

Dependence of Structural Breaks in Rating Transition Dynamics on Economic and Market Variations

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Abstract: The financial crisis of 2007-2008 has caused severe economic and political consequences over the world. An interesting question from this crisis is whether or to what extent such sharp changes or structural breaks in the market can be explained by economic and market fundamentals. To address this issue, we consider a model that extracts the information of market structural breaks from firms' credit rating records, and connects probabilities of market structural breaks to observed and latent economic variables. We also discuss the issue of selecting significant variables when the number of economic covariates is large. We then analyze market structural breaks that involve U.S. firms' credit rating records and historical data of economic and market fundamentals from 1986 to 2015. We find that the probabilities of structural breaks are positively correlated with changes of S&P500 returns and volatilities and changes of inflation, and negatively correlated with changes of corporate bond yield. The significance of other variables depends on the inclusion of latent variables in the study or not.

Keywords: Credit rating; Markov chain Monte Carlos; Stochastic approximation; Structural break; Variable selection

JEL Classification: C13, C41, G12, G20

1. Introduction

In economics, a structural break occurs when the fundamental relationship of economic variables changes or a new equilibrium in the system is attained. Econometricians usually define structural breaks as changes of parameters in time series models of economic variables (Hansen, 2001). During the last two decades, the U.S. credit market has experienced several market-wide structural breaks that were triggered by some social and economic events. One example is that, in the second half of 1998, the U.S. commercial paper markets experienced a severe disruption due to a series of events including the Russian's default, Brazil's currency crisis, and the downturn of the LTCM (Long-Term Capital Management L.P.). Another example is the 2007-2008 financial crisis that was resulted from a complex interaction of financial and regulatory policies such as "high risk lending by U.S. financial institutions, regulatory failures, inflated credit rating, high risk, poor quality financial products" (Permanent Subcommittee on Investigations, 2011). Given that market structural breaks may cause devastating economic, social and political consequence, an interesting question for the regulatory authority and financial practitioners is whether or to what extent such structural breaks can be

explained and forecasted by economic and market fundamentals.

To address this issue, we first need to find a proxy in the credit market that contains information on structural breaks. We note that credit ratings have been widely used in the credit market to measure the issuers' credit risk. As an information good provided by credit rating agencies (CRAs), credit ratings measure the ability of fulfilling the issuer's future financial obligation under current market conditions, and enhance capital market efficiency and transparency by reducing the information asymmetry (Langohr & Langohr, 2008). The economic literature has discussed several types of information revealed by firms' rating records, such as the conflict of interest between investors and CRAs (Skreta & Veldkamp, 2009; Opp *et al.*, 2013), the effect of CRA's reputational concern on rating quality (Mathis *et al.*, 2009; Mariano, 2012), the interaction between the business cycle and firms' incentives (Povel *et al.*, 2007), and so on. Among these discussion, Xing *et al.* (2012) argued that firms' rating transitions contain information on market structural breaks, and proposed a stochastic structural break model to extract market structural break information from firms' rating transition records.

Then the next step is to connect market structural breaks to economic and market fundamentals. Some analysis has shown the dependence of firms' credit exposure on macroeconomic conditions. For example, Bangia *et al.* (2002) and Nickell *et al.* (2000) found that rating transitions are dependent on economic regimes. Xie *et al.* (2008) showed that firms' default intensities were related to the performance of stock market. Carling *et al.* (2007) and Duffie *et al.* (2007) showed that firms' default probabilities are sensitive to some macroeconomic factors such as GDP growth rates and the yield curve spread. Figlewski *et al.* (2012) used 18 economic variables to represent general economic conditions and studied their impact on firms' credit rating transitions. Duffie *et al.* (2009) and Koopman *et al.* (2009) further showed dynamic latent components in economic conditions also play an important role in firms' credit risk. This indicates that, to study the connection between economic conditions and market structural breaks, both observable macroeconomic variables and dynamic latent factors should be included in the analysis.

Therefore, we consider a model with two components. The first component is an extension of Xing *et al.* (2012), and the second is a multiplicative intensity model that connects rates of market structural breaks to economic and market fundamentals. In particular, we assume that market structural breaks follow a nonhomogeneous Poisson process, and the rate of the process is determined by a set of economic variables via a multiplicative intensity model. Since all observations such as firms' rating records and economic fundamentals are collected in discrete time, our assumption implies that, in discrete time, the probability of structural breaks is associated with observable and latent economic variables via a logistic regression model. Note that, different from Xing *et al.* (2012) that focused on identifying market structural breaks from firms' rating transition records, our concentration here is to study whether or to what extent market structural breaks can be explained by observable and latent macroeconomic variables. To make inference on the effect of observable economic variables, we note that, due to the existence of dynamic latent variables, it is not easy to use the expectation-maximization algorithm in Xing *et al.* (2012) any more. Instead, we use a stochastic approximation algorithm with Markov chain Monte Carlo (MCMC) simulations that was introduced by Gu & Kong (1998) to estimate the effect of observable and latent economic variables. To compare the effect of the dynamic latent variables on structural breaks, we also consider a special case that all variables are observable. Furthermore, since more and more variables are collected to characterize various aspects of the economy in these years, we further discuss the case, when a large number of economic factors are incorporated in the study, how to use regularization methods to select and estimate statistically significant variables.

We then study the effect of a set of observable and latent economic factors on structural breaks in a credit market that consists of rated U.S. firms' from January 1986 to December 2015. The study

contains three parts. The first part measures the effect of observable and latent economic and market variables on structural breaks, and discuss the implied probabilities of structural breaks. We find that probabilities of structural breaks are highly correlated with variations of policy and macroeconomic variables, such as changes of money supply, changes of short- and long-term interest rates, and so on. Furthermore, the significance of the impact of changes of economic variables on structural break probabilities depends on whether latent variables are included in the set of covariates or not. For a comparison purpose, we perform an analysis when latent factors are omitted from the set of covariates in the second part. We then show how significant variables are selected for market structural breaks when a large number of covariates are involved in the study. In the third part, we use a sequential procedure to perform an out-of-sample study on the data.

The remainder of the paper is organized as follows. Section 2 describes the data in our analysis, which include US firms' rating transition records and time series of 20 macroeconomic variables. In Section 3, we introduce a structural break model with observable and latent economic covariates, and consider a special case that latent factors are omitted. Section 4 presents the inference procedure for the models in Section 3 and an extended procedure to select significant factors. Section 5 analyzes the data and studies the in-sample and out- of-sample performance of our model on rating transition records of U.S. firms and economic variables. It also discusses the estimation results and their economic implication. Section 6 provides some concluding remarks.

2. Data

The data in our analysis consist of Standard & Poor's monthly credit ratings of firms and 20 time series on the U.S. economy from January 1985 to December 2015. They are obtained from COMPUSTAT and the Federal Reserve Bank of St. Louis, respectively. The credit rating dataset contains 25,891 firms and their 3,001,063 ratings recorded at the end of each month. Applying the data cleaning procedure in Xing *et al.* (2012), we obtain $K = 8$ rating categories, *AAA*, *AA*, *A*, *BBB*, *BB*, *B*, *CCC*, and *D* (default), and 5,886 initial ratings and 7,964 transitions for 5,886 firms. Since there is only one rating transition in 1985, our analysis will focus on the records starting from January 1986. To connect structural breaks in rating transitions to economic and market variables, we note that studies have shown that firms' credit risk transitions are dependent on the performance of stock market, GDP growth rates, and the yield curve spread. In our study, we construct covariates from macroeconomic variables that represent conditions and directions of the economic and market conditions; see Figlewski *et al.* (2012). We also include a dynamic latent variable in the analysis, besides growth rates or lag-1 difference of the following observable series:

- (1) *Real GDP growth*. The U.S. real GDP in total is only available quarterly, so we constructed monthly series of real GDP by a linear interpolation. We then compute the growth rate of monthly real GDP for the model.
- (2) *Growth rate of industrial production*. As real GDP consists of economic activities from government, corporate and non-corporate business, and other sections that may not be directly related to the credit market, we include the growth rate of industrial production to strengthen the effect of activities from the corporate sector.
- (3) *Change rates of unemployment rate and mean duration*. The unemployment rate and mean duration indicate the overall health of the economy. High unemployment rate and long unemployment mean duration should decrease (or increase) the hazard of upgrade (or downgrade) transitions.

