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Dynamic Firm R&D Games: Manufacturing Costs and Reliability Paths

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Abstract: Consider a dynamic intra-industry trade model with two goods, two firms, and two countries in which product "reliability" is determined by R&D paths. This paper focuses on how a change in competitive conditions in terms of manufacturing costs affects the firms' decision about optimal reliability. Briefly, the main result of the paper is that when the manufacturing costs are similar and closely track each other, a lower manufacturing cost prompts both firms to increase their R&D and product reliability. But when the manufacturing costs are not similar, either before or after the change, the results are quite different. A profit maximizing firm will sometimes take advantage of a reduction in its own manufacturing cost by actually doing less R&D—and thus producing a less reliable product.

Keywords: Differential game, Innovation competition, Product reliability

JEL Classifications: D92; O31; F12

1. Introduction

Firms competing in global markets in many industries find their research and development programs critical to their long term success. Consider a dynamic two-good, two-firm, two-country intra-industry trade model. Suppose R&D makes a product more "reliable" so that it reduces the customer costs associated with an unreliable product as well as reducing the firm's manufacturing costs since fewer replacement units have to be produced. In maximizing profits over the planning period, each firm's R&D path depends on the market demands, how customers value reliability, its own costs, and the costs of its competitor. But although the R&D paths depend on the complete underlying structure of the model, the particular focus of the present paper is on the effect of an exogenous change in manufacturing costs on the R&D paths.

Suppose one firm experiences a reduction in the per unit marginal cost of manufacturing its product. The question is whether that will cause it to increase its R&D spending and produce a more reliable product. The short answer is that it depends on what happens to the costs of its competitor. Briefly, if the manufacturing costs are similar and closely track each other then, yes, a lower manufacturing cost will prompt the firm to increase its R&D and product reliability. But when the manufacturing costs are not similar, either before or after the change, the results are quite different. A profit maximizing firm will sometimes take advantage of a reduction in its own manufacturing cost by actually doing less R&D—and thus producing a less reliable product.

While the paper is agnostic about the reason for the exogenous reduction in manufacturing cost it might, for example, come from a fixed-cost one-off project, whether for original research, some sort of backward engineering, or adopting a technology developed by the government. (To be absolutely clear, such a one-off project is *not* what we mean by R&D in our model, our notion being that R&D is a continuous process of product improvement.) While beyond the scope of this paper, much of the public debate about the extent to which the government should support private sector R&D often essentially assumes there is a connection between giving firms technology to produce things more cheaply and thus allowing it to produce a better product for its customers. So, for example, suppose government scientists develop a new process which lowers manufacturing costs for firms—and it essentially just gives this technology to the firms. Opponents of industrial policy often argue that is essentially what the government is doing. Our results suggest that the firm will sometimes respond by improving its product, but sometimes it will do the opposite. One goal of the paper is to organize the parameters in such a way as to allow us to give a relatively straightforward analysis of when reliability goes up and when it goes down. This is done in Section 4.

There is beginning to be a literature with a formal model structure similar to that of the present paper. But as far as we are aware, the literature is either static (Haaland and Kind (2008, 2006); Gretz, Highfill, and Scott (2012, 2011)) or if dynamic, the R&D path is only endogenous for one firm, not for both (Highfill and McAsey (2010, 2010)). There is also an emerging literature on trade-type dynamic R&D paths somewhat further afield from the present paper. The focus has been either on export or invest type questions, falling in the FDI literature (for example see Petit, Sanna-Randaccio, and Sestini (2012), Sanna-Randaccio (2002)), whether firms should form R&D cooperative ventures (for example see Cellini and Lambertini (2009), Petit and Tolwinski (1999)) or spillovers (for example see Femminis and Martini (2011)). It should be mentioned that there is an extensive related empirical literature on firm-level R&D practices (for example see Lokshin, Belderbos, and Carree (2008)).

2. The Model

2.1 Demand-Side Theory

Assume a dynamic two-good, two-firm, two-country intra-industry trade model. At each moment in time the firms play a two-stage game with the order of play as follows. First, each firm chooses reliability (equivalently, R&D spending) holding the other firm's reliability constant, and then each firm decides its own quantity holding the quantity of the other firm constant. Subgame perfect Nash equilibria are computed using generalized backward induction. As such, the quantity decisions are analyzed first. R&D spending is strategic because each firm chooses its reliability in order to affect the other firm's reliability. Countries are denoted i = A, B and firms j = 1, 2. Firm 1 is located in country A in the sense that its R&D and production are conducted there. Similarly, firm 2 is located in country B. Denote firm 1's domestic sales in country A by $Q_{A1}(t)$ and their selling price by $P_{A1}(t)$. Denote firm 1's exports to country B by $Q_X(t)$ and their selling price by $P_X(t)$. Total sales for firm 1 are $Q_1(t) = Q_{A1}(t) + Q_X(t)$. Firm 2's domestic sales in country B are $Q_{B2}(t)$ and their selling price is $P_{B2}(t)$. Firm 2's exports to country A are imports from country A's point of view and are denoted $Q_M(t)$ with a selling price of $P_M(t)$. Total sales for firm 2 are $Q_2(t) = Q_M(t) + Q_{B2}(t)$. Total quantity demanded in country A is $Q_A(t) = Q_{A1}(t) + Q_M(t)$ while total quantity demanded in country B is $Q_B(t) = Q_X(t) + Q_{B2}(t)$.

The general setup for the demand side of the model assumes that customers base their behavior on the "full quality price" of the product which includes the purchase price and an expected cost associated with the product's reliability (in the case the product is not perfectly reliable). Customers whose reservation price for the product is greater than the full quality price will purchase it. Our measure of product reliability for the jth firm, denoted $R_j(t)$, is the probability that any given unit will be judged by the customer to be "reliable" or of acceptable quality. Alternatively, $1-R_j(t)$ is the probability that the product fails and is returned by the customer for exchange or repair. Product failure imposes costs on the customer that are not reimbursed by the firm. This cost to customers of product failure in country *i* is the parameter $K_i(t) = K_{i0}e^{rt}$, $K_{i0} > 0$ and $r > 0$, so the "expected customer cost of product failure" for a customer in country *i* purchasing from the jth firm is $(1-R_j(t))K_{i0}e^{rt}$. Customers know the probability that any arbitrary unit will fail, but not whether the particular unit they purchase will fail.

