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R&D Policy Competition with Process Innovation in a Multi-Product Duopoly[1]

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Abstract: This paper considers a reciprocal dumping model which consists of two countries, each owning a multi-product firm which sells products to both countries. The firms choose the R&D investment portfolio for their products, and a government may subsidize or tax its domestic firm for the R&D investment. It is shown that a firm invests more in R&D for its core (non-core) product if products are sufficiently differentiated (similar) to each other. Moreover, if a firm invests more in its non-core product than its core product, it does that to an extent such that the non-core product becomes the core product after the R&D process. Policy competition results in a unilateral incentive of a subsidy, and the stable optimal policy is always a subsidy. When two governments harmonize their policies, it is optimal for them to set subsidies to zero. The optimal subsidy in a duopoly is higher than that in a monopoly if and only if two governments' policies are strategic substitutes.

JEL Classifications: F12, F13, L13

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1. Introduction

Study of R&D policy competition between two countries has received wide attention in the literature, starting with Spencer and Brander (1983). By using a third-country model in which firms export their products to a third country, most researchers have shown that R&D subsidies can be a useful substitute when the use of an export subsidy is prohibited. Moreover, R&D policy competition tends to result in a standard prisoner's dilemma situation where both countries are harmed by excessively subsidizing their firms.[2] Although most studies have focused on the business stealing effect because of the third-country assumption, a small number of exceptions, including Leahy and Neary (2001) and Haaland and Kind (2006, 2008) have attempted to study R&D policy competition in the presence of consumer surplus effects by considering a reciprocal dumping model.

Most studies have considered the case of single-product duopoly; however, it is well known that most firms actually produce multiple products in the real world. Therefore, it is crucial to study R&D policy competition in the context of multiproduct firms. In this study, we adopt the model in Lin and Zhou (2013) with two multiproduct firms each producing two types of products; an example of which is that both Apple and HTC produce mobile phones (e.g., iPhone and HTC

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[2] For examples see Bagwell and Staiger(1994), Brander(1995), Neary and Leahy (2000), and Leahy and Neary (2001, 2009).

Desire) and tablet PCs (e.g., iPad and HTC Flyer). It is assumed that the same types of product of each firm are perfect substitutes, but that the different types of product of a particular firm are imperfect substitutes. Using our example, this implies that consumers view the iPhone and the iPad (or the HTC desire and the HTC Flyer) as imperfect substitutes but view the iPhone and the HTC Desire (or the iPad and the HTC Flyer) as perfect substitutes. Another setting in this model is that each firm is assumed to have its own core product which is characterized as having a lower unit cost, and the core products of the firms differ. In our example, this means that Apple's core product is its mobile phones whereas HTC's core product is tablet PCs. Because of the initial cost differential of two products, each firm's process R&D investment levels for two products will naturally be different; therefore, we can study a firm's R&D portfolio selection problem which does not exist for single-product firms or for multi-product firms whose products have the same cost.

This study is among the first to investigate the R&D policy competition in the context of multiproduct duopoly. We consider a reciprocal dumping model similar to that of Haaland and Kind (2008) in which the governments choose the optimal R&D subsidy (or tax) levels to maximize its welfare which consists of consumer surplus and the firm's net profit from subsidy payments. R&D investments are non-drastic and certain, and there is no spillover effect. We obtain certain noteworthy results; First, whether the firms invest more or less in their core products than in their non-core products depends on how differentiated the two products are. In fact, if the two products are sufficiently differentiated (similar), the firms' R&D investment levels for the core products are higher (lower) than for the non-core products. Moreover, if the firm invests more in its non-core product, it will do so to the extent that the post-R&D cost for the non-core product is lower than that of the core-product; this changes the non-core (core) product into the core (non-core) product after R&D investments. Second, we show that whether the governments' policies are strategic complements or substitutes depends on the degree of product differentiation and R&D cost parameter. Third, we solve for the stable optimal symmetric policy which is a subsidy regardless of parameter values. Moreover, we show that each government has a unilateral incentive to subsidize the firms. Lastly, we show that, depending on the degree of product differentiation, an R&D portfolio can be concentrated on one product or diversified between two products. We also show that the prisoner's dilemma result prevails in which both countries are worse off in policy competition as compared with the case in which they do not use subsidies. This result is consistent with that in the optimal export (R&D) subsidy literature. We also find that the level of optimal subsidy is zero when they cooperate and set a common level of subsidy to maximize the joint welfare.

The remainder of this study is organized as follows. Section 2 presents the basic model. Section 3 provides the benchmark case in which each firm is a monopolist in its own market segment. Section 4 analyzes the firms' R&D investment portfolio and optimal R&D policies. Section 5 presents a discussion of the policy cooperation between two governments and compares the optimal subsidies between policy competition and monopolist multiproduct firm. Section 6 offers concluding remarks.

2. Model

2.1 Demand Side

Without loss of generality, assume that Firm 1 is located in and owned by residents of Country 1, whereas Firm 2 is located in and owned by residents of Country 2. The utility function of Country i's representative consumer is given by

$$U_i = A_i(q_{iii}+q_{iji})+A_i\left(q_{jii}+q_{jji}\right)-\frac{1}{2}\left[\left(q_{iii}+q_{iji}\right)^2+\left(q_{jii}+q_{jji}\right)^2\right] \\ +\gamma(q_{iii}+q_{iji})(q_{jii}+q_{jji})+z, \qquad (1)$$

where $A_i > 0$, q_{ijk} is the good i produced by firm j and sold in country k, and z is the numeraire good with price normalized to unity. One can interpret A_i as the market size in country i, where the larger the value of A_i, the larger the demand for all goods. The degree of horizontal product differentiation for two goods is $\gamma \in [0,1]$ where two goods become independent when $\gamma \to 0$ and homogenous when $\gamma \to 1$.

The same type of goods produced by two firms are identical, so they have the same price. Denote P_{ii} and P_{ji} to be the prices of the two goods in country i, and the inverse demand functions are given by

$$\begin{aligned} P_{ii} &= A_i-\left(q_{iii}+q_{iji}\right)-\gamma\left(q_{jii}+q_{jji}\right) \\ P_{ji} &= A_i-\left(q_{jii}+q_{jji}\right)-\gamma\left(q_{iii}+q_{iji}\right). \end{aligned} \qquad (2)$$

The corresponding consumer surplus in country i is

$$\begin{aligned} CS_i &= U_i - P_{ii}\left(q_{iii}+q_{iji}\right)-P_{ji}\left(q_{jii}+q_{jji}\right)-z \\ &= \frac{1}{2}\left[\left(q_{iii}+q_{iji}\right)^2+\left(q_{jii}+q_{jji}\right)^2\right]+\gamma\left(q_{iii}+q_{iji}\right)\left(q_{jii}+q_{jji}\right). \end{aligned} \qquad (3)$$