- (4) *Inflation measured through CPI, PPI and oil prices.* Both the Consumer Price Index (CPI) and Producer Price Index (PPI) show the change in price of a set of goods and services, but they differ in the following aspects. First, the PPI focuses on the whole output of producers in the U.S., while the CPI only focuses on goods and services bought for consumption by urban U.S. residents. Second, sales and taxes are included in the calculation of the CPI, but not in that of the PPI. We use the percentage changes of the seasonally adjusted CPI and PPI to measure inflation faced by consumers and producers. We notice that the oil prices change significantly during the sample period, so we also compute the percentage change of the oil prices (i.e., West Texas intermediate spot oil prices) as a separate covariate.
- (5) *Growth rates of the government and consumer debts.* We consider the total public debt of the federal government and the total outstanding credit of consumer owned and securitized. Government debts play an important role in relaxing private credit and liquidity constraints (Woodford, 1990). A positive growth rate of government debts suggests a relaxation of private credit constraints. Accordingly, high government debt might lower down the overall level of credit contracts in the market, which may cause structural changes of the market. High consumer debt also reduces the demand for credit, which may increase consumers' defaults and is closely related to the 2007-2008 U.S. financial crisis. The government debt data are quarterly, so we use linear interpolations to obtain monthly data.
- (6) *Change rates of consumer sentiment.* Consumer sentiment measures economic agents' opinion on the overall health of the economy. Research has shown consumer sentiment can have a significant impact on consumers' and investor's behavior (Baker & Wurgler, 2006). We collect this from the University of Michigan Survey of Consumer Sentiment as the measure for consumer sentiment, and use the monthly percentage change of consumer sentiment to show economic agents' subjective beliefs and expectations about the economy. An increase of consumer sentiment indicates consumers' optimistic belief on economic prospects and consumers' willingness of more credit-spending, which tends to change the market environment and increase the probabilities of market structural breaks.
- (7) *Chicago Fed National Activity Index (CFNAI).* This composite series captures the overall economic conditions and summarizes the behavior of 85 economic series in the categories of production and income, employment, unemployment and hours, personal consumption and housing, and sales, orders and inventories. The CFNAI are published monthly in the form of a 3-month moving average by the Federal Reserve Bank of Chicago. A positive CFNAI value suggests that the economy is expanding faster than average.
- (8) *Short- and long-term interest rates.* The interest rate series consist of the monthly rates of 3-month Treasury Bill and 10-year Treasury Constant Maturity. High interest rates may increase difficulty in raising fund to make debt service payment and cause general tightness and structural changes in the economy. As an approximation to interest rate term structures, the difference of the long- and short-term interest rates reflects the forward-looking expectation of the tightness of the money market. Besides, the 3-month interest rates can be considered as a proxy for the U.S. monetary policy rate, and the change of 3-month interest rate is an approximation to the change of monetary policy. Both variables are found to be associated with increased default rates; see Duffie *et al.* (2007).
- (9) *Stock market performance measured by S&P 500 returns and volatility.* These two series show the stock market performance and indicate the general healthiness of the stock market. In particular, volatility shows the extent of stock market instability, and is computed as the

annualized standard deviation of S&P500 daily returns in every month. The S&P500 has been found negatively correlated with the default premium of corporate bonds (Xie *et al.*, 2008). Changes of S&P500 levels and volatilities indicate the instability of the economy, which in turn have positive effect on structural changes of the economy.

- (10) *Growth rate of money supply.* This is measured by the growth rate of the M2 series. As one of the monetary policy instruments, it can be affected by private demand for credit and liquidity. As discussed by Gertler *et al.* (2012), government monetary policy plays an important role in the stability and vulnerability of the financial system, which further affects the stability of the credit market.
- (11) *Growth rate of all U.S. commercial banks' net asset.* Assets and liabilities of commercial banks show the borrowing and lending activities in the economy and are important variables on the general healthiness of an economy. Banks' increasing assets and liabilities make the banking system more vulnerable (Gertler *et al.*, 2012). Hence it weakens the stability of the economy and increases the possibility of market structural changes.
- (12) *Corporate bond Aaa and Baa credit spreads.* Research has shown that credit spreads have significant predictive value for real output (Friedman & Kuttner, 1992). A rise in credit spreads can be used to predict the severity of a credit crisis. We measure the spread here as the difference between Moody's Aaa and Baa corporate bond yield and 10-year Treasury bonds, respectively.

We use these variables to construct 20 economic covariates, as shown in Table 1. In general, they can be classified into two categories, one representing the improving or worsening of general economic conditions, and the other describing current situations of financial market. As market structural breaks are essentially big and sharp changes of the market environment, their explanatory variables should reflect changes of certain economic features. Hence, we use the change or the lag-1 difference of these 20 series as explanatory variables. We shall note that the effect of these variables on market structural breaks may not be instantaneous, hence we consider two types of lagged variables in Section 5. One is the aggregation of these variables by imposing an exponentially weighted lag structure on the factors, and the other is the set of their current and lagged values (up to 24 months).

Table 1. Defined economic variables ($\Delta\{\cdot\}$ represents the lag-1 difference of the variable)

Variable	Definition
Y_1	$\Delta\{\text{S\&P500 monthly return}\}$
Y_2	$\Delta\{\text{S\&P500 monthly realized volatility}\}$
Y_3	$\Delta\{\text{3-month T-Bill rate}\}$
Y_4	$\Delta\{\text{10-year Treasury rate}\}$
Y_5	$\Delta\{\text{Moody's Aaa corporate bond yield}\}$
Y_6	$\Delta\{\text{Moody's Baa corporate bond yield}\}$
Y_7	$\Delta\{\text{monthly unemployment rate}\}$
Y_8	$\Delta\{\text{mean duration of unemployment}\}$
Y_9	$\Delta\{\text{the inflation rate measured by CPI}\}$
Y_{10}	$\Delta\{\text{the inflation rate measured by PPI}\}$
Y_{11}	$\Delta\{\text{the inflation rate measured oil price}\}$
Y_{12}	$\Delta\{\text{CFNAI}\}$
Y_{13}	growth rate of US real GDP
Y_{14}	growth rate of industrial production

Y_{15}	growth rate of M2 money stock
Y_{16}	growth rate of consumer sentiment
Y_{17}	growth rate of bank's total net asset
Y_{18}	growth rate of public debt of the U.S. Fed. Gov.
Y_{19}	growth rate of the total outstanding consumer credit
Y_{20}	growth rate of loans at all commercial banks

3. Model Specification

3.1 Firms' rating transitions and structural breaks in continuous time

Suppose that the credit market consists of n firms and these firms' rating transitions follow a K -state non-homogeneous continuous time Markov process, which can be represented as piecewise homogeneous continuous time Markov processes with unobserved structural breaks. Consider firms' rating transitions in the period $(0, T)$. Denote $P(s, t)$ the transition probability matrix of the piecewise homogeneous continuous time Markov processes over the period (s, t) , in which the ij 'th element of $P(s, t)$ represents the probability that a firm starting in state i at time s is in state j at time t , and $\Lambda(u)$, $u \in (s, t)$, the associated generator matrix. Since a homogeneous continuous time Markov process can be characterized via a constant generator matrix, the time-varying generator matrix $\Lambda(u)$ are piecewise constant. We also assume, for convenience, that firms' rating migration from state i to state j at the period (s, t) are conditional independent given the generator matrix $\Lambda(u)$ in the period (s, t) .