Reservation prices, $v_i(t)$, at any time t in country i are distributed uniformly on the interval $(W_{i0}e^{rt}, V_{i0}e^{rt})$ where $V_{i0} > W_{i0} > 0$. Customers whose reservation price satisfies the following condition will purchase the product:

$$v_A(t) \geq P_{A1}(t) + (1-R_1(t))K_{A0}e^{rt} = P_M(t) + (1-R_2(t))K_{A0}e^{rt} \quad (1)$$
$$v_B(t) \geq P_X(t) + (1-R_1(t))K_{B0}e^{rt} = P_{B2}(t) + (1-R_2(t))K_{B0}e^{rt} \quad (2)$$

for country A and B respectively. Customers are risk neutral in the sense that their buying decisions are based on the sum of the price and the expected customer cost of product failure, i.e., the middle and right expressions of (1) and (2) for the products of firm 1 and 2 respectively. While in general the firms' costs, qualities, and prices are not the same, for both firms to have positive sales it must be the case that the full quality price is the same for both firms. If this were not the case, customers would only buy from the firm with the lower full quality price. Finally, notice that all parameters are assumed to grow at an exponential rate, captured by e^{rt}; the growth rate may be related to the inflation rate.

Under these assumptions the market quantity demanded for country A is, using (1)

$$\begin{aligned} Q_A(t) = Q_{A1}(t) + Q_M(t) &= N_{A0}e^{st}\int_{P_{A1}(t)+(1-R_1(t))K_{A0}e^{rt}}^{V_{A0}e^{rt}} \frac{1}{V_{A0}e^{rt} - W_{A0}e^{rt}} dv_A(t) \\ &= \frac{N_{A0}}{V_{A0} - W_{A0}} e^{(s-r)t}(V_{A0}e^{rt} - P_{A1}(t) - (1-R_1(t))K_{A0}e^{rt}) \end{aligned} \quad (3)$$

Note that (1) also implies that

$$\begin{aligned} Q_A(t) = Q_{A1}(t) + Q_M(t) &= N_{A0}e^{st}\int_{P_M(t)+(1-R_2(t))K_{A0}e^{rt}}^{V_{A0}e^{rt}} \frac{1}{V_{A0}e^{rt} - W_{A0}e^{rt}} dv_A(t) \\ &= \frac{N_{A0}}{V_{A0} - W_{A0}} e^{(s-r)t}(V_{A0}e^{rt} - P_M(t) - (1-R_2(t))K_{A0}e^{rt}) \end{aligned} \quad (4)$$

as well. Similarly for country B equation (2) implies

$$Q_B(t) = Q_X(t) + Q_{B2}(t) = N_{B0}e^{st}\int_{P_X(t)+(1-R_1(t))K_{B0}e^{rt}}^{V_{B0}e^{rt}} \frac{1}{V_{B0}e^{rt} - W_{B0}e^{rt}} dv_B(t)$$
$$= \frac{N_{B0}}{V_{B0} - W_{B0}} e^{(s-r)t}(V_{B0}e^{rt} - P_X(t) - (1 - R_1(t))K_{B0}e^{rt}) \tag{5}$$

and

$$Q_B(t) = Q_X(t) + Q_{B2}(t) = N_{B0}e^{st}\int_{P_{B2}(t)+(1-R_2(t))K_{B0}e^{rt}}^{V_{B0}e^{rt}} \frac{1}{V_{B0}e^{rt} - W_{B0}e^{rt}} dv_B(t)$$
$$= \frac{N_{B0}}{V_{B0} - W_{B0}} e^{(s-r)t}(V_{B0}e^{rt} - P_{B2}(t) - (1 - R_2(t))K_{B0}e^{rt}) \tag{6}$$

In both market demand functions, the potential market size N_{i0} grows exponentially at a rate of e^{st}; this growth rate may be related to the population growth rate. We also assume that on the demand side of the model the two economies are copies of each other, that is define

$$\text{Customer Value Ratio} = \frac{V_{B0}}{V_{A0}} = \frac{W_{B0}}{W_{A0}} = \frac{N_{B0}}{N_{A0}} = \frac{K_{B0}}{K_{A0}} = \rho_{CV}\ . \tag{7}$$

To say the same thing in slightly different terms, $V_{A0} = V_0$, $V_{B0} = \rho_{CV}V_0$ and so forth for the other parameters . (Even though suppressed in the notation, parameters are the initial value of an exponentially growing parameter path.) For the interpretations, when $\rho_{CV} > 1$ the "Customer Value" parameters $V, W, N,$ and K are greater in country B than in country A and conversely for $\rho_{CV} < 1$.

Using this notation and solving for the (indirect) demand functions yields

$$P_{A1}(t) = \frac{V_0 - W_0}{N_0} e^{rt}\left(V_0 - K_0(1 - R_1(t)) - e^{-st}\left(Q_{A1}(t) + Q_M(t)\right)\right) \tag{8}$$

$$P_M(t) = \frac{V_0 - W_0}{N_0} e^{rt}\left(V_0 - K_0(1 - R_2(t)) - e^{-st}\left(Q_{A1}(t) + Q_M(t)\right)\right) \tag{9}$$

$$P_X(t) = \frac{V_0 - W_0}{N_0} e^{rt}\left(\rho_{CV}\left(V_0 - K_0(1 - R_1(t))\right) - e^{-st}\left(Q_{B2}(t) + Q_X(t)\right)\right) \tag{10}$$

$$P_{B2}(t) = \frac{V_0 - W_0}{N_0} e^{rt}\left(\rho_{CV}\left(V_0 - K_0(1 - R_2(t))\right) - e^{-st}\left(Q_{B2}(t) + Q_X(t)\right)\right) \tag{11}$$

