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Age-Efficiency and Replacement Requirements for Measures of Capital Services

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Abstract: Measures of capital input depend on assumptions that are typically made without clear direction from economic theory, such as the best method for incorporating the physical deterioration of assets into measures of capital stocks and capital services. Compared with alternative age-efficiency profiles that might be assumed, a geometric profile has the advantages of general applicability, internal consistency, ease of use in data construction, and most importantly its robust approximation to the physical deterioration of assets given the types and sources of data that are typically available to researchers for constructing such measures.

Keywords: Capital services; Age-efficiency profiles; Deterioration; Depreciation; Geometric profile
JEL Classification: D24, E22

1. Introduction

Economists use measures of the quantity of capital services to estimate production functions, measure productivity, and inform economic policy. Studies by Griliches (1963), Griliches and Jorgenson (1966), Coen (1968), and Hall (1968) developed much of the theory related to measuring capital input as a service flow; however, the construction of these measures often depends on assumptions that are made without a clear direction from economic theory (OECD 2009; Schreyer 2003), such as the appropriate age-efficiency profiles to use when constructing the measures.

The age-efficiency profile of an asset (or group of assets) refers to the deterioration of its productive capacity as it ages, a concept that is separate from but related to economic depreciation. It is also important to note the distinction between the physical deterioration of an asset such as a specific piece of machinery, and the physical deterioration of the stock of a heterogeneous group of assets, which may each have very different patterns. In this paper, we examine the replacement requirements to add and maintain a unit of productivity capacity to the capital stock, comparing the assumptions of using a geometric, hyperbolic, and linear functional form for the age-efficiency profile.[1] We argue that the productive efficiency of the stock (and therefore the service flow) of capital is well-approximated using a constant geometric rate of deterioration applied to an estimate of the stock, regardless of the pattern of deterioration for any individual pieces of machinery or

[1] Note that we are assuming that capital stocks and capital service flows are proportional, sometimes called the proportionality assumption.

other elements of the capital stock. Furthermore, the assumption of a geometric age-efficiency profile with a constant rate of deterioration has the advantages of general applicability, internal consistency, ease of use in data construction, and most importantly its robust approximation to the deterioration of assets given the types and sources of data that are typically available to researchers for constructing such measures.

2. Economic Depreciation and Age-efficiency Profiles

While many economists assume that the economic depreciation of an asset follows a geometric pattern over time, some make different assumptions about the physical deterioration of assets when constructing a measure of the quantity of capital services. For example, the U.S. Bureau of Labor Statistics (BLS) assumes a hyperbolic age-efficiency profile when measuring capital for the purpose of calculating indexes of multi-factor productivity, as indicated by Dean and Harper (1998) and Harper (1999). Measures of capital services for U.S. agriculture constructed by the USDA Economic Research Service utilize a hyperbolic deterioration assumption based on evidence from Penson *et al*. (1977) and Romain *et al*. (1987).[2] Similarly, the OECD uses a hyperbolic age-efficiency profile in constructing measures of capital services and multi-factor productivity (Schreyer 2003). Alternatively, measures of capital services in U.S. agriculture used in studies by Alston *et al*. (2011) and Andersen *et al*. (2011), and described in Pardey *et al*. (2009), were constructed assuming a geometric age-efficiency profile. Such assumptions can have important consequences for measures of capital input and their economic implications.

Hulten and Wykoff (1981, 1996) and Jorgenson (1996) provide extensive treatments of the issues surrounding the measurement of economic depreciation. These studies examine changes in the market value of assets over time to infer rates and patterns of depreciation. The change in the market value of an asset over time, measured in current dollars, can be attributed to two main sources, depreciation and re-valuation (capital gains and losses). Depreciation is the reported loss in the constant-dollar value of the asset measured over time. Commonly, it is assumed that an asset loses value over its life either by a constant proportion each period (geometric depreciation), by a constant amount each period (straight-line depreciation), or not at all until the end of its useful life, at which time 100 percent depreciation applies (a one-hoss-shay pattern). Another possibility is to assume that depreciation follows a concave (hyperbolic) pattern, with a rate that increases over the service life of the asset.

The only components of depreciation that are of interest for measuring the loss in the productive capability of capital assets are those that affect the productivity of capital: namely, physical deterioration (i.e., physical as opposed to other forms of economic depreciation). The empirical evidence suggests that the value of most durable assets declines geometrically, a view that is widely supported in the literature; however, a geometric decline in the value of an asset could be consistent with a hyperbolic, linear, or geometric decline in its physical capabilities. This is because the physical deterioration of an asset is only one component of economic depreciation.[3]

[2] The USDA data and construction methods can be found online at: https://www.ers.usda.gov/data-products/agricultural-productivity-in-the-us/.

[3] Economic depreciation includes physical deterioration, exhaustion, obsolescence, changes in the supply and demand for the asset, and changes in the general price level.