We now characterize the dynamics of market structural breaks. Denote $N(t)$ the number of market structural breaks up to time t , then $N(t)$ can be assumed to follow a Poisson process. Different from Xing *et al.* (2012) that assumed the rate of Poisson process is constant, we assume that the Poisson process has a time-varying rate $\eta(t)$. We assume that market structural breaks are related to some exogenous macroeconomic variables and try to connect them to $\eta(t)$. Specifically, assume that explanatory variables for market structural breaks consist of a set of observable economic covariates $X_1(t), \dots, X_q(t)$ and a latent factor $U(t)$, and (X_1, \dots, X_q) are independent of U . Denote the information sets $F_t = \sigma\{X(s)|0 \leq s \leq t\}$ and $G_t = F_t \cup \sigma\{U(s)|0 \leq s \leq t\}$. Then the intensity of the nonhomogeneous Poisson process $N(t)$ can be assumed to follow a multiplicative model (Andersen & Gill, 1982),

$$E\{dN(t)|\mathcal{G}_{t-}\} = \eta(t)dt, \quad \eta(t) = \eta_0 \exp \left\{ U(t) + \sum_{i=1}^q \theta_i X_i(t) \right\}, \quad (1)$$

in which $dN(t)$ is the increment $N\{(t+dt)-\} - N(t-)$ over the small interval $[t, t+dt)$, and η_0 and $\theta_1, \dots, \theta_q$ are unknown parameters.

Given the market structural break process $N(t)$, the generator matrices between two adjacent structural breaks are constant. Then the generator matrix at time t can be written as $\Lambda(t) = Q_{N(t)}$. We further assume that matrices Q_1, Q_2, \dots are independent and identically distributed random matrices such that the off-diagonal elements $\lambda^{(ij)}$ follow independently a Gamma prior distribution with shape parameter α_{ij} and rate parameter β_i for $(i, j) \in K$ and $K = \{(i, j) | i \neq j, 1 \leq i \leq K-1, 1 \leq j \leq K\}$. Note that the elements of the last row in the generator matrix, representing the rating migrations from the default category to others, are usually assumed to be zero, so we don't need to model the dynamics of those elements.

3.2 Firms' rating transitions and structural breaks in discrete time

As ratings are assigned in discrete times, we consider an evenly spaced partition for the period $(0,$

T), $0 = t_0 < t_1 < \dots < t_L = T$, and assume that structural breaks can only occur at the times t_1, \dots, t_L . Let $I_l = 1$ and $I_l = N(t_l^-) - N(t_{l-1}^-)$ for $l = 2, \dots, L$ indicate if there is a structural break at t_{l-1} , then conditional on G_t , I_l are independent Bernoulli random variables with success probability

$$p_l = 1 - \exp\left(-\int_{t_{l-1}}^{t_l} \eta(t) dt\right).$$

Note that, when the partition is fine enough,

$$\frac{p_l}{1-p_l} \approx \int_{t_{l-1}}^{t_l} \exp\left\{U(s) + \sum_{i=1}^q \theta_i X_i(s)\right\} \eta_0 ds = \exp\left\{\theta_0 + U(\xi) + \sum_{i=1}^q \theta_i X_i(\xi)\right\}, \quad (2)$$

in which the approximation is based on the first-order Taylor expansion, and the equality is based on the first mean value theorem for integration, $\xi \in [t_{l-1}, t_l]$ and $\theta_0 = \log(t_l - t_{l-1}) + \log \eta_0$. Since $U(t)$ and $X_i(t)$ do not change much in the small interval (t_{l-1}, t_l) , $U(\xi)$ and $X_i(\xi)$ in (2) can be replaced by $U_{l-1} := U(t_{l-1})$ and $X_{i,l-1} := X_i(t_{l-1})$. This is reasonable since our data are monthly and many macroeconomic variables are measured in quarters or years so do not change much within a month. Then we obtain a logistic type regression model with dynamic latent variables for probabilities of structural breaks,

$$\log \frac{p_l}{1-p_l} = U_{l-1} + X'_{l-1} \theta, \quad (3)$$

where $X_{l-1} = (1, X_{1,l-1}, \dots, X_{q,l-1})'$ and $\theta = (\theta_0, \theta_1, \dots, \theta_p)'$. For latent dynamic variables U_{l-1} , we assume that they follow a first-order autoregressive process, i.e.,

$$U_l = aU_{l-1} + \epsilon_l, \quad (4)$$

in which ϵ_l are independent and identically distributed normal random variables with mean 0 and variance v^2 . Then conditional on the indicators I_l , which take value 1 with probability p_l at time t_l , the generators of firms' rating transitions matrices may undergo a jump at time t_l . For example, if $I_l = 1$, the generator matrix $\Lambda(t_{l-1})$ may jump to a new level, and elements of the post-change generator matrix $\Lambda(t_l)$ follow the prior distribution $\text{Gamma}(\alpha_{ij}, \beta_i)$; otherwise, $\Lambda(t_l) = \Lambda(t_{l-1})$. Then the transition matrix $P(t_{l-1}, t_l)$ can be generated from $\Lambda(t_{l-1})$.

4. Inference Procedures

Denote Y_l all firms' rating transition records from 0 to time t_l , and X_l the collection of observed economic variables from time 0 to time t_l , and let $\Theta = \{\theta_0, \dots, \theta_p, a, v\}$. Since variables $U := \{U_1, \dots, U_L\}$ are not observed, the likelihood function of Θ is expressed as

$$L(\Theta | \mathcal{Y}_L, \mathcal{X}_{L-1}) = \int f_{\Theta}(\mathcal{Y}_L | \mathcal{X}_{L-1}, U) g_{\Theta}(U) dU, \quad (5)$$

in which $f_{\Theta}(\cdot | \mathcal{X}_{L-1}, U)$ is the probability density function of Y_L conditional on (\mathcal{X}_{L-1}, U) and $g_{\Theta}(\cdot)$ is the density function of U given by

$$\log g_{\Theta}(U) = -(2\nu^2)^{-1} \sum_{l=1}^L (U_l - aU_{l-1})^2 - \frac{L}{2} \log(2\pi\nu^2).$$

Let $p_{m,l}$ be the conditional probability that the most recent structural break time before time t_l is t_{m-1} given observed rating history Y_l , observed covariates X_{l-1} , and a path of latent variable (U_1, \dots, U_l) . Using similar arguments as in Xing *et al.* (2012, p.88), we obtain an expression for the conditional density $f_{\Theta}(Y_L | \mathcal{X}_{L-1}, U)$, that is,

$$\log f_{\Theta}(\mathcal{Y}_L | \mathcal{X}_{L-1}, U) = \sum_{l=1}^L \log \left\{ \sum_{m=1}^l p_{m,l}^* \right\},$$

where $p_{m,l}^*$ can be computed recursively as follows

$$p_{m,l}^* = \begin{cases} p_l f_{l,l} / f_{0,0} & m = l, \\ (1 - p_l) p_{m,l-1} f_{m,l} / f_{m,l-1} & m < l, \end{cases} \quad (6)$$

the probability $p_{m,l-1}$, $f_{m,l}$ and $f_{0,0}$ are respectively defined as $p_{m,l-1} = p_{m,l-1}^* / \sum_{m=1}^l p_{m,l-1}^*$,

$$f_{m,l} = \prod_{i,j \in \mathcal{K}} \Gamma(K_{t_{m-1}, t_l}^{(i,j)} + \alpha_{ij}) / (S_{t_{m-1}, t_l}^{(i)} + \beta_i)^{(K_{t_{m-1}, t_l}^{(i,j)} + \alpha_{ij})}, \quad f_{0,0} = \prod_{i,j \in \mathcal{K}} \Gamma(\alpha_{ij}) / \beta_i^{\alpha_{ij}}.$$

Note that p_l is given by (3), $K_{s,t}^{(i,j)}$ the number of transitions from category i to category j , $S_{s,t}^{(i,j)}$ the amount of time that firms spend in category i , hence given a path of covariates (X_1, \dots, X_q) and latent factor U , the conditional density $\log f_{\Theta}(Y_L | X_{L-1}, U)$ can be computed explicitly. However, since factor U is latent, the estimation method in Xing *et al.* (2012) is not applicable here, and we next use a stochastic approximation procedure to make inference on model parameters.