2.2 Supply Side

The firms can invest in cost-reducing (process) R&D, and the unit costs of two products for two firms are

$$\begin{aligned} c_{ii} &= c - x_{ii} \\ c_{ji} &= c+\delta - x_{ji} \end{aligned}, i, j = 1,2, i \neq j, \qquad (4)$$

where $c > 0, \delta > 0$, x_{ii} and x_{ji} represent the R&D investment on two goods of Firm i, and c_{ii} and c_{ji} represent the unit costs of two goods after Firm i's investment. This setting assumes, without the loss of generality, that Firm 1's (2's) initial production cost of good 1 (2) is equal to c, which is lower than that of good 2 (1), $c+\delta$. The product i is the core product for Firm i. Note that the two firms are symmetric in that they have the same initial costs for their core products, as well as their non-core products. We assume that each firm's cost for conducting process R&D is the same and given by the standard quadratic function $\beta x_{1i}^2 + \beta x_{2i}^2$. Also, each firm receives the per-unit R&D subsidies of s_i from its domestic government. Thus, profit function of firm i is given by

$$\Pi_i = \pi_i + s_i\left(x_{1i}+x_{2i}\right)-\beta x_{1i}^2-\beta x_{2i}^2, \qquad (5)$$

where π_i are profits from the sales obtained by Firm i in two countries, given by

$$\pi_i = \pi_{i1}+\pi_{i2} = (P_{1i}-c_{1i})q_{1i1}+(P_{2i}-c_{2i})q_{2i1} \\ +(P_{1j}-c_{1i})q_{1i2}+(P_{2j}-c_{2i})q_{2i2}, \qquad (6)$$

Government i is assumed to maximize the welfare in country i, which is given by the sum of domestic consumer surplus and firm i's net profit from R&D subsidies:

$$W_i = CS_i + \Pi_i - s_i(x_{1i} + x_{2i}). \qquad (7)$$

We consider a three-stage game whereby the governments choose s_1 and s_2 at the first stage. The firms choose $(x_{11}, x_{21}, x_{12}, x_{22})$ at the second stage. At the last stage, the firms compete in product markets in Cournot fashion.

3. R&D Subsidies to a Monopoly

We assume that the firms are monopolists in their own market segments in which the products produced by one firm are independent from those produced by the other firm. To make the comparison between optimal subsidies under in a monopoly and in a Cournot duopoly meaningful, we assume that the monopolists sell their products to both countries. We consider the case in which two countries are intrinsically symmetric such that $A_1 = A_2 = A$, so both countries have the same market size. Therefore, the two markets are the same in the view of the firms, and the firms' output of a particular type of good will be the same in both countries. We denote q_{ii} and q_{ji}, $i, j = 1,2,\ j \neq i$, as Firm i's output of two goods in each market, such that $q_{ii} = q_{ii1} = q_{ii2}$ and $q_{ji} = q_{ji1} = q_{ji2}$, and P_i as the price of good i. Using (2), Firm i faces inverse demands for two goods in both countries, given as

$$P_1 = A - q_{1i} - \gamma q_{2i}, \qquad P_2 = A - q_{2i} - \gamma q_{1i}$$

and its operating profit obtained from the sales in both countries is

$$\pi_i = 2\left((P_1 - c_{1i})q_{1i} + (P_2 - c_{2i})q_{2i}\right).$$

In the market stage, given the R&D investments x_{1i} and x_{2i}, operating profit maximizing outputs are determined by setting $\frac{\partial \pi_i}{\partial q_{1i}} = \frac{\partial \pi_i}{\partial q_{2i}} = 0$. This yields monopoly outputs

$$\begin{aligned} q_{ii}^M(x_{ii}, x_{ji}) &= \frac{(1-\gamma)(2A - 2c - \delta) + (1+\gamma)\delta + 2x_{ii} - 2\gamma x_{ji}}{4(1-\gamma^2)} \\ q_{ji}^M(x_{ii}, x_{ji}) &= \frac{(1-\gamma)(2A - 2c - \delta) - (1+\gamma)\delta - 2\gamma x_{ii} + 2x_{ji}}{4(1-\gamma^2)}. \end{aligned} \qquad (8)$$

It can be easily verified that the second order condition is satisfied in this case. First, note that $q_{ii} > q_{ji}$ for $x_{ii} = x_{ji} = 0$, and this implies that the the monopolists' outputs for the core products are higher than those for the non-core products in the absence of R&D investments. Second, the difference between its core and non-core output levels is equal to $\frac{1}{2(1-\gamma)}\delta$, implying that the larger the initial cost differential δ is, the larger the difference between outputs of core and non-core products is. Lastly, one can see that $\frac{\partial q_{ii}}{\partial x_{ii}} = \frac{\partial q_{ji}}{\partial x_{ji}} = 2 > 0$ and $\frac{\partial q_{ii}}{\partial x_{ji}} = \frac{\partial q_{ji}}{\partial x_{ii}} = -2\gamma < 0$;

therefore, as the cost of one type of goods decreases, resulting from R&D, a multiproduct monopolist raises the output level for this type and decreases the output level for the other type.

We shall assume that the output levels for two products are positive in the absence of R&D investments. Because $q_{ii} > q_{ji}$ for $x_{ii} = x_{ji} = 0$, it is sufficient to assume that $q_{ji} > 0$. In fact, this is equivalent to assuming that $\gamma < \frac{(A-c-\delta)}{A-c}$. The parameter γ can only take a positive value, we thus require that $A > c+\delta$.

In the second stage, the multiproduct monopolist, taking as given the R&D subsidies provided by the government, chooses x_{ii} and x_{ji} to maximize its profit given as (5), and the first order conditions are

$$\frac{\partial \Pi_i}{\partial x_{ii}} = \frac{(1-\gamma)(A-c)+\gamma\delta-(2\beta-2\beta\gamma^2-1)x_{ii}-\gamma x_{ji}+(1-\gamma^2)s_i}{1-\gamma^2} = 0$$
$$\frac{\partial \Pi}{\partial x_{ji}} = \frac{(1-\gamma)(A-c)-\delta-\gamma x_{ii}-(2\beta-2\beta\gamma^2-1)x_{ji}+(1-\gamma^2)s_i}{1-\gamma^2} = 0.$$

Solving the system of first order conditions for x_{ii} and x_{ji} yields the monopolist's optimal R&D portfolio:

$$x_{ii}^M(s_i) = x_c^M(s_i) = \frac{(2\beta-2\beta\gamma-1)(2A-2c-\delta)+(2\beta+2\beta\gamma-1)\delta+2(1+\gamma)(2\beta-2\beta\gamma-1)s_i}{2(2\beta-2\beta\gamma-1)(2\beta+2\beta\gamma-1)}$$
$$x_{ji}^M(s_i) = x_{nc}^M(s_i) = \frac{(2\beta-2\beta\gamma-1)(2A-2c-\delta)-(2\beta+2\beta\gamma-1)\delta+2(1+\gamma)(2\beta-2\beta\gamma-1)s_i}{2(2\beta-2\beta\gamma-1)(2\beta+2\beta\gamma-1)}, \quad (9)$$

where $x_c^M(s_i)$ and $x_{nc}^M(s_i)$ denote the optimal R&D investments for the core and non-core products, respectively.