The assumption of a constant geometric rate of deterioration, δ, is widely used in the literature on capital measurement because it is simple to apply and provides a good approximation to physical deterioration of the productive stock of capital, which might include assets of different deterioration patterns, service lives, and vintages. Also, the assumption of geometric deterioration provides an internal consistency between estimates of the productive stocks of assets and their rental rates, when the same δ is used in the calculation of both the stocks (and flows) and the rental rates. Jorgenson (1995, p. 218) argued that the ". . . available empirical evidence supports the use of geometric decline in efficiency as a useful approximation to replacement requirements and depreciation." Additionally, Hulten (1990, p.142) wrote: "The studies of Fraumeni and Jorgenson (1986), Jorgenson, Gollop, and Fraumeni (1987), Boskin, Robinson, and Huber (1989), and Boskin, Robinson, and Roberts (1989) use the Hulten-Wykoff estimates of δ more or less directly. These studies accept the best geometric approximation and use the self-dual property of geometric depreciation to calculate stocks and user costs using the same δ ."

Studies by Schreyer (2003) and Biatour *et al*. (2007) investigated the implications of different age-efficiency profiles for measures of capital services, concluding that estimates of capital services are affected by the use of different age-efficiency profiles and the decision of which profile to use should be based on the nuances of the application, and that more research was needed on the subject.

3. Replacement Requirements for Capital Services

While the empirical evidence supports assuming a geometric pattern for economic depreciation, less information is available on the patterns of the physical deterioration of assets, which is the relevant concept when measuring capital inputs for empirical studies of production and productivity. Perhaps this is one reason why there is a lack of consensus in the literature on the appropriate pattern to employ when measuring capital usage. Part of the disagreement relates to the distinction between the decline in efficiency of an individual asset and the decline in efficiency of the stock of a heterogeneous group of assets. Berndt (1990, p.155), wrote: "…because of varying vintage composition over time, the average efficiency (deterioration) function of an entire cohort can be quite different from the individual efficiency functions; while each asset in a stock might, for example, follow the one-hoss-shay form, the cohort as a whole can follow a rather different age-efficiency (deterioration) pattern."

The pattern of deterioration of an aggregate measure of capital can be different from that of the individual assets, and this is true when aggregating assets with identical service profiles but different vintages, assets with the same vintage but different service profiles, or a combination of different service profiles and vintages. Heterogeneity of the assets in the various cohorts will impart convexities in the deterioration pattern in the physical capacity of the aggregate; however, there is an additional and less discussed reason to choose a geometric age-efficiency profile that applies to individual assets that is separate from the aggregation issue. Consider the physical deterioration of an individual asset from the perspective of what is required to maintain the productive capacity of the asset over time, commonly called the replacement requirements.[4] If a producer wants to permanently add one unit of productive capacity, they will incur a stream of

[4] The change in the capital stock each period is defined as investment minus replacement requirements.

replacement costs, R_t , which are necessary to maintain the additional unit. The infinite stream of replacements (which includes replacements of replacements) generated by an initial investment approaches a fixed proportion of the accumulated capital stock. Griliches and Jorgenson (1967, p. 255) wrote:

> "*The appropriate model for replacement of investment goods is not the distribution over time of replacements for a given investment, but rather the distribution over time of the infinite stream of investment generated by a given investment. The distribution of replacements for such an infinite stream approaches a constant fraction of the accumulated stock of investment goods for any "survival curve" of individual pieces of equipment and for an initial age distribution of the accumulated stock, whether the stock is constant or growing*."

The replacement requirements for an asset are determined by a mortality distribution, which is a combination of an assumed service life and deterioration pattern as defined in Coen (1975). We can define a mortality distribution for a homogeneous class of capital with an assumed two-year service life that loses a fraction of productive capacity during each year of use, equal to d_1 in year one and d_2 in year two, with $d_1 + d_2 = 1$. Also note that we can define three age-efficiency profiles including hyperbolic $(d_1 < d_2)$, linear ($d_1 = d_2$), and geometric ($d_1 > d_2$). The permanent addition of one unit of productive capacity to the stock requires a stream of R_t that can be expressed as a second-order homogeneous difference equation, $R_t = d_1 R_{t-1} + d_2 R_{t-2}$, with initial conditions $R_0 = 1$, $R_{-1} = 0$, and solution: [5]

$$R_t = \frac{1-(-d_2)^{t+1}}{1+d_2} \qquad (1)$$

Note that d_1 and d_2 can imply a hyperbolic loss in productive capacity over the assumed service life, yet the stream of replacements (including replacement of replacements) converges to a constant fraction of the accumulated stock:

$$\lim_{t\to\infty} R_t = (1 + d_2)^{-1} \qquad (2)$$

Figure 1 shows R_t over a 20-year period with an assumed geometric, linear, and hyperbolic decline for the individual asset.

Each of the R_t series converges to a constant proportion of the initial investment, with the geometric series converging fastest, and the hyperbolic slowest. The result is that, regardless of the assumed age-efficiency profile, the entire stream of replacement investments that is required to maintain the initial investment approaches a constant rate. If that one unit of productive capacity is added permanently, replacement at time *t* is equal to deterioration at *t*, and the physical deterioration of the accumulated capital stock converges to a constant fraction of the stock, regardless of the pattern of deterioration of the initial investment.[6] This provides another reason why the assumption of geometric deterioration provides a good approximation to the decline in physical productivity of aggregate capital, whether that aggregate be an agglomeration of identical assets, or assets with the

[5] See Appendix for the derivation of equation (1).