2.2 Manufacturing Costs

Consider next the firm's production costs. Each firm *j* has a per unit manufacturing cost of $mc_{j0}e^{rt}$, $mc_{j0} > 0$. It is assumed that the units of the product that fail are returned by the customer and replaced (or if repaired that the repair cost is the same as the replacement cost) so that the per unit manufacturing costs are

$$c_1(t) = e^{rt}mc_{10}(1 + (1 - R_1(t)) - \sigma R_2(t)) = e^{rt}mc_0(2 - R_1(t) - \sigma R_2(t))$$
$$c_2(t) = e^{rt}mc_{20}(1 + (1 - R_2(t)) - \sigma R_1(t)) = e^{rt}mc_0\rho_{MC}(2 - R_2(t) - \sigma R_1(t)). \tag{12}$$

Stated slightly differently, for firm 1 the manufacturing cost of the original units is $mc_{10}e^{rt}Q_1(t)$ (where $Q_1(t) = Q_{A1}(t) + Q_X(t)$) ; the (expected) cost of replacing or repairing the defective units is $mc_{10}e^{rt}(1-R_1(t))Q_1(t)$, where $(1-R_1(t))Q_1(t)$ is the expected number of defective units. These costs are reduced by $\sigma e^{rt}R_2(t)$ which captures the inter-firm spillovers, i.e., the reduction in costs for firm 1 from improvements in its competitor's quality. We assume spillovers are small. Firm 2 is analogous. Notice that the marginal costs grow at the same rate as the demand parameters; further, suppose

$$Cost\ Ratio = \frac{mc_{20}}{mc_{10}} = \rho_{MC}. \qquad (13)$$

That is, firm 2's costs are greater than firm 1's if and only if the "Cost Ratio" $\rho_{MC} > 1$.

Equivalently, $mc_{10} = mc_0$ and $mc_{20} = \rho_{MC}\, mc_0$. The right hand side equations in (12) reflect this notation.

Define "Variable Profits," that is, instantaneous (discounted) profits before R&D expenditures:

$$\begin{aligned} VP_1(t) &= e^{-\rho t}\left((P_{A1}(t) - c_1(t))Q_{A1}(t) + (P_X(t) - c_1(t))Q_X(t)\right) \\ VP_2(t) &= e^{-\rho t}\left((P_M(t) - c_2(t))Q_M(t) + (P_{B2}(t) - c_2(t))Q_{B2}(t)\right) \end{aligned} \qquad (14)$$

where ρ is the discount rate. Substituting (8), (9), (10), (11) and (12) into (14), variable profits are written as functions of both firm's quantities and reliabilities (and the parameters).

Using backward induction, the quantities (as functions of the reliabilities) can now be described using the quantity first order conditions: $\partial VP_1(t)/\partial Q_{A1}(t) = 0$, $\partial VP_1(t)/\partial Q_X(t) = 0$, $\partial VP_2(t)/\partial Q_{B2}(t) = 0$, $\partial VP_2(t)/\partial Q_M(t) = 0$. Substituting the quantities into the variable profit functions means that variable profits are simply of a function of the reliabilities. Please see the appendix for one example.

Finally, the trade balance from country A's point of view is

$$TB_A(t) = P_X(t)Q_X(t) - P_M(t)Q_M(t) \qquad (15)$$

and the cumulative trade balance is

$$CTB_A(t) = \int_0^t TB_A(\tau)e^{-\rho\tau}d\tau \qquad (16)$$

2.3 Reliability and Research and Development

Improvements in reliability require research and development; expenditure on R&D at time t for firm j is $E_j(t) \geq 0$. Assume that such expenditure produces an improvement in the reliability of the firm's product, but is subject to diminishing marginal returns.

Specifically, suppose

$$\frac{dR_j(t)}{dt} = z_j(1-R_j(t))\sqrt{E_j(t)} \qquad (17)$$

where $z_j > 0$ and $0 \leq R_j(t) \leq 1$. Note that the $R_j(t)$ are the state variables, the $E_j(t)$ the controls. The assumption of a quadratic relationship between quality improvement and R&D spending which is independent of the quantity produced is common in the literature, for example see Haaland and Kind (2008) and Gretz, Highfill, and Scott (2012). But here the relationship is modified by the term $(1-R_j(t))$ which implies that the closer the reliability is to 100% at a given time the less

productive a given level of R&D expenditure will be. Recall that $R_j(t)$ is a probability and so is between zero and one. The expenditure on R&D is typically rather large, and so the constant z_j needs to reduce $\sqrt{E_j(t)}$ by several orders of magnitude and is typically a small fraction. Finally, although for the sake of simplicity we refer to $E_j(t)$ as R&D expenditure, it is really the component of expenditure which varies with reliability. There would normally be many fixed-cost R&D expenditures.

3. Profit Maximization Problem of the Firms

The optimal control problem for firm *j* can now be stated. The objective for each firm is to maximize profits, which are the (discounted) instantaneous variable profits minus the (discounted) instantaneous expenditure on R&D. Choose $E_j(t)$ to maximize the integral:

$$\text{Max}_{E_j(t)} \int_0^T \left(VP_j(R_1(t), R_2(t), t) - e^{-\rho t} E_j(t) \right) dt \qquad (18)$$

subject to

$$\frac{dR_j(t)}{dt} = z_j(1 - R_j(t))\sqrt{E_j(t)} \qquad (19)$$

$$R_j(0) = R_{j0}, \quad 0 \le R_j(t) \le 1, \quad j = 1, 2 \qquad (20)$$

recalling that ρ is the discount rate. An equilibrium is the solution to (18), (19), and (20) for *j* = 1, 2. After performing the calculations in the preceding sections, it turns out that $VP_j(R_1(t), R_2(t), t)$ is a cumbersome quadratic expression in $R_1(t)$ and $R_2(t)$. For the interested readers, one of these is reproduced in the Appendix. Software that performs algebraic manipulations, e.g., *Mathematica*, is helpful in producing such expressions.