We find that the second order condition is satisfied if and only if $(2\beta-2\beta\gamma-1)>0$, and rearranging terms, the inequality becomes $\gamma < \bar{\gamma}^M = \frac{2\beta-1}{2\beta}$.[3] We assume that $\beta > \frac{1}{2}$ to ensure

[3] First note that $\frac{\partial \Pi_i}{\partial x_{ii}^2} = \frac{\partial \Pi_i}{\partial x_{ji}^2} = -\frac{1}{1-\gamma^2}(2\beta-2\beta\gamma^2-1)$ and $\frac{\partial \Pi_i}{\partial x_{ji}\partial x_{ii}} = \frac{\partial \Pi_i}{\partial x_{ii}\partial x_{ji}} = -\frac{\gamma}{1-\gamma^2}$. The second order condition requires that $\frac{\partial \Pi_i}{\partial x_{ii}^2} < 0$, $\frac{\partial \Pi_i}{\partial x_{ji}^2} < 0$, and $\frac{\partial \Pi_i}{\partial x_{ii}^2}\frac{\partial \Pi_i}{\partial x_{ji}^2} - \frac{\partial \Pi_i}{\partial x_{ji}\partial x_{ii}}\frac{\partial \Pi_i}{\partial x_{ii}\partial x_{ji}} > 0$. One can see that for $\frac{\partial \Pi_i}{\partial x_{ii}^2} < 0$ and $\frac{\partial \Pi_i}{\partial x_{ji}^2} < 0$ to be true, we require that $(2\beta-2\beta\gamma^2-1) > 0$. Moreover, we obtain $\frac{\partial \Pi_i}{\partial x_{ii}^2}\frac{\partial \Pi_i}{\partial x_{ji}^2} - \frac{\partial \Pi_i}{\partial x_{ji}\partial x_{ii}}\frac{\partial \Pi_i}{\partial x_{ii}\partial x_{ji}} = \frac{1}{1-\gamma^2}(2\beta-2\beta\gamma-1)(2\beta+2\beta\gamma-1)$, and for it to be positive, it must be true that either $(2\beta-2\beta\gamma-1) > 0$ or $(2\beta+2\beta\gamma-1) < 0$. It is easy to see that $(2\beta+2\beta\gamma-1) > (2\beta-2\beta\gamma^2-1)$, and the condition for $(2\beta-2\beta\gamma^2-1) > 0$ implies that $(2\beta+2\beta\gamma-1) > 0$; therefore, the second order condition is satisfied if and only if $(2\beta-2\beta\gamma-1) > 0$.

that $\overline{\gamma}^M$ is positive. In addition, for $\beta > \frac{1}{2}$, we assume that the goods are sufficiently differentiated from each other, such that $\gamma < \overline{\gamma}^M$ to guarantee that the second order condition is satisfied.

We should determine the non-negativity conditions for quantities and R&D investments when $s_i = 0$. First note that $x_c^M > x_{nc}^M$ such that $x_{nc}^M > 0$ implies that $x_c^M > 0$. Moreover, substituting (9) into (8) yields the outputs under optimal R&D portfolio:

$$q_{ii}^M(s_i) = \frac{\beta(2\beta - 2\gamma - 1)(2A - 2c - \delta) + \beta(2\beta + 2\beta\gamma - 1)\delta + (2\beta - 2\gamma - 1)s_i}{2(2\beta + 2\gamma - 1)(2\beta - 2\gamma - 1)}$$
$$q_{ji}^M(s_i) = \frac{\beta(2\beta - 2\gamma - 1)(2A - 2c - \delta) - \beta(2\beta + 2\beta\gamma - 1)\delta + (2\beta - 2\gamma - 1)s_i}{2(2\beta + 2\gamma - 1)(2\beta - 2\gamma - 1)}, \qquad (10)$$

and it is easy to see that $q_{ii}^M(s_i = 0) = \beta x_c^M(s_i = 0)$ and $q_{ji}^M(s_i = 0) = \beta x_{nc}^M(s_i = 0)$; therefore, outputs are positive if and only if R&D investments are positive, and the condition for $x_{nc}^M(s_i = 0) > 0$ guarantees that outputs and R&D investments for all goods are positive. The conditions for non-negativity of post-R&D unit costs must also be determined. The initial unit cost for the core product is lower than that for the non-core product, and the monopolist conducts more process R&D for its core product than for its non-core product. The post-R&D unit cost for the core product must be lower than that for the non-core product; $c_{ii}(x_{ii}^M(s_i = 0)) < c_{ji}(x_{ji}^M(s_i = 0))$, and the condition for $c_{ji}(x_{ji}^M(s_i = 0)) > 0$ implies that $c_{ii}(x_{ii}^M(s_i = 0)) > 0$.

The condition for $x_{nc}^M(s_i = 0) > 0$ requires that the market size, A, is bounded from below:

$$A > c + \frac{2\beta - 1}{2\beta - 2\beta\gamma - 1}\delta.$$

This lower bound for A is clearly positive for $\gamma < \overline{\gamma}^M$, Moreover, it can be easily verified that $(c + \frac{2\beta - 1}{2\beta - 2\beta\gamma - 1}\delta) > (c + \delta)$, such that $A > c + \frac{2\beta - 1}{2\beta - 2\beta\gamma - 1}\delta$ implies that $A > c + \delta$, which is the requirement for outputs to be positive in the absence of process R&D investments.

The condition for $c_{ii}(x_{ii}^M(s_i = 0)) > 0$ requires that A is bounded from above:

$$A < 2\beta(\gamma + 1)c - \frac{2\beta\gamma}{(2\beta - 2\beta\gamma - 1)}\delta.$$

The upper bound must be larger than the lower bound, otherwise, no value of A exists, such that the non-negative conditions can be satisfied. We observe that it will be true if and only if $c > \frac{1}{(2\beta - 2\beta\gamma - 1)}\delta$. Thus we make the following assumption:

Assumption 1 *For any value of* $\beta > \frac{1}{2}$, *we assume that* $\gamma < \overline{\gamma}^M \equiv \frac{2\beta - 1}{2\beta}$. *Moreover, we assume* *that* c *is sufficiently large, relative to* δ *, that* $c > \frac{1}{(2\beta - 2\beta\gamma - 1)}\delta$ *, and* A *is bounded from below and above, such that* $c + \frac{2\beta - 1}{2\beta - 2\beta\gamma - 1}\delta < A < 2\beta(1+\gamma)c - \frac{2\beta\gamma}{2\beta - 2\beta\gamma - 1}\delta.$

In the first stage, the governments in each country choose the subsidy level s_i to maximize its welfare, as given by (7). The first order condition for this problem is

$$\frac{\partial W_i}{\partial s_i} = \frac{\partial(\Pi_i - s_i(x_{ii} + x_{ji}))}{\partial s_i} + \frac{\partial CS_i}{\partial s_i} = 0.$$

