existing explanations and instead uses contingent-claims valuation in an economy with a complete set of asset markets and common knowledge of market characteristics. When lenders share a common perception that an observable demographic trait distinguishing one group of borrowers from groups is directly related to the relative degree of risk posed to lenders by the characteristics of the properties which secure the mortgages of these borrowers, then we show, in the presence of rational lenders and absent any source of market failure, that mortgage market equilibria will invariably exhibit discrimination in the loan terms and cost of credit to the members of this group. Such discrimination occurs even when, under standard underwriting procedures, the representative members of different groups have similar measures of individual credit risk.

Our model and results also have implications for both empirical applications of financial economics based on most databases currently available and current data and for the design of regulations, based on these applications, intended to mitigate both the efficiency and welfare inequities of lending discrimination. More specifically, they imply a potential fragility of evidence of discrimination based on current econometric analyses of HMDA or similar data.

HMDA data, for example, fails to include observations of any features of those properties or other assets securing mortgage loans relevant to the collateral risk to which borrowers are exposed. If the design of econometric tests requires holding constant or equal the credit risk of given pairs or groups of sample borrowers, then the omission of observations or variables measuring the collateral risk from those features could mean a failure to properly control for the full measure of the credit risk posed to a lender from an individual borrower.

Current statistical evidence of lending discrimination, as a consequence of both results, is subject to misinterpretation and is, in and of itself, an insufficient basis on which to design or adopt financial regulations intended to enhance the economic efficiency of residential mortgage lending. Using only traditional economic models and empirical evidence which may be less than robust in ascertaining the nature of the incentives to which lenders respond in actual mortgage markets, redistributive policies may also be effective in their purpose or may alternatively reduce the welfare of all borrowers through a distortion of the incentives to lend and borrow.

It is important to note that our results in no way deny the existence or possible ubiquity of various types of lending discrimination, such as the disparate treatment of minority mortgage applicants. Neither does it deny that such discrimination can and likely does often arise from demographic bias in lender preferences, errors in lending decisions arising from behavioural limitations or from various sources of market failure. What our results do, however, is demonstrate that these qualities are only sufficient, rather than necessary, conditions for lending discrimination to occur. Evidence of lending discrimination, as a result, cannot imply that lenders are irrational, prejudiced, systematically err in their lending decisions or otherwise perversely respond, relative to standard economic predictions, to market incentives. Nor can such evidence imply the presence of any remediable failures in credit markets.

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