As described in Kamien and Schwartz (1991) there are two Hamiltonians for the (open-loop) optimization problem. Because the notation can be burdensome if not presented explicitly, we give both Hamiltonians, thereby introducing the notation that will be used.

$$H_1 = VP_1(R_1(t), R_2(t), t) - e^{-\rho t} E_1(t) + \lambda_a(t) z_1 (1 - R_1(t))\sqrt{E_1(t)} + \lambda_b(t) z_2 (1 - R_2(t))\sqrt{E_2(t)}$$

and

$$H_2 = VP_2(R_1(t), R_2(t), t) - e^{-\rho t} E_2(t) + \lambda_c(t) z_1 (1 - R_1(t))\sqrt{E_1(t)} + \lambda_d(t) z_2 (1 - R_2(t))\sqrt{E_2(t)}.$$

The first order conditions are $\frac{\partial H_j}{\partial E_j} = 0$ which give $0 = -e^{-\rho t} + \frac{\lambda_a(t) z_1 (1 - R_1(t))}{2\sqrt{E_1(t)}}$ and $0 = -e^{-\rho t} + \frac{\lambda_d(t) z_2 (1 - R_2(t))}{2\sqrt{E_2(t)}}$. These equations can be solved for $E_j(t)$ but it is the square root that will be useful in the co-state equation. So we have

$$\sqrt{E_1(t)} = \frac{1}{2} \lambda_a(t) z_1 (1 - R_1(t)) e^{\rho t}$$

and

$$\sqrt{E_2(t)} = \frac{1}{2}\lambda_d(t) z_2 (1 - R_2(t)) e^{\rho t}.$$

The state equations can now be written in terms of the state variables $R_j(t)$ and the co-state variables $\lambda_*(t)$ as

$$dR_1 / dt = z_1(1 - R_1(t))\left(\frac{1}{2}\lambda_a(t) z_1 (1 - R_1(t)) e^{\rho t}\right) = \frac{1}{2}\lambda_a(t) z_1^2 (1 - R_1(t))^2 e^{\rho t}$$

and similarly

$$dR_2 / dt = z_2(1 - R_2(t))\left(\frac{1}{2}\lambda_d(t) z_2 (1 - R_2(t)) e^{\rho t}\right) = \frac{1}{2}\lambda_d(t) z_2^2 (1 - R_2(t))^2 e^{\rho t}.$$

There are four co-state equations all of the form $d\lambda / dt = -\frac{\partial H}{\partial R}$. More specifically they are

$$\frac{\partial \lambda_a}{\partial t} = -\frac{\partial H_1}{\partial R_1}, \quad \frac{\partial \lambda_b}{\partial t} = -\frac{\partial H_1}{\partial R_2}, \quad \frac{\partial \lambda_c}{\partial t} = -\frac{\partial H_2}{\partial R_1}, \text{ and } \frac{\partial \lambda_d}{\partial t} = -\frac{\partial H_2}{\partial R_2}.$$

For the purpose of illustration, here is the first of these.

$$\begin{aligned}
\frac{\partial \lambda_a}{\partial t} &= -\frac{\partial H_1}{\partial R_1} = -\frac{\partial}{\partial R_1}\left[VP_1 - e^{-\rho t} E_1 + \lambda_a z_1 (1 - R_1)\sqrt{E_1} + \lambda_b z_2 (1 - R_2)\sqrt{E_2}\right] \\
&= -\frac{\partial VP_1}{\partial R_1} + \lambda_a z_1 \sqrt{E_1} = -\frac{\partial VP_1}{\partial R_1} + \lambda_a z_1 \left(\frac{1}{2}\lambda_a z_1 (1 - R_1) e^{\rho t}\right) \\
&= -\frac{\partial VP_1}{\partial R_1} + \frac{1}{2}\lambda_a^2 z_1^2 (1 - R_1) e^{\rho t}.
\end{aligned}$$

The other three equations are similar, with the appropriate permutation of subscripts. The result is a system of six first-order, ordinary differential equations. The state variables have initial conditions $R_1(0) = R_{10}$ and $R_2(0) = R_{20}$. The co-state equations have final conditions $\lambda_p(T) = 0, \;\; p = a, b, c, d$.

4. Results

Analysis of the optimal solutions will rely on numerical methods. The numerical algorithm used is referred to as the Forward-Backward Sweep Method and uses a Runge-Kutta order 4 differential equation routine to solve the optimality conditions above. See Lenhart and Workman (2007) for a discussion of the method. We restrict our attention to interior solutions, so our first task is to identify the relevant parameter sets. Suppose the maximum reservation price for country A is $V_0 = 200$, and the minimum is $W_0 = 100$. The potential market size is $N_0 = 100$. The customer cost of product failure is $K_0 = 100$ and the per unit manufacturing cost is $mc_0 = 100$. (There is nothing complicated here; we just set everything at 100, shifting the range of reservation prices up so that the minimum reservation price was also 100. Other parameter values are found in the Appendix.) The initial reliability level in both countries is $R_1(0) = 0.9 = R_2(0)$; the planning period is three years. Holding these constant (except for the comparative statics result below), our notation defines country B's parameters as a multiple of country A's. This is done in (7) and (13). The ratio of the demand side variables is denoted ρ_{CV}, so that for example $V_{B0} = \rho_{CV} V_{A0} = \rho_{CV} V_0$.

Similarly, the ratio of the marginal manufacturing costs is denoted as ρ_{MC}, so that $mc_{20} = \rho_{MC}\, mc_{10} = \rho_{MC}\, mc_0$. The latter is relatively more important in the discussion that follows because of our focus on manufacturing costs. The Appendix gives all sets of (ρ_{CV}, ρ_{MC}) that generate an interior solution, and briefly states the binding constraints.

4.1 The Basic Example

Considering now only interior solutions, our primary interest is in the reliability paths, that is, the state variable paths. When $\rho_{CV} = 1.2$ and $\rho_{MC} = 1.3$ the reliability paths, $R_i(t)$ (Figure 1), the R&D expenditure paths, $E_i(t)$ (Figure 2), and the co-state variable paths, $\lambda_p(t)$ (Figure 3) are shown next. (The parameters $\rho_{CV} = 1.2$ and $\rho_{MC} = 1.3$ were chosen because they are about the middle of set of interior solutions for the two main examples which follow; please see Figure 7 in the Appendix.)