Because the firm is a monopolist in both markets, the subsidy granted by the government does not generate the familiar business-stealing effect as indicated in Spencer and Brander (1983). The subsidy lowers the monopolist's net profit from R&D subsidies, as shown by

$$\frac{\partial(\Pi_i - s_i(x_{ii} + x_{ji}))}{\partial s_i} = -\frac{2(1+\gamma)}{2\beta + 2\beta\gamma - 1} s_i < 0 \text{ for } s_i > 0.$$

Therefore, if the government grants any subsidy, it must be because the increase in consumer surplus outweighs the loss from the firm's net profit from R&D subsidies. Using (3) and excluding the outputs from the other firm, we observe that

$$\frac{\partial CS_i}{\partial s_i} = \frac{1+\gamma}{2(2\beta + 2\beta\gamma - 1)}(q_{ii}^M + q_{ji}^M) > 0.$$

Solving the first order condition for s_i yields the optimal subsidy in a monopoly:

$$s^M = \frac{\beta}{8\beta + 8\beta\gamma - 5}(2A - 2c - \delta). \qquad (11)$$

Moreover, the second order condition is

$$\frac{\partial^2 W_i}{\partial s_i^2} = -\frac{(1+\gamma)(8\beta + 8\beta\gamma - 5)}{2(2\beta + 2\beta\gamma - 1)^2} < 0,$$

which will be satisfied if and only if $(8\beta + 8\beta\gamma - 5) > 0$, or equivalently $\gamma > \underline{\gamma}^M \equiv \frac{5 - 8\beta}{8\beta}$.
Assuming that the second order condition is satisfied, it is clear that the optimal policy requires a subsidy.

Using (11), (9), and (8), R&D investments and outputs are given by:

$$\begin{aligned}
x_c^M &= \frac{5(2\beta - 2\beta\gamma - 1)(2A - 2c - \delta) + (8\beta + 8\beta\gamma - 5)\delta}{2(8\beta + 8\beta\gamma - 5)(2\beta - 2\beta\gamma - 1)} \\
x_{nc}^M &= \frac{5(2\beta - 2\beta\gamma - 1)(2A - 2c - \delta) - (8\beta + 8\beta\gamma - 5)\delta}{2(8\beta + 8\beta\gamma - 5)(2\beta - 2\beta\gamma - 1)} \\
q_{ii}^M &= \frac{4\beta(2\beta - 2\beta\gamma - 1)(2A - 2c - \delta) + \beta(8\beta + 8\beta\gamma - 5)\delta}{2(2\beta - 2\beta\gamma - 1)(8\beta + 8\beta\gamma - 5)} \\
q_{ji}^M &= \frac{4\beta(2\beta - 2\beta\gamma - 1)(2A - 2c - \delta) - \beta(8\beta + 8\beta\gamma - 5)\delta}{2(2\beta - 2\beta\gamma - 1)(8\beta + 8\beta\gamma - 5)}.
\end{aligned} \qquad (12)$$

From (9) and (10), it is clear that the outputs and R&D investments for both products will necessarily be positive with optimal subsidy s^M given that Assumption 1 holds; however, one has to modify the non-negativity condition of post-R&D unit cost for the core product because the optimal subsidy granted by the government will further lower the unit cost for the core product (i.e., $c - x_c^M > 0$). Therefore, we make the following assumption:[4]

Assumption 2 *For any value of* $\frac{9}{16} \approx 0.563 < \beta < \frac{5}{8} \approx 0.625$*, we assume that* $\frac{5-8\beta}{8\beta} \equiv \underline{\gamma}^M < \gamma < \bar{\gamma}^M \equiv \frac{2\beta-1}{2\beta}$*, and for* $\beta > \frac{5}{8}$ *, we assume that* $0 < \gamma < \bar{\gamma}^M$*. Furhtermore, we assume that c is sufficiently large such that* $c > \frac{(9\beta + 9\beta\gamma - 5)}{(8\beta + 8\beta\gamma - 5)(2\beta - 2\beta\gamma - 1)}\delta$ *, and A is bounded both from below and above,*

$$\left(c + \frac{(2\beta - 1)}{(2\beta(1-\gamma) - 1)}\delta\right) < A < \left(\frac{8\beta(\gamma+1)}{5}c - \frac{\beta(9\gamma - 1)}{5(2\beta - 2\beta\gamma - 1)}\delta\right).$$

Considering Assumption 2 and differentiating x_{nc}^M with respect to γ, we find that

$$\frac{\partial x_{nc}^M}{\partial \gamma} = -\beta \frac{\left(20(2\beta - 2\beta\gamma - 1)^2 (2A - 2c - \delta) + (8\beta + 8\beta\gamma - 5)^2 \delta\right)}{(8\beta + 8\beta\gamma - 5)^2 (2\beta - 2\beta\gamma - 1)^2},$$

which is clearly negative, indicating that the R&D investment for non-core product decreases as two goods become more similar. Although $\frac{\partial x_c^M}{\partial \gamma}$ can take a positive or negative value, depending on the parameter values, we find that

$$\frac{\partial \left(x_c^M - x_{nc}^M\right)}{\partial \gamma} = \frac{2\beta}{(2\beta - 2\beta\gamma - 1)^2}\delta$$

$$\frac{\partial}{\partial \gamma}\left(x_c^M / x_{nc}^M\right) = \frac{20\beta(16\beta - 9)(2A - 2c - \delta)\delta}{\left(5(2\beta - 2\beta\gamma - 1)(2A - 2c - \delta) - (8\beta + 8\beta\gamma - 5)\delta\right)^2}.$$

One can see that $\frac{\partial \left(x_c^M - x_{nc}^M\right)}{\partial \gamma}$ and $\frac{\partial}{\partial \gamma}\left(x_c^M / x_{nc}^M\right)$ are both positive, considering Assumption 2. Thus, it implies that the monopolist's R&D portfolio becomes more specialized toward its core product if the two products become closer substitutes. We thus derive the following proposition.

Proposition 1 *Given Assumption 2, and the multiproduct firms monopolized in their own market, (i) each government subsidize its firm with* $s^M = \frac{\beta}{8\beta + 8\beta\gamma - 5}(2A - 2c - \delta)$ *(ii) the monopolist always conducts more process R&D for its core product than its non-core product (iii) the R&D investment for the non-core product decreases as two products become more similar (iv) the R&D portfolio becomes more specialized toward core product if two goods are more similar, i.e.,*

[4] See Appendix A.1 for the derivation of these conditions.

$\frac{\partial\left(x_c^M - x_{nc}^M\right)}{\partial\gamma} > 0$ *and* $\frac{\partial}{\partial\gamma}\left(x_c^M / x_{nc}^M\right) > 0$.

Although the result that the monopolist always invests more in its core product than in its non-core product may seem intuitive, we will show in the next section that it is not necessarily the case when one consider multiproduct duopolists competing in the final good markets. In fact, depending on the values of γ and β, we find a firm may invest more or less in its core product than in its non-core product.