4.1 Stochastic approximations with MCMC simulations for latent variables

Assume that $\hat{\Theta}$ maximize the likelihood function (5). Then $\hat{\Theta}$ solves the first-order condition

$$\frac{\partial}{\partial \Theta} L(\Theta | \mathcal{Y}_L, \mathcal{X}_{L-1}) = \int \frac{\partial h_{\Theta}(\mathcal{Y}_L, U | \mathcal{X}_{L-1})}{\partial \Theta} dU = 0, \quad (7)$$

where $h_{\Theta}(Y_L, U | X_{L-1}) = f_{\Theta}(Y_L | X_{L-1}, U) g_{\Theta}(U)$ represents the joint density function of Y_L and U conditional on X_{L-1} . Because the function $f_{\Theta}(Y_L | X_{L-1}, U)$ is expressed recursively and factors U are unobserved, it is difficult to find an analytic solution for equation (7). We thus consider the method of stochastic approximation introduced by Robbins & Monro (1951) to find $\hat{\Theta}$. We also note that, due to the ‘‘curse of dimensionality’’, it is not practical to use numerical integration in the algorithm. We hence choose the stochastic approximation algorithm with MCMC method proposed by Gu & Kong (1998) to solve (7) above.

4.2 EM algorithm when all factors are observable

If all factors are observable, equations (1) and (3) can be simplified by letting $U_l \equiv 0$, and the likelihood (5) becomes $f_{\Theta}(Y_L | X_L, U \equiv 0)$. To find an estimate for θ , a simple way is to consider an expectation-maximization (EM) algorithm by treating the generator matrices $\{\Lambda(t)\}$ as missing data. The EM algorithm in the Appendix shows that it is enough to maximize

$$\ell_e(\theta) := \sum_{l=1}^L \left\{ (X'_{l-1} \theta) y_l - \log [1 + \exp(X'_{l-1} \theta)] \right\}, \quad (8)$$

over the space Θ , in which $y_l = P(\Lambda(t_l) \neq \Lambda(t_{l-1}) | Y_L, X_{L-1}) = P(I_l = 1 | Y_L, X_{L-1})$. Note that y_l is the conditional probability given firms’ rating transition records and observed macroeconomic variables, it aggregates and extracts structural break information from both observed economic factors and firms’ rating records.

Note that if the number of factors, q , is small, expression (8) can be maximized by an iteratively reweighted least squares procedure so that a maximum likelihood estimate for θ can be obtained; see Lai & Xing (2008, Section 4.1). If q is large, maximizing (8) over Θ might not be easy due to ‘‘the curse of dimensionality’’, hence our goal should shift from estimating all the θ ’s to selecting and estimating a small number of significant variables. Specifically, we will minimize the following penalized function

$$\tilde{\ell}_e(\theta) = -\frac{1}{L} E(\ell_e(\theta) | \mathcal{Y}_L, \mathcal{X}_{L-1}) + \gamma \Phi(\theta), \quad (9)$$

in which the penalty function $\Phi(\theta)$ is a weighted average of L1 and squared L2 norms as

$$\Phi(\theta) = \phi \sum_{j=1}^q |\theta_j| + (1 - \phi) \sum_{j=1}^q \theta_j^2;$$

see Zou & Hastie (2005). The penalty $\Phi(\theta)$ is a combination of a L_1 and squared L_2 penalty on θ . It includes both sparsity and smoothness with respect to the correlated structure of θ , and allows us to control the amount of regularization for sparsity and smoothness at the same time through the tuning parameters γ and ϕ , respectively. In particular, when $\phi = 0$, it is equivalent to impose some prior distribution on θ , and when $\phi = 1$, the penalty reduces to the L_1 norm of θ and can shrink the estimates of most θ_i 's to 0; see Tibshirani (1996). As our purpose is to select significant variables from a number of covariates, we choose ϕ to be close or equal to 1.

4.3 Firms' rating transitions and structural breaks probabilities

Using the above procedures, we obtain an estimate for $\hat{\Theta}$ and simulated paths for $\{U_i\}$ (note that in the degenerated case of Section 4.2, $U_i \equiv 0$). Then we can compute the structural break probabilities p_l ($1 \leq l \leq L$). Given p_l , we can compute the posterior distribution of generators of rating-transition matrices $\Lambda(t_l)$ ($1 \leq l \leq L$). Specifically, the posterior distribution of the (i, j) th element of $\Lambda(t_l)$ given (Y_L, X_L, U_{L+1}) is expressed as

$$\sum_{m=1}^L p_{m,L} \text{Gamma}(K_{t_{m-1}, t_L}^{(i,j)} + \alpha_{ij}, S_{t_{m-1}, t_L}^{(i)} + \beta_i),$$

where the weight can be computed via (6). Then the element $\Lambda(t_l)^{(i,j)}$ in $\Lambda(t_l)$ can be estimated by its posterior mean. Furthermore, given the estimated generators $\Lambda(t_l)$ ($1 \leq l \leq L$), we can compute the rating transition matrices $P(t_{l-1}, t_l)$ ($1 \leq l \leq L$).

Besides the in-sample estimates, we also compute out-of-sample prediction. Specifically, once the covariates X_L are observed, we can first simulate the latent variable U_{L+1} using the AR(1) model (4) and estimated latent variables U_L , and then compute a prediction for the structural break probability in the next period (t_L, t_{L+1}) ,

$$\hat{p}_{L+1} = \exp(X'_L \hat{\theta} + U_{L+1}) / [1 + \exp(X'_L \hat{\theta} + U_{L+1})].$$

Furthermore, the distribution of the (i, j) th element of the generator matrix $\Lambda(t_{L+1})$ conditional on (Y_L, X_L, U_{L+1}) is given by

$$\hat{p}_{L+1} \text{Gamma}(\alpha_{ij}, \beta_i) + (1 - \hat{p}_{L+1}) \sum_{m=1}^L p_{m,L} \text{Gamma}(K_{t_{m-1}, t_L}^{(i,j)} + \alpha_{ij}, S_{t_{m-1}, t_L}^{(i)} + \beta_i),$$

and then $\Lambda(t_{L+1})^{(i,j)}$ can be estimated by its mean, and the predicted transition matrix $P(t_L, t_{L+1})$ can be computed from the predicted $\hat{\Lambda}(t_{L+1})$. Note that the predicted structural break probability \hat{p}_{L+1} only tells us the change of having a structural break, it doesn't say anything about the direction of the structural break. However, the predicted generator matrix $\hat{\Lambda}(t_{L+1})$ or the predicted transition matrix $\hat{P}(t_L, t_{L+1})$ can tell us the direction of the structural break.