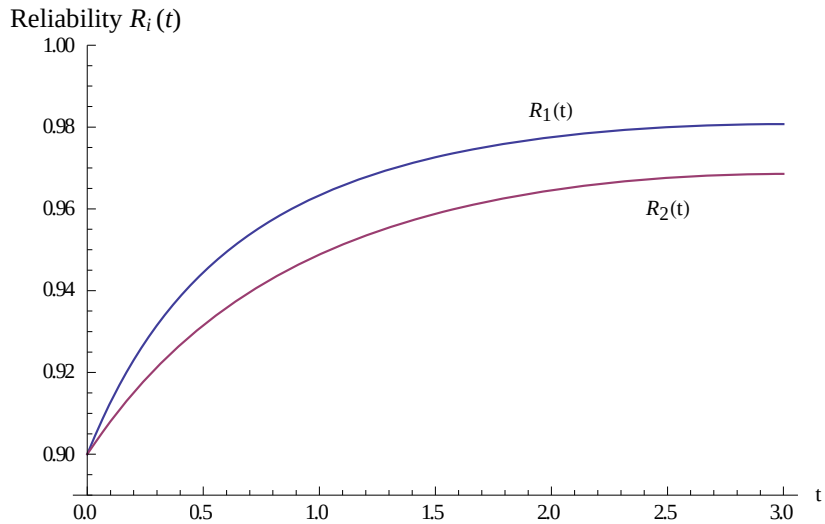

Figure 1 Reliability paths

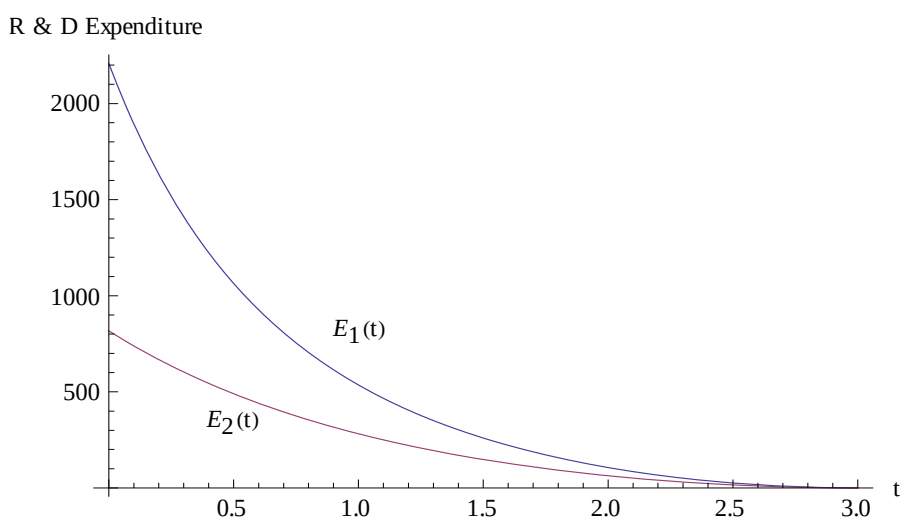

Figure 2 R&D expenditure paths

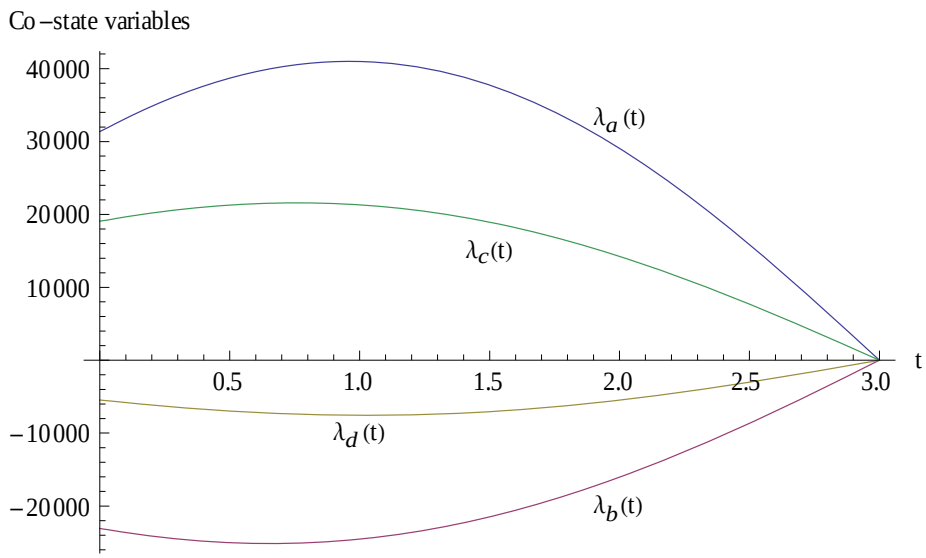

Figure 3 Shadow-price paths

Briefly, because $\rho_{MC} = 1.3 > 1$, firm 2 has a manufacturing cost disadvantage, $mc_{20} > mc_{10}$; firm 1 has a manufacturing cost advantage. Therefore firm 1 responds by doing more R&D and producing a more reliable product than firm 2 does. As for the co-state variables, for each firm one co-state variable is something like a shadow price of its own reliability (and thus positive) while the other is the shadow price of the other firm's reliability (and thus negative). Thus $\lambda_a(t)$ is firm 1's shadow price of its own reliability $R_1(t)$ while $\lambda_b(t)$ is firm 1's shadow price of its competitor's reliability $R_2(t)$. These shadow prices are larger in absolute value that the analogous shadow prices for firm 2.