4. R&D Policy Competition

In this section, we make the same assumption that $A_1 = A_2 = A$. At the last stage, firms, taking as given the firms' R&D investments, simultaneously choose quantities to maximize their profits. Denote q_{ii} and q_{ji}, $i, j = 1,2,\ j \neq i$, as Firm i's outputs of two goods in each market, such that $q_{ii} = q_{ii1} = q_{ii2}$ and $q_{ji} = q_{ji1} = q_{ji2}$. Given the quantities of the other firm $\left(q_{ij}, q_{jj}\right)$, Firm i chooses (q_{ii}, q_{ji}) to maximize its operating profit π_i given by (6). Setting $\frac{\partial \pi_i}{\partial q_{1i}} = \frac{\partial \pi_i}{\partial q_{2i}} = 0$ for $i = 1,2,$ andsolving for equilibrium output, we obtain the Cournot output[5]:

$$\begin{aligned} q_{ii}^*(\mathbf{x}) &= \frac{(1-\gamma)(2A-2c-\delta)+3(1+\gamma)\delta+4x_{ii}-4\gamma x_{ji}-2x_{ij}+2\gamma x_{jj}}{6(1-\gamma^2)} \\ q_{ji}^*(\mathbf{x}) &= \frac{(1-\gamma)(2A-2c-\delta)-3(1+\gamma)\delta-4\gamma x_{ii}+4x_{ji}+2\gamma x_{ij}-2x_{jj}}{6(1-\gamma^2)}, \end{aligned} \tag{13}$$

for $i.j = 1,2$, $j \neq i$, and $\mathbf{x} = (x_{11}, x_{21}, x_{12}, x_{22})$ is the vector of two firms' R&D investment profiles.

It is easy to verified that for $x_{ii} = x_{ij} = 0$, $i, j = 1,2$, $j \neq i$, $q_{ii}^*(\mathbf{x}) - q_{ji}^*(\mathbf{x}) = \frac{\delta}{1-\gamma} > 0$. This implies that if no R&D investment is conducted, each firm sell more core product than non-core product. To ensure for interior equilibrium, we must find conditions which guarantee that $q_{ji}^*(\mathbf{x = 0}) > 0$. This condition is reduced to $A > c + \frac{2+\gamma}{1-\gamma}\delta$. The own-product effect of R&D can be seen from $\frac{\partial q_{ii}}{\partial x_{ii}} = 2 > 0$ and $\frac{\partial q_{ij}}{\partial x_{ii}} = -1 < 0$, for $j \neq i$, and this implies that an increase in a firm's R&D investment for a particular product raises this firm's output level for this product and

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5 The second order condition of Cournot outputs are satisfied: $\frac{\partial^2 \pi_1}{\partial q_{11}^2} = \frac{\partial^2 \pi_1}{\partial q_{21}^2} = -4 < 0$ and $\frac{\partial^2 \pi_1}{\partial q_{21} q_{11}} = \frac{\partial^2 \pi_1}{\partial q_{11} q_{21}} = -4\gamma$, and $\frac{\partial^2 \pi_1}{\partial q_{11}^2}\frac{\partial^2 \pi_1}{\partial q_{21}^2} - \left(\frac{\partial^2 \pi_1}{\partial q_{21} q_{11}}\right)^2 = 16(1-\gamma^2) > 0.$

decreases the rival's output level for the same product. The cross-product effect of R&D is shown by $\frac{\partial q_{ji}}{\partial x_{ii}} = -2\gamma < 0$, and $\frac{\partial q_{jj}}{\partial x_{ii}} = \gamma > 0$, and this implies that a firm's R&D for Product 1 lowers that firm's output level of Product 2, and increases its rival's output level for Product 2. It is argued that the cross-product effect is the main reason why the firms' R&D investments are strategic complements rather than strategic substitutes.

In the R&D stage, Firm i, taking as given the government's R&D subsidy, s_i, and Firm j's R&D portfolio (x_{1j}, x_{2j}), chooses x_{1i} and x_{2i} to maximize its profit:

$$\Pi_i = \pi_i(\mathbf{q}^*(\mathbf{x})) + s_i(x_{1i} + x_{2i}) - \beta x_{1i}^2 - \beta x_{2i}^2,$$

where $\mathbf{q}^*(\mathbf{x})$ is a vector of two firms' output level of two products, given by (13). Setting $\frac{\partial \Pi_i}{\partial x_{ii}} = \frac{\partial \Pi_i}{\partial x_{ji}} = 0$ yields Firm i's reaction functions:

$$\begin{aligned} x_{ii} &= \frac{4\Phi(2A-2c-\delta)+12(9\beta+9\beta\gamma-8)\delta-8(9\beta-8)x_{ij}+72\beta\gamma x_{jj}+9(1+\gamma)\Phi s_i}{2(9\beta+9\beta\gamma-8)\Phi} \\ x_{ji} &= \frac{4\Phi(2A-2c-\delta)-12(9\beta+9\beta\gamma-8)\delta+72\beta\gamma x_{ij}-8(9\beta-8)x_{jj}+9(1+\gamma)\Phi s_i}{2(9\beta+9\beta\gamma-8)\Phi}, \end{aligned} \tag{14}$$

where $\Phi = (9\beta - 9\beta\gamma - 8) > 0$ as required by the second order conditions[6]. For Φ to be positive, we require $\beta > \frac{8}{9}$, otherwise, it admits no $\gamma \in (0,1)$, such that $\Phi > 0$. For $\beta > \frac{8}{9}$, we must further assume that $\gamma < \bar{\gamma}^{PC} \equiv \frac{9\beta - 8}{9\beta}$, where the superscript of this upper bound on γ denotes policy competition. Moreover, it is evident that $\frac{\partial x_{ii}}{\partial s_i} = \frac{\partial x_{ji}}{\partial s_i} > 0$ because $\Phi > 0$, and this implies that the subsidy provided by the government increases the domestic firms' R&D investment in both products by the same amount. Furthermore, the fact that $\frac{\partial x_{ii}}{\partial x_{ij}} = \frac{\partial x_{ji}}{\partial x_{jj}} < 0$ and $\frac{\partial x_{ii}}{\partial x_{jj}} = \frac{\partial x_{ji}}{\partial x_{ij}} > 0$ implies that two firms' R&D investments in the same product are strategic substitutes and those across products are strategic complements.