5. Data Analysis

For convenience, we denote the beginning of January 1986 as time 0 and the end of December 2015 as time T , and partition the sample period from January 1986 to December 2015 to $L = 360$ intervals so that each interval corresponds to a calendar month. Then we use the transformed monthly series $\{Y_{i,t}; t = 1, \dots, L, i = 1, \dots, 20\}$ discussed in Section 2 to construct aggregated variables as follows,

$$X_{i,t} = \frac{\sum_{k=1}^H \delta^{k-1} Y_{i,t-k}}{\sum_{k=1}^H \delta^{k-1}},$$

where δ is the decay factor and H is the length of the lag window. As there are no rules to choose parameters H and δ , we use $\delta = 0.8, 0.9, 1.0$ and $H = 12, 24$. The result doesn't show any significant difference, so we only report the analysis of using $\delta = 0.9$ and $H = 18$. Table 2 shows the means, variances, and correlations of these variables. We note that most correlations are small and moderate, except the high correlations of (X_5, X_6) (0.867), (X_{13}, X_{14}) (0.844), (X_7, X_{14}) (-0.801), and (X_7, X_{13}) (-0.701). Moderate correlations include the ones among interest rates and bond credit spreads (-0.551, -0.656, -0.494, and -0.525), the changes of the 3-month T-Bill and unemployment rates (-0.604), the changes of unemployment rate and mean duration (0.579), and the changes of unemployment rate and growth rate of the total outstanding consumer credit (-0.506). Figure 1 on page 11 shows the time series plots of X_1, \dots, X_{20} . We can see that some of these variables such as X_7, X_{14} , and X_{19} have some sharp changes over time, and these changes may have an impact on firms' risk exposures which further lead to some structural changes in the market. We also note that the variances of these variables are significantly different. Hence, to avoid the dominated effect of factors with large variance, we use the centered and normalized $X_{i\cdot}$ (still denoted as $X_{i\cdot}$) in the following analysis.

Table 2. Correlation among macroeconomic variables

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}
X_1	1	-.383	.004	-.092	-.014	.002	.081	.027	-.071	-.046	.001	-.05	-.044	-.096	-.008	-.079	.056	.07	-.067	-.036
X_2	-.383	1	-.176	-.088	.415	.503	-.022	-.14	-.244	.042	-.206	-.287	-.106	-.031	.18	.08	-.096	-.031	.315	.139
X_3	.004	-.176	1	.537	-.551	-.494	-.604	-.331	.135	.071	.041	.071	.505	.532	-.338	.297	-.044	-.286	.291	.292
X_4	-.092	-.088	.537	1	-.656	-.525	-.217	-.084	.151	.045	.114	.165	.182	.259	-.30	.084	-.267	-.253	.044	-.022
X_5	-.014	.415	-.551	-.656	1	.867	.251	-.057	-.214	.002	-.181	-.456	-.322	-.311	.315	-.011	.041	.027	.269	.06
X_6	.002	.503	-.494	-.525	.867	1	.253	-.159	-.283	-.017	-.201	-.575	-.392	-.4	.388	-.003	.010	-.005	.39	.131
X_7	.081	-.022	-.604	-.217	.251	.253	1	.579	-.072	-.125	.037	-.015	-.701	-.801	.152	-.395	.115	.47	-.474	-.506
X_8	.027	-.14	-.331	-.084	-.057	-.159	.579	1	.022	-.024	.042	.317	-.374	-.291	-.074	-.569	.055	.513	-.737	-.583
X_9	-.071	-.244	.135	.151	-.214	-.283	-.072	.022	1	.058	.479	.14	.144	.117	-.041	.078	-.038	-.071	-.062	.013
X_{10}	-.046	.042	.071	.045	.002	-.017	-.125	-.024	.058	1	-.025	-.052	.083	.121	-.005	.063	.002	-.023	.058	.088
X_{11}	.001	-.206	.041	.114	-.181	-.201	.037	.042	.479	-.025	1	.137	.066	.023	-.02	.03	-.032	-.034	-.096	-.037
X_{12}	-.050	-.287	.071	.165	-.456	-.575	-.015	.317	.14	-.052	.137	1	.237	.245	-.188	-.124	.006	.116	-.453	-.243
X_{13}	-.044	-.106	.505	.182	-.322	-.392	-.701	-.374	.144	.083	.066	.237	1	.844	-.253	.67	.08	-.445	.145	.433
X_{14}	-.096	-.031	.532	.259	-.311	-.4	-.801	-.291	.117	.121	.023	.245	.844	1	-.352	.466	-.026	-.437	.213	.352
X_{15}	-.008	.18	-.338	-.30	.315	.388	.152	-.074	-.041	-.005	-.02	-.188	-.253	-.352	1	.07	-.119	-.067	.21	.104
X_{16}	-.079	.08	.297	.084	-.011	-.003	-.395	-.569	.078	.063	.03	-.124	.67	.466	.07	1	.178	-.635	.323	.633
X_{17}	.056	-.096	-.044	-.267	.041	.01	.115	.055	-.038	.002	-.032	.006	.08	-.026	-.119	.178	1	-.059	-.076	.079
X_{18}	.07	-.031	-.286	-.253	.027	-.005	.47	.513	-.071	-.023	-.034	.116	-.445	-.437	-.067	-.635	-.059	1	-.434	-.528
X_{19}	-.067	.315	.291	.044	.269	.39	-.474	-.737	-.062	.058	-.096	-.453	.145	.213	.21	.323	-.076	-.434	1	.524
X_{20}	-.036	.139	.292	-.022	.06	.131	-.506	-.583	.013	.088	-.037	-.243	.433	.352	.104	.633	.079	-.528	.524	1
Mean	-.003	0	-.008	-.007	0	0	-.001	.004	-.001	-.001	-.002	-.001	.016	.012	.029	5.45	.039	.035	.019	.025
S.D.	.535	.031	.034	.018	.006	.009	.010	.030	.003	.002	.099	.006	.01	.021	.011	.684	.018	.018	.033	.015

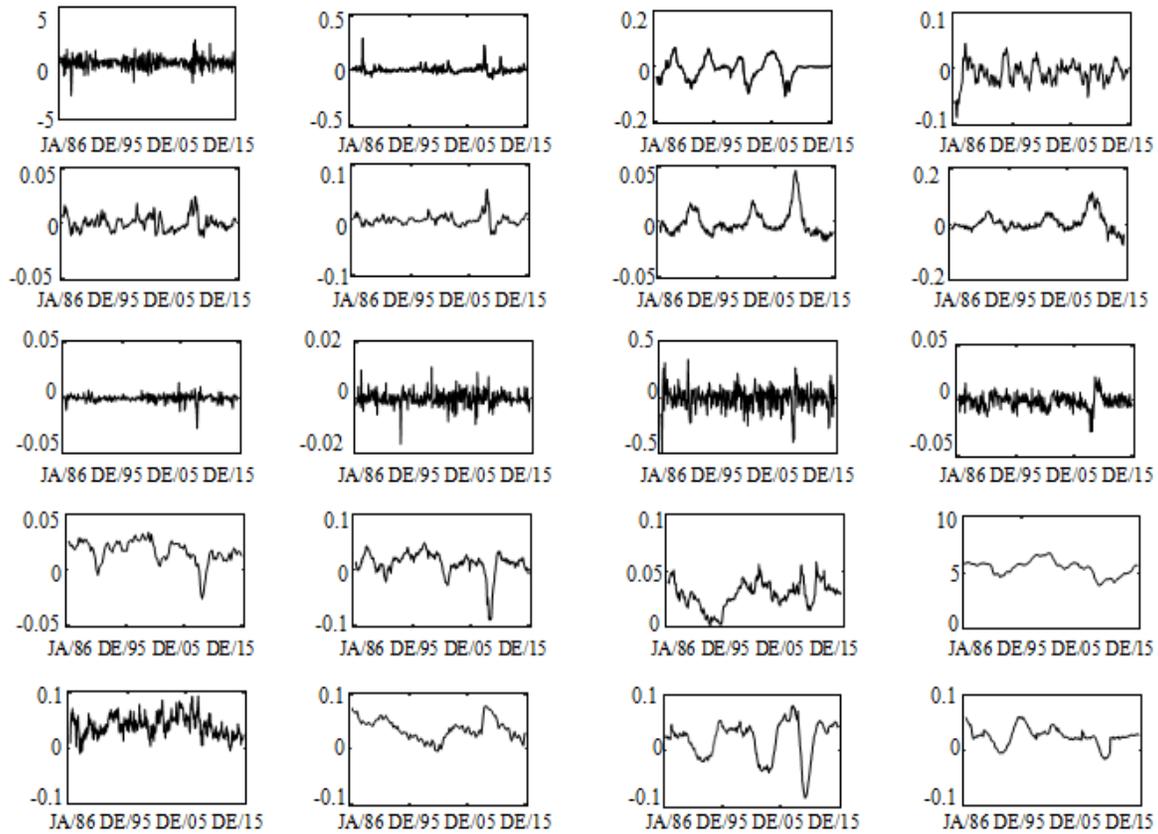


Figure 1. Time series plot of X_1, \dots, X_{20} (from left to right and top to bottom)