Discussion of international trade issues in the political arena often focuses on the current account balance as a measure of whether a country is performing well. Trade economists are sometimes skeptical of the trade balance as a measure of anything important, but economists who have to deal daily with politicians and pundits usually have learned to at least address trade account issues. Gregory Tassey, for example, the Senior Economist for the U.S. National Institute of Standards and Technology cites the negative U.S. trade balance in "advanced technology products" as evidence that the U.S. is not fostering the development of appropriate R&D strategies as compared to such countries as Japan, Germany, Korea, and Taiwan (Tassey (2010)[15]). Our model provides sufficient structure that we can compute the trade balance. Recalling from equation (15) that the trade balance is from country A's point of view and that firm 1 is located in country A, higher R&D and reliability paths are associated with a positive and increasing trade balance, as shown in Figure 4.

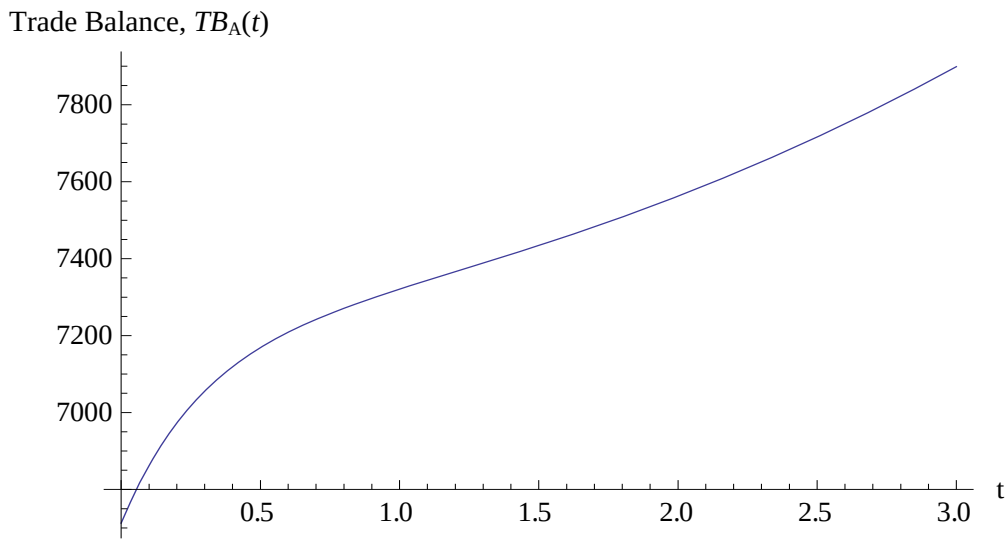

Figure 4 Trade balance path for Country A

4.2 Exogenous Changes in Manufacturing Costs

Suppose firm 1 experiences a reduction in the per unit marginal cost of manufacturing its product. While the paper is agnostic about the reason for the exogenous reduction in manufacturing cost it might, for example, come from a fixed-cost one-off project, whether for original research, some sort of backward engineering, or adopting a technology developed by the government. By assumption this reduction in manufacturing costs is not caused by the optimal R&D paths of the previous section. Rather the question is the reverse. How does a change in mc_{10} affect the optimal reliability paths? In general, the answer depends on how mc_{20} changes.

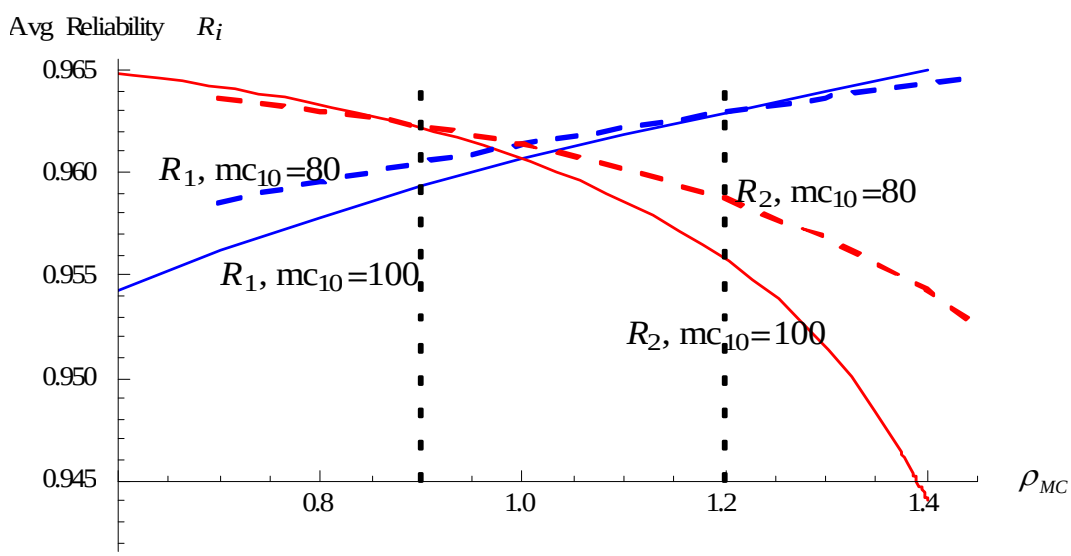

Figure 5 Average reliabilities as a function of the cost ratio ρ_{MC}

We will use the cost ratio parameter ρ_{MC} to organize our thinking. Recall briefly, that firm 2 has a manufacturing cost advantage or simply a cost advantage when $\rho_{MC} < 1$. For example, when

$\rho_{MC} = .9$, firm 2 has a manufacturing cost of 90 dollars per unit as compared to firm 1's manufacturing cost of 100 dollars per unit. When $\rho_{MC} = 1.2$, on the other hand firm 1 has a manufacturing cost of 100 and firm 2 has a manufacturing cost of 120 dollars per unit. I.e., firm 1 has the cost advantage.

The reliabilities can be expressed as functions of ρ_{MC} ; changes in mc_{10} can be shown as shifts. Please see Figure 5.

The cost ratio parameter ρ_{MC} is on the horizontal axis. At $\rho_{MC} = 1$ the manufacturing costs are the same and the reliabilities are the same. The left side of the figure ($\rho_{MC} < 1$) is where firm 2 has the manufacturing cost advantage while the right side ($\rho_{MC} > 1$) is where firm 1 has the manufacturing cost advantage.