Denote $x_c^* = x_{ii}^*$ and $x_{nc}^* = x_{ji}^*$ as the R&D investment for the core and non-core products, respectively, and solving two firms' reaction functions yields Firm i's equilibrium R&D

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[6] The second order condition requires that: $\frac{\partial^2 \Pi_i}{\partial x_{ii}^2} < 0$, $\frac{\partial^2 \Pi_i}{\partial x_{ji}^2} < 0$, and $\frac{\partial^2 \Pi_i}{\partial x_{ii}^2}\frac{\partial^2 \Pi_i}{\partial x_{ji}^2} < \frac{\partial^2 \Pi_i}{\partial x_{ii} x_{ji}}\frac{\partial^2 \Pi_i}{\partial x_{ji} x_{ii}}$. First note that $\frac{\partial^2 \Pi_i}{\partial x_{ii}^2} = \frac{\partial^2 \Pi_i}{\partial x_{ji}^2} = -\frac{2(9\beta(1-\gamma^2)-8)}{9(1-\gamma^2)}$ and $\frac{\partial^2 \Pi_i}{\partial x_{ii}\partial x_{ji}} = \frac{\partial^2 \Pi_i}{\partial x_{ji}\partial x_{ii}} = -\frac{16\gamma}{9(1-\gamma^2)}$. It thus requires that $(9\beta(1-\gamma^2)-8) > 8\gamma \Rightarrow 9\beta(1-\gamma) - 8 \equiv \Phi > 0.$

investments:

$$x_{ii}^* = x_c^*(s_i, s_j) = \frac{4(3\beta+3\beta\gamma-4)\Theta(2A-2c-\delta)+4\Delta\delta+3(1+\gamma)(9\beta+9\beta\gamma-8)\Theta s_i -12(1+\gamma)\Theta s_j}{2\Theta\Delta}$$
$$x_{ji}^* = x_{nc}^*(s_i, s_j) = \frac{4(3\beta+3\beta\gamma-4)\Theta(2A-2c-\delta)-4\Delta\delta+3(1+\gamma)(9\beta+9\beta\gamma-8)\Theta s_i -12(1+\gamma)\Theta s_j}{2\Theta\Delta}, \quad (15)$$

where $\Theta = (3\beta - 3\beta\gamma - 4)$ and $\Delta = (3\beta + 3\beta\gamma - 4)(9\beta + 9\beta\gamma - 4)$. Taking difference of $x_c^*(s_i, s_j)$ and $x_{nc}^*(s_i, s_j)$, we obtain that

$$x_c^*(s_i, s_j) - x_{nc}^*(s_i, s_j) = \frac{4}{\Theta}\delta, \quad (16)$$

and this suggests that the difference in investment levels for the two products is independent of subsidies granted by the governments.

Substituting (15) into (13), the Cournot quantities under optimal R&D investments are

$$q_{ii}^*(s_i, s_j) = \frac{3\beta(3\beta+3\beta\gamma-4)\Theta(2A-2c-\delta)+3\beta\Delta\delta+6(3\beta+3\beta\gamma-2)\Theta s_i -9\beta(1+\gamma)\Theta s_j}{2\Theta\Delta}$$
$$q_{ji}^*(s_i, s_j) = \frac{3\beta(3\beta+3\beta\gamma-4)\Theta(2A-2c-\delta)-3\beta\Delta\delta+6(3\beta+3\beta\gamma-2)\Theta s_i -9\beta(1+\gamma)\Theta s_j}{2\Theta\Delta}. \quad (17)$$

Considering $s_1 = s_2 = 0$ enables us to compare our results directly with that of Lin and Zhou (2013). We find that $x_{ii}^*(s_i, s_j) - x_{ji}^*(s_i, s_j) = \frac{4}{\Theta}\delta$; thus, the firm's R&D investment level for its core product is larger than that for its non-core product if and only if $\Theta = (3\beta - 3\beta\gamma - 4) > 0$. If $\beta < \frac{4}{3}$, then $\Theta < 0$ for all $\gamma \in (0,1)$. By contrast, if $\beta > \frac{4}{3}$, then $\Theta > 0$ if and only if $\gamma < \gamma_{\Theta=0} \equiv \frac{3\beta - 4}{3\beta}$. It is easy to verify that $\gamma_{\Theta=0} < \bar{\gamma}^{PC}$ for $\beta > \frac{4}{3}$; therefore, we can always find a value of γ, such that $\gamma_{\Theta=0} < \gamma < \bar{\gamma}^{PC}$ in which $\Theta < 0$, and this implies that the firm invest more R&D to reduce the cost of its non-core product than its core product.

To understand this result, using (14) and subtracting the two firms' reaction functions provide

$$(x_{ii} - x_{ji}) = \frac{12}{\Phi}\delta + \frac{4}{\Phi}(x_{jj} - x_{ij}), i = 1,2,\ j \neq i.$$

If we draw two firms' reaction functions, R_1 and R_2, on a graph with $(x_{22} - x_{12})$ on the X-axis and $(x_{11} - x_{21})$ on the Y-axis, two scenarios arises, depending on the slopes of these two reaction functions. As shown in Figure 1 below, if the slope of R_1 (R_2)is smaller (larger) than unity, i.e., $\frac{4}{\Phi} < 1$, two reaction functions intersects on the quadrant I as demonstrated in Figure 1a, and it is evident that $(x_{11} - x_{21}) > 0$ and $(x_{22} - x_{12}) > 0$. It can be verified that $\frac{4}{\Phi} < 1$ implies $3\Theta > 0$ or $\Theta > 0$; therefore, each firm invests more in its core-product than in its non-core product in this case. By contrast, if $\Theta < 0$, the slope of R_1 (R_2) is larger (smaller) than unity, the equilibrium results in the quadrant IV as demonstrated in Figure 1b. In this case, each firm invests more in its non-core product than its core product; $(x_{ii} - x_{ji}) < 0$ and $(x_{jj} - x_{ij}) < 0$.

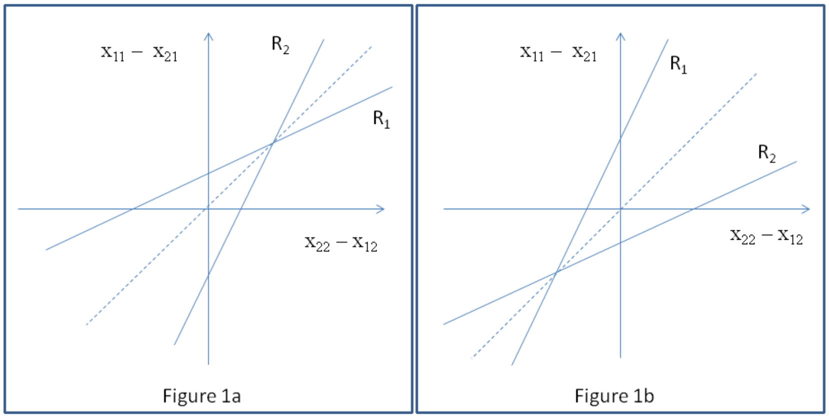

Figure 1. Reaction functions of two competing firms

Moreover, subtracting the firm's output levels of two products yields

$$q_{ii}^*(s_i, s_j) - q_{ji}^*(s_i, s_j) = \frac{3\beta\delta}{\Theta}.$$

Thus, it is evident that $q_{ii}^*(s_i, s_j) < q_{ji}^*(s_i, s_j)$ if $\Theta < 0$, implying that the firms sell more of their non-core products than their core products. This result might seem to contradict the one found previously that each firm's output level of its core product is larger than that of its non-core product. However, it is because, in the case of $\Theta < 0$, the firm invest so much more in its non-core product than in its core product such that the cost of originally a non-core product becomes smaller than that of originally a core product. To illustrate this result, first determine the post R&D costs for two products, which are provided as follows:

$$c_{ii}(x_{ii}^*) = c - x_{ii}^* = -\frac{4}{(9\beta + 9\beta\gamma - 4)}A + \frac{9\beta(\gamma+1)}{9\beta + 9\beta\gamma - 4}c - \frac{12\beta(2\gamma+1)}{\Theta(9\beta + 9\beta\gamma - 4)}\delta,$$

$$c_{ji}(x_{ji}^*) = c + \delta - x_{ji}^* = -\frac{4}{(9\beta + 9\beta\gamma - 4)}A + \frac{9\beta(\gamma+1)}{9\beta + 9\beta\gamma - 4}c + \frac{3\beta(9\beta - 4\gamma - 9\beta\gamma^2 - 8)}{(9\beta + 9\beta\gamma - 4)\Theta}\delta.$$