5.1 Effects of observed aggregated factors and latent variables

To study the effects of observed and unobserved variables on market structural breaks, we first carry out studies without and with latent variables, which are equivalent to assume that $U_l \equiv 0$ and $U_l \sim (4)$, respectively. In both studies, we first use the method of moments to have a rough estimate for prior parameters $\{\alpha_{ij}, \beta_i; 1 \leq i \leq K-1, 1 \leq j \leq K\}$ of Gamma distribution. Then we use the EM and stochastic approximation algorithms with MCMC simulations in Sections 4.2 and 4.1, respectively, to make inference on parameters Θ .

Table 3 on page 12 shows the estimated coefficients, t -statistics, and corresponding p-values in both cases. In the case of no latent variables (i.e., $U_l \equiv 0$), aggregated variables X_3 (changes of 3-month T-Bill rates), X_5 (changes of Moody's Aaa corporate bond yields), X_{12} (changes of CFNAI), X_{14} (growth rates of industrial production), X_{15} (growth rates of M2 money stock) and X_{19} (growth rates of the total outstanding consumer credit) are not significant. In the case that U_l follows equation (4), the aggregated variables X_4 (changes of 10-year Treasury rates), X_{14} , X_{19} are not significant. In both studies, X_{14} and X_{19} are not significant. The effect of other variables in the two studies are very different. For instance, the case of $U_l \equiv 0$ shows that X_3 are not significant, while the case of $U_l \sim (4)$ indicates that X_4 are not significant. Besides, the estimated coefficients for X_1, X_2 , and X_9 in $U_l \equiv 0$ are similar to those of $U_l \sim (4)$, but the signs of estimated coefficients for $X_7, X_8, X_{10}, X_{11}, X_{16}, X_{17}, X_{20}$ are opposite in both studies. Hence, after dynamic latent factors U_l are incorporated into the model, variables X_3, X_5, X_{12}, X_{15} become significant, while variable X_4 becomes insignificant. This finding on structural breaks is related to those in Koopman *et al.* (2009), where after a dynamic latent factor is

introduced into the study of dependence of firms' risk exposure on economic variables, the effect of some observable variables changes significantly.

The effect of estimated coefficients can be further interpreted in the following way. Take X_2 and X_{18} in the case $U_l \equiv 0$ as an example, their coefficients show that one unit increase of X_2 (changes of S&P 500 monthly realized volatility) and X_{18} (growth rates of public debt of the U.S. Federal government) could increase the odds ratio of structural break probabilities by a factor of $e^{1.651} \approx 5.21$ and $e^{1.243} \approx 3.47$, respectively. We also note that the estimated autoregressive coefficient for latent factors U is -0.439 , suggesting the latent dynamic factors are moderately persistent. On the other hand, the error variance of the latent factor is 6.148 , hence the unconditional variance of U is about $6.148/(1 - (-0.439)^2) = 7.616$ as all other covariates in the analysis are standardized (i.e., the variance is 1). The bottom panel of Figure 2 shows the range of simulated $\{U_l\}$ for $l = 1, \dots, L$. From the time series plot of $\{U_l\}$, we can see that the effect of latent dynamic factors U is comparable to the observed economic factors.

Table 3. Estimated parameters without and with latent variables.

	$U_l \equiv 0$			$U \sim (4)$		
	Estimate	t-Stat	p-value	Estimate	t-Stat	p-value
θ_0	-15.467	-33.767	.000	15.619	56.299	.000
θ_1	0.789	2.514	.011	0.850	5.402	.000
θ_2	1.651	3.259	.000	1.920	10.605	.000
θ_3	-0.489	-0.701	.482	5.403	20.956	.000
θ_4	-2.329	-3.226	.000	0.179	0.746	.456
θ_5	-0.256	-0.244	.806	3.069	7.161	.000
θ_6	-2.575	-3.078	.002	-1.271	-2.364	.018
θ_7	7.720	6.528	.000	-1.517	-3.488	.000
θ_8	-6.127	-6.315	.000	1.469	3.114	.002
θ_9	1.103	2.507	.012	1.207	5.782	.000
θ_{10}	-0.789	-2.144	.032	2.155	14.916	.000
θ_{11}	-1.667	-4.581	.000	1.672	11.133	.000
θ_{12}	0.043	0.098	.920	0.893	4.276	.000
θ_{13}	-1.428	-1.513	.130	1.235	4.102	.000
θ_{14}	-0.084	-0.079	.936	-0.520	-1.219	.224
θ_{15}	0.561	0.895	.370	7.066	27.185	.000
θ_{16}	9.550	9.089	.000	-9.907	-21.117	.000
θ_{17}	-2.339	-4.906	.000	0.570	3.356	.000
θ_{18}	1.243	1.871	.062	-4.477	-17.910	.000
θ_{19}	0.418	0.447	.654	0.030	0.092	.926
θ_{20}	-5.400	-7.981	.000	1.421	8.779	.000
a	N/A			-0.439	-11.331	.000
σ^2	N/A			6.148	38.933	.000
v	N/A					

To further see the difference between the cases of $U_l \equiv 0$ and $U_l \sim (4)$, we estimate structural break probabilities p_l and rating transition probabilities for $l = \text{January 1986}, \dots, \text{December 2015}$ in both cases. In the case of $U_l \sim (4)$, since the estimation procedure involves the step of drawing m paths of $\{U_l; l = 1, \dots, L\}$ from some posterior distributions, we simply take the value $U_l = 0$ when we use the

estimated coefficients to compute the structural break and rating transition probabilities. In particular, structural break probabilities are computed as discussed in Section 4.2 and shown in the top and middle panels of Figure 2. We also mark three periods of economic recessions (July 1990 — March 1991, March 2001 — November 2001, and December 2007 — June 2009) announced by NBER as shaded areas in the top and middle panels of Figure 2 so that we can see that the periods of structural breaks are not necessarily coincident with beginning or ending periods of economic recessions. Both the top and middle panels show that most structural break probabilities \hat{p}_l are almost zero except at a few periods. Specifically there are seven months at which \hat{p}_l are much larger than zero.