The vertical axis shows the reliability for each firm. Reliabilities for the original set of parameters are shown by the solid curves. Since we want to report an overall reliability for a given ρ_{MC}, and the reliabilities are changing over time we report the average reliability computed by

$$\text{Average Reliability}_j = \frac{1}{T}\int_0^T R_j(t)dt \qquad (21)$$

(Because values of $R_j(t)$ are only available for discrete values of *t*, a numerical procedure (Simpson's Rule) is used to approximate the value of the integral.) It follows that the curve labeled $R_2, mc_{10} = 100$, for example, in Figure 5 is a map of these average values as a function of ρ_{MC}. Since the values of ρ_{MC} are restricted to 0.2, 0.4 ,…, 1.4, the data points have been interpolated by a smooth curve.

Looking at the solid curves, reading from left to right in Figure 5, firm 1's manufacturing cost disadvantage is first shrinking to zero (at $\rho_{MC} = 1$) and then its manufacturing cost advantage is growing. Thus for our example, on the far left ($\rho_{MC} < .9$) firm 2 has the reliability advantage; on the far right ($\rho_{MC} > 1.2$) firm 1 has the reliability advantage. The reliabilities converge and are equal at $\rho_{MC} = 1$. So for a given marginal cost for firm 1, (at $mc_{10} = 100$ for the solid curves) the firm with the higher marginal cost has the lower reliability. Figure 5 can be thought of as a type of sensitivity analysis. The case reported in the previous section assumed $\rho_{MC} = 1.3$. The points on the solid curves for each firm above $\rho_{MC} = 1.3$ are the average of the reliabilities reported in Fig. 1. Thus one way of thinking of Figure 5 is that it reports the average reliabilities for other values of the cost ratio parameter.

Now suppose firm 1's marginal cost falls to $mc_{10} = 80$ as shown by the dashed curves in the figure. It is certainly the case that what happens to the two firms' reliabilities depends on firm 2's marginal cost. What the figure does is help us see what happens when the cost ratio parameter does not change. For the simple examples above, when $\rho_{MC} = .9$, firm 2 now has a marginal cost of \$72/unit as compared to firm 1's \$80/unit. When $\rho_{MC} = 1.2$ firm 2 now has a manufacturing cost of \$96/unit while firm 1's is 80 dollars.

For all marginal cost ratios, lower overall manufacturing costs (dashed curves) lead to a smaller difference in reliabilities between the firms as compared to the base case with the solid curves. Further, considering the marginal cost ratios "near" $\rho_{MC} = 1$ both firm's reliabilities are higher than

in the base case. As long as the firms' manufacturing costs are "similar" (i.e., $\rho_{MC} \approx 1$) lower overall costs lead to higher reliabilities. But when the marginal cost ratios are "farther" from $\rho_{MC} = 1$ the firm with the cost advantage has a higher reliability with lower absolute costs. For example, on the far right where firm 1 has a larger cost advantage the reliability when its costs are low (the dashed line) is actually lower than for the base case for a given value of the cost ratio parameter ρ_{MC}. The analogous result holds for firm 2 on the far left.

So when the firms' manufacturing costs are very dissimilar, a reduction in its own marginal cost holding the cost ratio parameter constant (dashed curve compared to solid on the far right or the far left) leads the firm with the cost advantage to optimally produce a less reliable product. In summary, a reduction in a given firm's manufacturing cost only causes it to produce a more reliable product when the difference in costs between firms is low. When the difference in costs is high the firm with a manufacturing cost advantage will use the reduction in its marginal cost as the occasion to actually produce a less reliable product.

The following Figure 6 shows what happens to the cumulative trade balance (16) from country A's point of view for the same thought experiment, a change in the marginal cost as reported above.

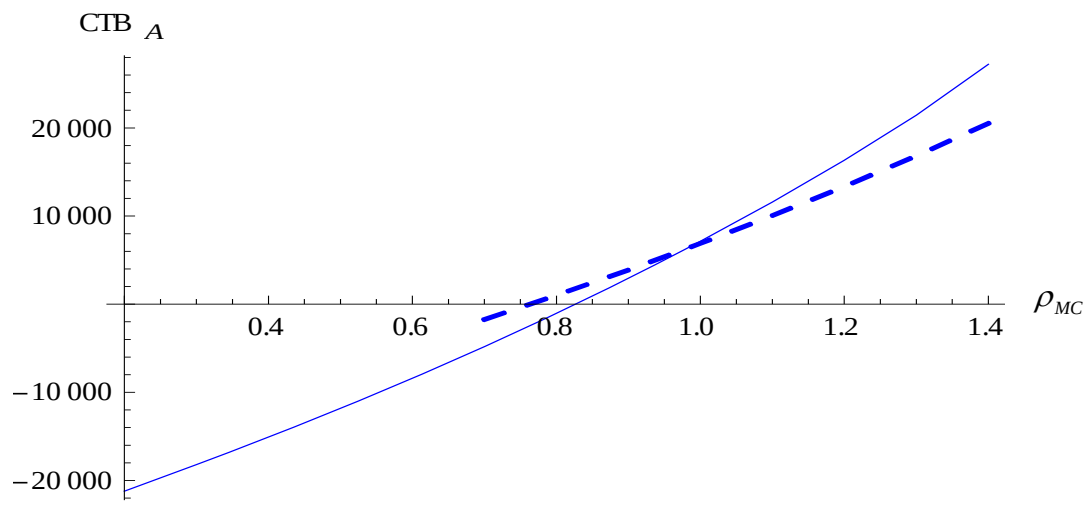

Figure 6 Cumulative trade balance for Country A as a function of the cost ratio ρ_{MC}

Recalling the definition of ρ_{MC}, on the far left firm 1 has the largest manufacturing cost disadvantage and it will have a lower reliability than firm 2 (for either the solid or dashed curves). But, looking at the solid curve, as firm 1's cost disadvantage gets smaller its home country (i.e., country A) begins to run a trade balance surplus for some interval of time, and as ρ_{MC} increases even further the cumulative trade balance over the entire planning period becomes positive. Comparing now the solid and dashed curves, recall that the solid curve is for $mc_0 = 100$ (i.e., firm 1's marginal cost is 100, while firm 2's is $\rho_{MC} * 100$) while the dashed curve is for $mc_0 = 80$ (with an analogous interpretation). The solid and dashed curves do not quite intersect at 1, but essentially when firm 1 has the cost advantage its home country (i.e., country A) has a higher cumulative trade balance, and conversely. Figure 6 can also be thought of as a type of sensitivity analysis. At the value of the cost ratio parameter assumed in the previous section, $\rho_{MC} = 1.3$, it gives the cumulative trade balance for the trade balance path reported in Figure 4. Figure 6 reports the cumulative trade balance for other values of the cost ratio parameter.