[6] It is important to note that the same results hold for assets with service lives longer than two periods.

same patterns of deterioration but different vintages, or of assets with different patterns of deterioration but with the same vintage, or of different patterns of deterioration and different vintages.

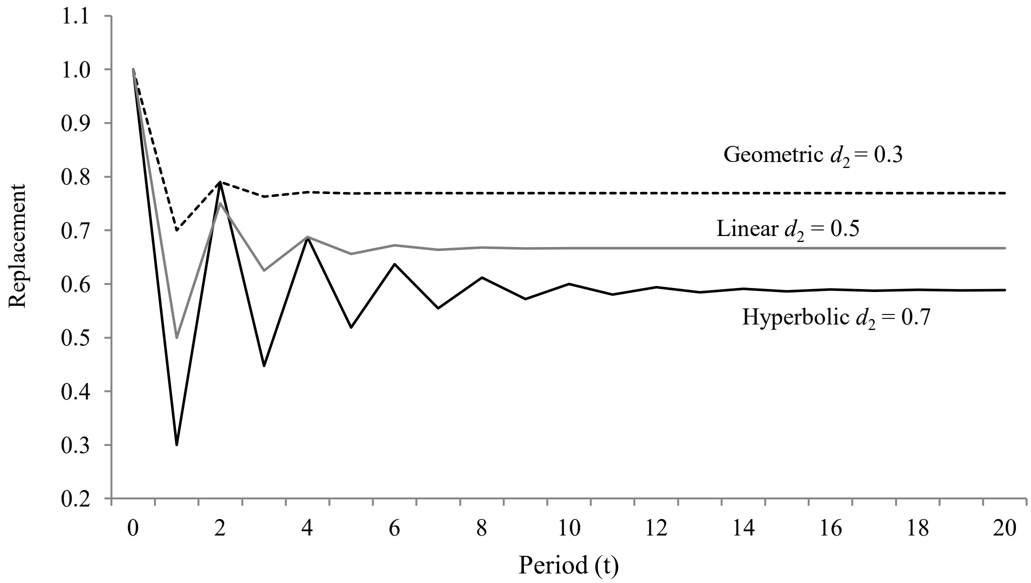

Figure 1. Replacement Requirements, R_t , for the permanent addition of one unit of capital stock

4. Conclusion

The primary sources that are available for constructing measures of capital input are either data on investment in new assets, or data on the counts of existing assets, where the assets are typically heterogeneous to some extent. The broad categories of assets typically reported in statistics from various government agencies include assets of different quality, type, and vintage, and this is an unavoidable data constraint even when working with investment data. This heterogeneity of reported assets contributes to a geometric pattern in the deterioration in the productive capacity of a measure of aggregate capital services. While other age-efficiency profiles might apply to an individual asset, they are unlikely to apply for an aggregate measure that includes heterogeneous assets. Furthermore, if we equate the physical deterioration in the capital stock to the replacement requirements to add and maintain a unit of productive capacity, then the physical deterioration approaches a constant proportion of the stock regardless of the assumed age-efficiency profile.

The assumption of a geometric age-efficiency profile is convenient from an empirical perspective, provides an internal consistency between estimates of capital stocks and their rental rates, and provides a robust approximation of the deterioration in the productive capacity of a capital aggregate with the additional benefit that it simplifies many of the calculations involved in measuring capital services and their rental rates.

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Appendix

Adding one unit of productive capacity to an asset with a two-year service life requires replacement investment at the end of period t that can be expressed as a homogeneous second-order difference equation, $R_t = d_1 R_{t-1} + d_2 R_{t-2}$, with $d_1 + d_2 = 1$ and the initial conditions $R_0 = 1$ and $R_{-1} = 0$. Using recursive substitution this second-order difference equation can be expressed as:

$R_{-1} = 0$

$R_0 = 1$

$R_1 = d_1 R_0 + d_2 R_{-1} = d_1 = 1 - d_2$

$R_2 = d_1 R_1 + d_2 R_0 = (1 - d_2)^2 + d_2 = 1 - d_2 + d_2^2$

$R_3 = d_1 R_2 + d_2 R_1 = (1 - d_2)(1 - d_2 + d_2^2) + d_2(1 - d_2) = 1 - d_2 + d_2^2 - d_2^3$

$$\vdots$$

$R_T = \sum_{t=0}^{T} (-d_2)^t = R_0 \times \frac{1-(-d_2)^{T+1}}{1-(-d_2)} = \frac{1-(-d_2)^{T+1}}{1+d_2}$ for any T = 0, 1, 2, 3, ...

since R_T is a convergent geometric series with the common ratio of $(-d_2)$ which is $0 < |-d_2| < 1$.