Subtracting $c_{ji}(x_{ji}^*)$ from $c_{ii}(x_{ii}^*)$ shows that $c_{ii}(x_{ii}^*) - c_{ji}(x_{ji}^*) = -\frac{3\beta(1-\gamma)}{\Theta}\delta$, which is positive if $\Theta < 0$. In this case, good 1 (2) becomes the non-core (core) product for Firm 1 after the investments.

The condition for values of parameters A, c, and δ must be determined to ensure that the R&D investments and Cournot quantities are positive for $s_i = s_j = 0$. From (15) and (17), it is easy to see that quantities are positive if and only if R&D investments are positive in the absence of subsidy. For $\gamma < \gamma_{\Theta=0}$ it is true that $\Theta > 0$, implying that the investment level for the core product is higher than that for the non-core product; therefore, we require that $x_{nc}^*(s_i = s_j = 0) > 0$. By contrast, if $\gamma_{\Theta=0} < \gamma < \bar{\gamma}^{PC}$ such that $\Theta < 0$, then x_c^* is smaller than x_{nc}^*, and we require that $x_c^*(s_i = s_j = 0) > 0$. These non-negativity conditions for R&D investments and post-R&D

costs are satisfied by the following assumption:[7]

Assumption 3 It is assumed that $c > 2\delta$ for $\frac{8}{9} < \beta < \frac{4}{3}$ and $\beta > 2$, and $c > \frac{4}{(3\beta - 4)}\delta$ for $\frac{4}{3} < \beta < 2$. For $\gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)} < \gamma < \overset{-}{\gamma}^{PC} \equiv \frac{9\beta - 8}{9\beta}$ such that $\Theta < 0$, we assume that $c - \frac{3\beta(2\gamma+1)}{\Theta}\delta < A < \frac{9\beta(\gamma+1)}{4}c + \frac{3\beta\left(9\beta\left(1-\gamma^2\right)-(4\gamma+8)\right)}{4\Theta}\delta$. For $0 < \gamma < \gamma_{\Theta} - \frac{4\delta}{3c\beta}$ such that $\Theta > 0$, we assume that $c + \frac{6\beta + 3\beta\gamma - 4}{\Theta}\delta < A < \frac{9\beta(1+\gamma)}{4}c - \frac{3\beta(2\gamma+1)}{\Theta}\delta$.

We summarize the results regarding the firms' optimal R&D investment without subsidies in the following proposition:

Proposition 2 *Suppose that the governments do not grant subsidy (i.e., $s_i = s_j = 0$), and considering Assumption 3,*

(i) the firms' invest more (less) in their core (non-core) products if $0 < \gamma < \gamma_{\Theta} - \frac{4\delta}{3c\beta}$.

(ii) the firms invest more (less) in their non-core (core) products, and change them into core (non-core) products, that is $x_c^* < x_{nc}^*$ *and* $c_{ii}(x_c^*) > c_{ji}(x_{nc}^*)$, *if* $\gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)} < \gamma < \overset{-}{\gamma}^{PC}$.

(iii) the non-negativity conditions of R&D investments and post-R&D are violated if $\gamma_{\Theta} - \frac{4\delta}{3c\beta} < \gamma < \gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)}$.

These results are in sharp contrast to that of Lin and Zhou (2013) who conclude that the duopolists also invest more in its core product than in its non-core product. The reason why they reach such conclusion is that they only consider the case of $0 < \gamma < \gamma_{\Theta} - \frac{4\delta}{3c\beta}$. By contrast, this study also include the case of $\gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)} < \gamma < \overset{-}{\gamma}^{PC}$ and demonstrate the possibility that the firm invest more in its non-core product than its core product. Moreover, the investment level differential is sufficiently large that it changes the non-core product to a core product (with a lower unit cost after R&D) in this case .

To understand (iii) of Proposition 2, consider first the case where γ approaches $\gamma_{\Theta=0}$ from above. From (15), it is easy to see that as γ approaches $\gamma_{\Theta=0}$, Θ approaches ∞, and x_c^* (x_{nc}^*) approaches infinity (negative infinity). In this case, the non-negativity conditions for R&D investments and post-R&D costs will be violated, and that is why one needs to consider the case

[7] See Appendix A.2 for the proof.

where γ is sufficiently different from $\gamma_{\Theta=0}$.

At the policy stage, Government i, taking as given the R&D subsidy (tax) imposed by Government j, s_j, chooses its R&D policy, s_i to maximize the welfare given by (7). To get a better understanding of the effect of subsidy/tax on a country's welfare, we decompose the first order condition as the following

$$\frac{\partial W_i}{\partial s_i} = \frac{\partial \pi_i}{\partial s_i} - \frac{\partial \beta(x_{ii}^2 + x_{ji}^2)}{\partial s_i} + \frac{\partial CS_i}{\partial s_i} = 0.$$

The effect of subsidy on the firm's operating profit is given by

$$\frac{\partial \pi_i}{\partial s_i} = \frac{3}{(3\beta+3\beta\gamma-4)(9\beta+9\beta\gamma-4)} \begin{bmatrix} 2(3\beta+3\beta\gamma-2)(2A-2c-\delta) \\ -(\gamma+1)(9\beta+9\beta\gamma-8)(q_{11}+q_{21}) \\ -2(\gamma+1)(3\beta+3\beta\gamma-2)(q_{12}+q_{22}) \end{bmatrix}.$$

The sign of $\frac{\partial \pi_i}{\partial s_i}$ can be positive or negative depending on the value of s_j, and it is easy to verify that $\frac{\partial \pi_i}{\partial s_i} > 0$ for $s_i = s_j = 0$.[8]

The effect of subsidy on the firm's R&D cost and consumer surplus are

$$\frac{\partial \beta(x_{ii}^2 + x_{ji}^2)}{\partial s_i} = \frac{3\beta(\gamma+1)(9\beta+9\beta\gamma-8)}{(3\beta+3\beta\gamma-4)(9\beta+9\beta\gamma-4)}(x_{11}+x_{21}),$$

and

$$\frac{\partial CS_i}{\partial s_i} = \frac{3(\gamma+1)}{2(9\beta+9\beta\gamma-4)}(q_{11}+q_{21}+q_{12}+q_{22})$$

respectively. It is intuitive that a subsidy induce a firm to increase its R&D investment level; thus, its cost for R&D investment increases. Similar to the case of multiproduct monopolist, a subsidy provided by the government increases the consumer surplus for a country because the price of products are lower due to a higher investment in cost-reducing R&D induced by the subsidy.