5. Conclusion

The goal of the paper is to characterize R&D, reliability, and shadow price paths for firms playing a dynamic R&D game in the context of intra-industry trade with two goods, two firms, and two countries. R&D in the model improves a product's reliability which benefits the firm by reducing its manufacturing costs and benefits customers by reducing their expected cost of customer failure. (There is a small spillover benefit of a given firm's R&D to its competitors' reliability as well.) In addition, we explored the effect of an exogenous decrease in the marginal manufacturing cost parameter on each firm's reliability. Our results suggest that when marginal costs are similar, reductions in manufacturing costs will prompt the firms to improve the reliability of their products. The result does not hold when manufacturing costs are sufficiently different. While beyond the scope of this paper, this suggests that a common criticism of industrial policy—that it benefits the firms but not their customers—might sometimes be reasonable. Future work might explore the effect of an exogenous change in the customer cost of product failure parameter in order to shed more light on this issue.

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Appendix

The goal of the appendix is to do a type of sensitivity analysis by describing the (ρ_{CV}, ρ_{MC}) pairs that generate an interior solution of the optimal control problem equations (18), (19), and (20). See Figure 7 below. The ovals (0) show the pairs that generate interior solutions for the base case when $mc_{10} = 100$ while the plus signs (+) show the pairs that give interior solutions when $mc_{10} = 80$. The other parameter assumptions are for the inflation, growth, and discount rates $r = s = \rho = 0.025$, the inter-firm spillover parameter is $\sigma = 0.1$, the parameter to change orders of magnitude in the co-state equation is $z = 0.03$. (These parameters are the same for both firms.)

The pairs (ρ_{CV}, ρ_{MC}) with both an oval and a plus sign give interior solutions for both parameters sets. Although we will omit the details for brevity, we briefly summarize the binding constraint along each border. The active constraint along the top horizontal border is given by $Q_M(t) \geq 0$. That is, for pairs above the top border at some time during the planning period quantity imported was negative. The top left diagonal is for the $Q_{B2}(t) \geq 0$ constraint. The bottom left diagonal is for the $Q_X(t) \geq 0$ constraint; the bottom horizontal is for $Q_{A1}(t) \geq 0$. The right hand diagonal reflects the requirement that the full quality price in country B , see (2), never falls below the lower bound of the range of reservation prices, W_{B0}. The analogous constraint is never binding for country A.

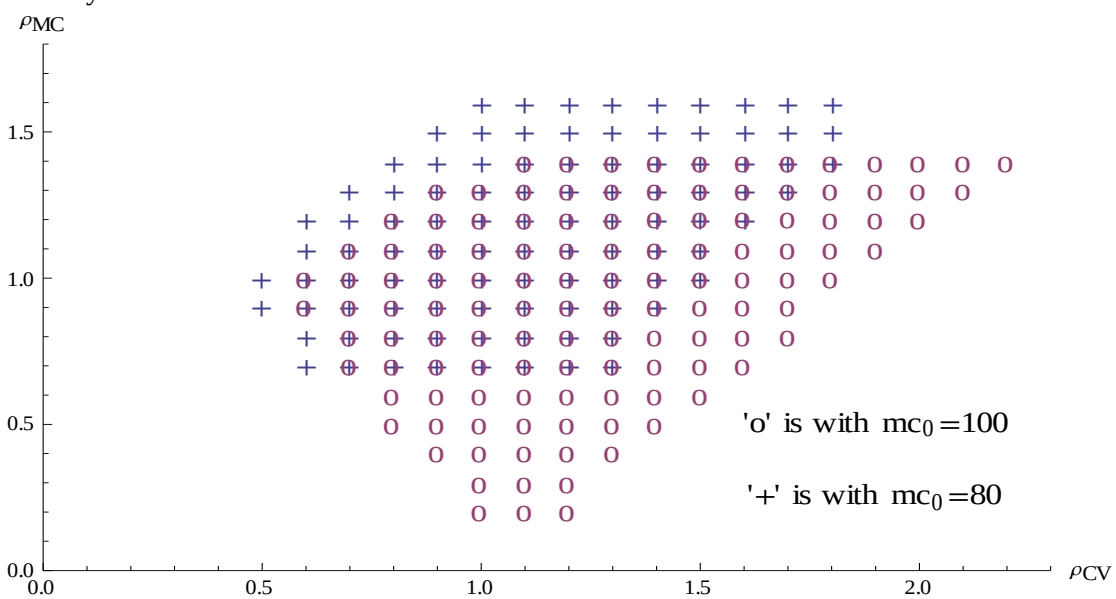

Figure 7 Parameters yielding interior solutions

The expressions for variable profits $VP_j(R_1(t), R_2(t), t)$ are cumbersome, yet quadratic expressions in $R_1(t)$ and $R_2(t)$. An example is

$$VP_1 = \frac{e^{(r+s-\rho)t} N_0}{9(V_0 - W_0)} [(K_0(-1 + 2R_1 - R_2) + V_0 - mc_0(4 + (-2 + R_2)\rho_{MC} - 2R_2\sigma + R_1(-2 + \rho_{MC}\sigma)))^2$$
$$+((K_0(1 - 2R_1 + R_2) - V_0)\rho_{CV} + mc_0(4 + (-2 + R_2)\rho_{MC} - 2R_2\sigma + R_1(-2 + \rho_{MC}\sigma)))^2].$$