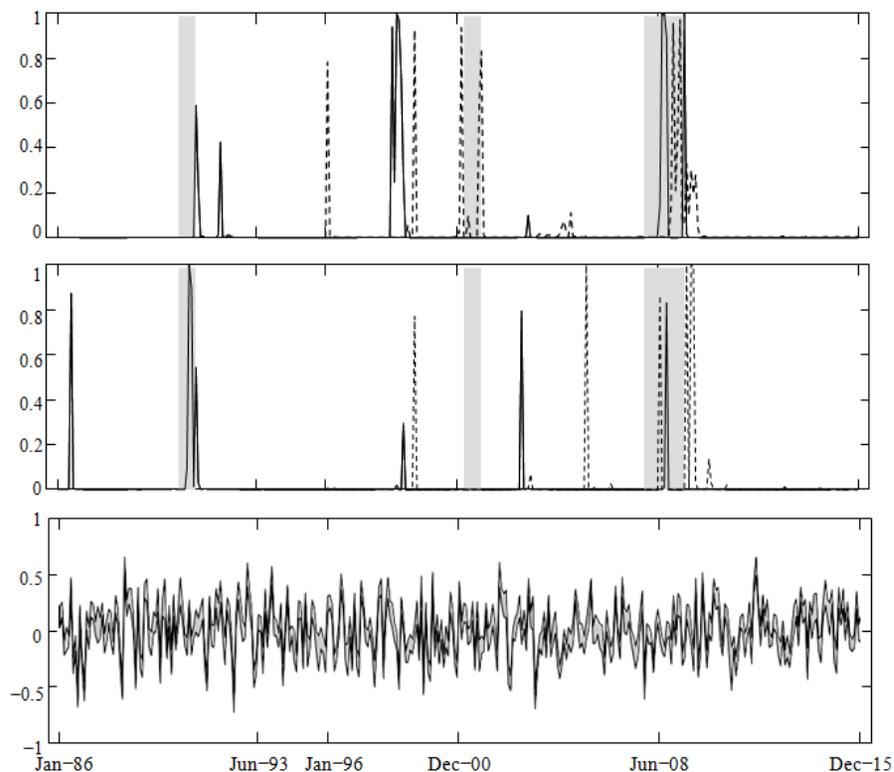


Figure 2. Top: Estimated (solid) and predicted (dotted) probabilities of structural breaks when $U_l \equiv 0$. Middle: Estimated (solid) and predicted (dotted) probabilities of structural breaks when $U_l \sim (4)$. Bottom: The range of simulated U_l 's when $U_l \sim (4)$.

These periods are December 1990 — March 1991, July 1998 — September 1998, June 2009 — August 2009 in the top panel and July 1986, December 1990 — January 1991, December 1998, May 2003, October 2008 in the middle panel. To show the paths of U_l simulated from the MCMC step, we plot the curves of the minimum and maximum of simulated U_l at each time point t_l . The range of U_l at each time point l is very narrow and the band fluctuates around its mean level. We further compare the estimated transition probabilities in the case of $U_l \equiv 0$ and $U_l \sim (4)$. Figure 3 on the next page shows the estimated transition probabilities of AAA \rightarrow C, BBB \rightarrow AAA, and B \rightarrow D in both cases. They are roughly in the same magnitude in both studies, but the existence of latent variable yields more volatile transition probabilities during 1998 and 2000. Note that Figure 3 indicates that, when the estimated structural break probabilities are high, firms' transition probabilities do have changes.

5.2 Variable selection when latent variables are omitted

We next study the issue of selecting important factors if the observed covariate set contains too

many factors. We consider two sets of economic covariates, one set includes the 20 aggregated covariates $\{X_1, \dots, X_{20}\}$, and the other treats Y 's and their lags (up to 24 months) as different variables, that is, $X_{24(i-1)+h,t} = Y_{i,t-h}$, $h = 1, \dots, 24$, $i = 1, \dots, 20$. We use the penalized estimation method in Section 4.2 to select and estimate the regression coefficients. As the procedure involves weight parameter ϕ and tuning parameter γ , we choose ϕ to be 0.9 and 1 so that the effect of unimportant factors can be shrunk to 0, and $\gamma = e^{-1/2}, e^{-1}, \dots, e^{-10}$ ¹.

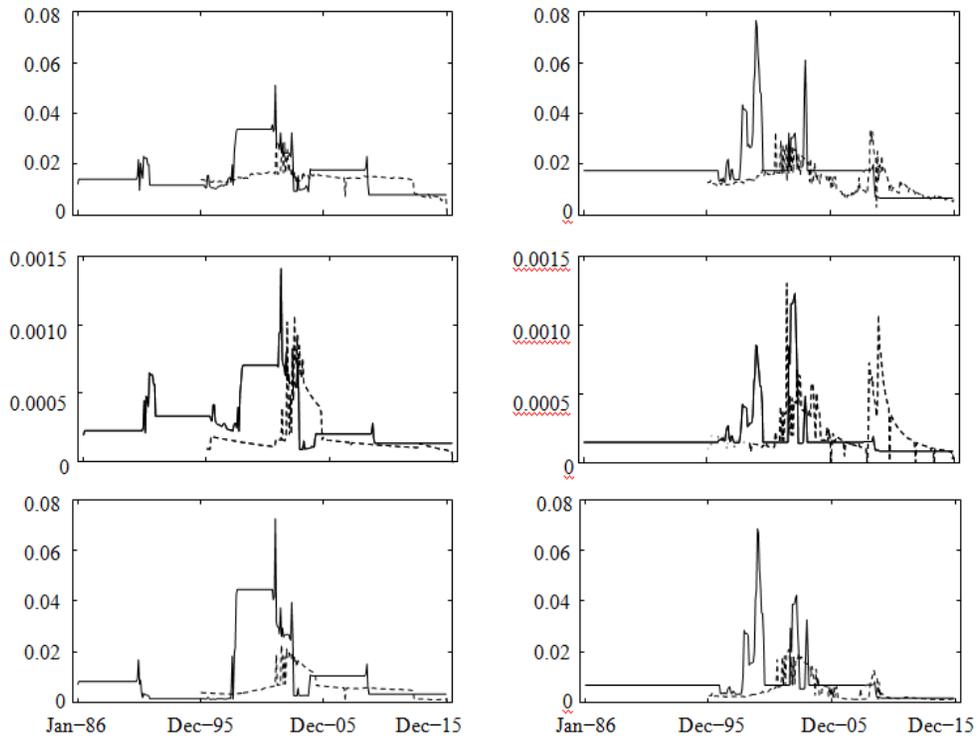


Figure 3. Estimated (solid) and predicted (dotted) rating transition probabilities of AAA → AA (Top), BBB → AAA (Middle), and B → D (Bottom) when $U_i \equiv 0$ (Left) and $U_i \sim (4)$ (Right)

Figure 4 plots the estimated coefficients in the regularized regression for $\{X_{i,t}\}$ (the top two panels) and $X_{24(i-1)+h,t}$ (the bottom two panels), respectively. In these plots, regression coefficients are shrunk to zero when the tuning parameter γ is larger than e^{-3} . In the top two panels, the covariates X_4, X_{14}, X_{15} , and X_{18} are selected and others are shrunk to 0 for $\gamma = e^{-5.5}$. Note that these variables represent variations of 10-year Treasury rates, growth rate of the industrial production, growth rate of M2 money stock, and growth rate of public debt of the U.S. Federal government. In the bottom panels that lagged variables are considered as separate factors, we focus on the study of $\gamma = e^{-2}$. When $\phi = 1$, selected variables includes lagged variables of $X_{12}, X_{13}, X_{15}, X_{16}, X_{19}$ and X_{20} . These selected variables suggest not only which economic aspects are important for market structural breaks, but also how far these variables should be traced back. When $\phi = 0.9$, the result and its interpretation are similar except that more variables are selected, comparing to the case of $\phi = 1$. Note that, the sets of selected variables are not only different for using aggregated and lagged variables, but also from that of

¹ For the values of ϕ larger than 0.8, the selected variables are similar, so we only present the results of $\phi = 0.9$ and 1.

significant variables estimated in Section 5.1. Such inconsistency suggests that the variable selection procedures here might be sensitive to the input covariates.

5.3 Out-of-sample forecasts of market structural breaks

We perform an out-of-sample study for market structural breaks using significant variables estimated in Section 5.1. Specifically, we remove variables $X_3, X_5, X_{12}, X_{14}, X_{15}$ and X_{19} in the case without latent variables (i.e., $U_l \equiv 0$) and remove variables X_4, X_{14} , and X_{19} in the case with latent variables (i.e., $U_l \sim (4)$). For the purpose of comparison, we use the in-sample estimates of p_l in Section 5.1 as a benchmark. We then carry out a forecasting procedure as follows. At each month t_L , we use firms' rating transition and economic factors ($Y_{(0,t_L)}, F_{t_L-1}$) as the training sample to estimate the model parameters, and then compute the one-month ahead forecast p_{L+1} and generator matrices of rating transitions for the next period (t_L, t_{L+1}) as discussed in Section 4.3. We implement this sequentially for $t_L = \text{January 1996}, \dots, \text{December 2015}$ and plot the forecasted p_{L+1} (dotted lines) and firms' rating transition probabilities (dotted lines) in the top and middle panels of Figures 2 and 3.

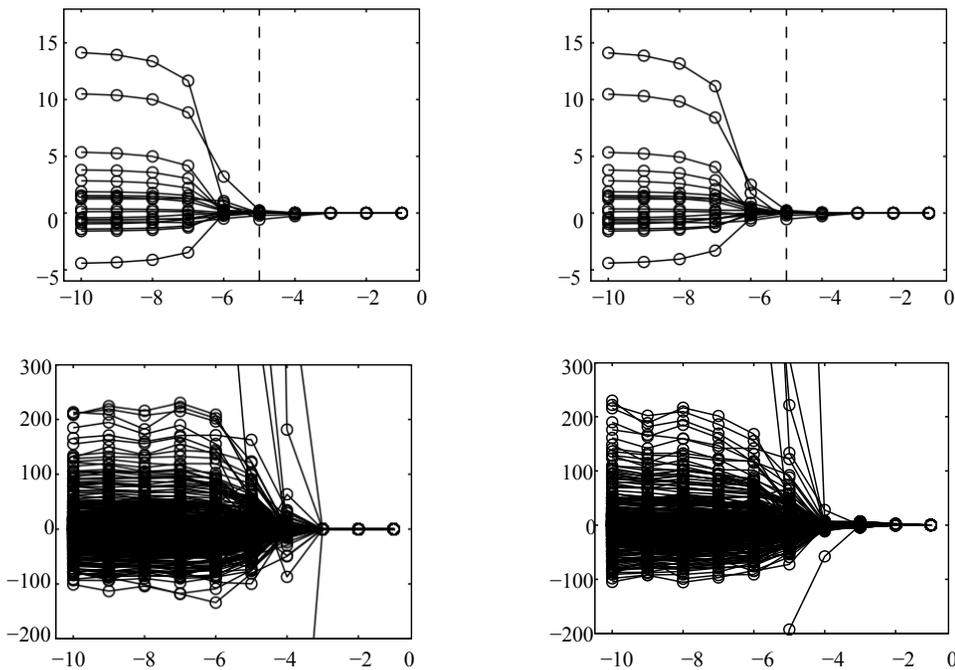


Figure 4. Regularized model parameter paths of $\{X_{i,t}; i = 1, \dots, 20\}$ (top) and $\{\tilde{X}_{24(i-1)+h,t}; h = 1, \dots, 24, i = 1, \dots, 20\}$ (bottom) versus $\log \gamma$. The left and right panels corresponds to the cases of $\varphi = 1$ and 0.9 , respectively. A vertical line is drawn at $\log \gamma = -5$.

In the case without latent variables, the predicted structural break probabilities have seven peaks, which are February 1996 (0.783), May 1999 (0.927), February 2001 (0.936), November 2001 (0.832), January 2009 (0.956), April 2009 (0.971), September 2009 (0.301). In the case with latent variables, the predicted structural break probabilities have only four peaks, which are May 1999 (0.772), October 2005 (0.999), July 2008 (0.864), September 2009 (0.999). We note that both studies show that the probabilities of structural breaks at May 1999 and September 2009 are large. To interpret this result, we note that May 1999 is several months after the occurrence of a series of disruptive events such as the Russian's default, Brazil's currency crisis and the downturn of the LTCM, and September 2009 is only three months after the end of economic recession announced by the NBER. These predicted structural

breaks may indicate that the credit market might start to turn over during those periods. To further demonstrate the predicted structural break probabilities are reasonable, we plot predicted firms' rating transition probabilities in Figure 3. We can see that when predicted structural break probabilities are high, predicted transition probabilities do show big changes over time. For other periods, the predicted probabilities of structural breaks are very different with and without dynamic latent variables. For instance, the forecasted structural breaks without latent variables capture the economic recession during March 2001—November 2001, while the forecast with latent variables does not. For October 2005, it is the period that the U.S. housing bubble began to burst, causing house prices stop rising and begin to decline. This period was captured by the prediction with latent variables, but not by the forecasts without latent variables.

Combining the predictions for probabilities of structural breaks and firms' rating transition probabilities may have some interesting implications for policy makers and investors. For instance, in the predictions with latent variables, the predicted probability of structural break at July 2008 is over 0.8, while the predicted transition probabilities of AAA \rightarrow AA and BBB \rightarrow AA change significantly, while the predicted transition probability of B \rightarrow D does not change much. This indicates that the credit exposures of firms with investment grade ratings may change significantly, investors of holding positions on those firms should accordingly adjust their portfolios to mitigate the potential loss. For policy makers, based on the sequentially estimated coefficients of economic variables on probabilities of structural breaks, they may consider using some policy tools, for example, adjust the value of X_{15} (the aggregated M2 money stock) or the value of X_{18} (the aggregated growth rate of public debt of the U.S. Federal government), and so on, to influence the chance of further structural breaks.

Although we can use historical events to interpret the out-of-sample predictions with or without latent variables, the analysis above also suggests some issues. For example, in some periods that predicted high probabilities of structural breaks do not match the in-sample estimates of probabilities structural breaks, it is not clear that this predicted structural breaks are simply false alarms, or they can be considered as examples of Lucas' critique on macroeconomic policymaking (Lucas, 1976), hence the potential structural breaks were eliminated by some economic policy interventions. On the other hand, when the in-sample estimates of high probabilities of structural breaks are missed in the out-of-sample study, it might suggest the estimated model from the in-sample analysis is not good enough for the out-of-sample prediction.

6. Concluding Remarks

To study the dependence of market structural breaks on variations of economic and market fundamentals, we consider a model that integrates Xing *et al.* (2012)'s model on structural breaks of firms' rating transition dynamics and a multiplicative model for intensities of Poisson process. Using the discrete time version of the model, we show how to extract the information of market structural breaks from firms' rating records and connects probabilities of structural breaks with observed and dynamic latent variables. We then use a stochastic approximation algorithm with MCMC simulations and an EM algorithm to make inference on the cases with and without latent variables, respectively.

We use the above model and inference methods to study the relationship between the probabilities of structural break in the market consisting of U.S. firms and variations of economic and market fundamentals. Our analysis shows that some market or economic variables indeed have significant impact on market structural break probabilities, no matter dynamic latent factors are included or not in the study. However, significant variables when latent factors included are different from those in the case without latent factors. We also consider the issue of selecting fewer

explanatory variables when a large number of factors are available for the study. In particular, we select and estimate significant variables by the regularization method in statistics. Furthermore, we investigate the out-of-sample forecasts of market structural breaks. We find that some predicted market structural breaks with high probabilities are consistent with the in-sample analysis. We also estimate and predict the probabilities of firms' rating transitions with and without latent factors included.

The results in the paper also provide some indications for further studies. First, the current model assumes firms are homogeneous, while in the real world, firms are heterogeneous and some firm-specific variables cannot be observed even. An interesting and challenging issue is whether structural breaks can still be forecasted if heterogeneous firms are used. Second, this study does not consider the feedback effect of structural break, hence how to incorporate such effect is an intriguing problem.

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