Setting $s_i = s_j = 0$ and the firm's profit net of subsidy is given by

$$\frac{\partial \pi_i}{\partial s_i} - \frac{\partial \beta(x_{ii}^2 + x_{ji}^2)}{\partial s_i} = -3\frac{(27\beta^2(\gamma+1)^2 - 8\beta(\gamma+1) - 16)(2A-2c-\delta)}{(3\beta+3\beta\gamma-4)(9\beta+9\beta\gamma-4)^2}.$$

It can be verified that the expression $(27\beta^2(\gamma+1)^2 - 8\beta(\gamma+1) - 16)$ is positive for all the relevant parameter values. Thus, If the government has any incentive to subsidize the firm's R&D investment, it must be the case that the gain in consumer surplus outweighs the loss in the firm's profit net of subsidy. This result is similar to the one obtained in the case of multiproduct

[8] Differentiating π_i with respect to s_i and setting $s_i = s_j = 0$, we get $\frac{\partial \pi_i}{\partial s_i} = \frac{(3\beta+3\beta\gamma-4)}{(9\beta+9\beta\gamma-4)^2}(2\text{A - }2c-\delta)$, which is positive.

monopolist.

Using (3), (5), (6), (13), and (15), and setting $\frac{\partial W_i}{\partial s_i} = 0$, we obtain Government i's reaction function:

$$s_i(s_j) = \frac{2\beta(3\beta + 3\beta\gamma - 4)}{3\Omega}(2A - 2c - \delta) + \frac{\Psi}{\Omega} s_j, \qquad (18)$$

where

$$\Psi \equiv (\beta + \beta\gamma - 4)$$
$$\Omega \equiv (18\beta^2)\gamma^2 + \beta(36\beta - 41)\gamma + (18\beta^2 - 41\beta + 20).$$

It can be easily verified that the second order condition for this problem requires that $\Omega > 0$, and this will be true for $\beta > \frac{8}{9}$ if and only if the value of γ is greater than a lower bound, $\underline{\gamma}^{PC} = \frac{(-36\beta + 41 + \sqrt{241})}{36\beta}$[9]. This lower bound is decreasing in β and takes a negative value for $\beta > \frac{\sqrt{241} + 41}{36}$; 1.570; thus, if $\beta > \frac{\sqrt{241} + 41}{36}$, $\Omega > 0$, for all $\gamma \in (0, \bar{\gamma}^{PC})$. Comparing with the upper bound and the lower bound obtained from the two second order conditions, we obtain that $\bar{\gamma}^{PC} - \underline{\gamma}^{PC} = \frac{72\beta - \sqrt{241} - 73}{36\beta}$, which is increasing in β and is positive if and only if $\beta > \frac{73 + \sqrt{241}}{72} \approx 1.230$. Because we require that the inequality $\underline{\gamma}^{PC} < \gamma < \bar{\gamma}^{PC}$ is satisfied, we conduct the remaining analysis assuming that $\beta > \frac{73 + \sqrt{241}}{72}$.

As can be seen from (14), whether two governments' policies are strategic substitutes or strategic complements depends on the sign of $\frac{\Psi}{\Omega}$. Provided that the second order condition is satisfied such that $\Omega > 0$, the governments' policies are strategic substitutes if $\Psi < 0$ and

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[9] The expression Ω is U-shaped in γ with two roots $\gamma_1 = \frac{-36\beta + 41 - \sqrt{241}}{36\beta}$ and $\gamma_2 = \frac{-36\beta + 41 + \sqrt{241}}{36\beta}$, and $\Omega > 0$ if and only if $\gamma < \gamma_1$ or $\gamma > \gamma_2$. It is easy to show that $\gamma_1 < 0$ for $\beta > \frac{41 - \sqrt{241}}{36} \approx 0.707$, and the second order condition in the R&D stage requires that $\beta > \frac{8}{9} \approx 0.889$. We can rule out the possibility of $\gamma < \gamma_1$. Therefore, $\Omega > 0$ if and only if $\gamma > \gamma_2 \equiv \gamma_s^{SOC} = \frac{-36\beta + 41 + \sqrt{241}}{36\beta}$.

strategic complements if $\Psi > 0$.[10] This result differs from that obtained in Haaland and Kind (2008), which showed that the governments' subsidies are always strategic substitutes in a single-product firm scenario.

Setting $s_j = 0$ in (18), we obtain

$$s_i(s_j = 0) = \frac{2\beta(3\beta + 3\beta\gamma - 4)}{3\Omega}(2A - 2c - \delta).$$

Therefore, whether the government has a unilateral incentive to impose R&D subsidies or taxes depends on the sign of $(3\beta + 3\beta\gamma - 4)$. The term $(3\beta + 3\beta\gamma - 4)$ is positive (negative) if $\gamma > (<) \frac{4 - 3\beta}{3\beta}$. It can be verified that the second order condition requires that $\gamma > \frac{4 - 3\beta}{3\beta}$; therefore, we conclude that the government has a unilateral incentive to subsidize (instead of tax) the firms' R&D investment.[11]

Using (18) and simultaneously solving the two governments' reaction functions yields the symmetric optimal policy:

$$s^{PC} = \frac{\beta}{9(\beta + \beta\gamma - 1)}(2A - 2c - \delta). \qquad (19)$$

It is clear that the term $(\beta + \beta\gamma - 1)$ is positive for all $\gamma \in (0,1)$ when $\beta > \frac{73 + \sqrt{241}}{72}$; 1.230. Therefore, s^{PC} is positive, implying that the symmetric optimal policy is a subsidy. Moreover, this optimal subsidy is increasing in A and decreasing in β, γ, c, and δ.

When Ψ is negative (i.e., two governments' reactions functions are downward sloping), the symmetric equilibrium is stable if and only if $\left|\frac{\Psi}{\Omega}\right| < 1$. We consider only the stable symmetric equilibrium in this study and determine the conditions for such case are $\beta > \frac{9 + \sqrt{7}}{9}$; 1.294 and

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[10] It is easy to verify that $\Psi = 0$ at $\gamma = \gamma_{\Psi=0} \equiv \frac{4 - \beta}{\beta}$, and $\Psi > (<) 0$ for $\gamma > (<) \gamma_{\Psi=0}$. Moreover, $\gamma_{\Psi=0}$ is decreasing in β and takes a positive (negative) value if $\beta < (>)$ 4. Comparing $\gamma_{\Psi=0}$ with $\underline{\gamma}^{PC}$, we observe that $\gamma_{\Psi=0} > \underline{\gamma}^{PC}$ for all $\beta > 0$. Comparing $\gamma_{\Psi=0}$ and $\bar{\gamma}^{PC}$, we observe that $\gamma_{\Psi=0} - \bar{\gamma}^{PC} = \frac{2}{9\beta}(22 - 9\beta)$ which is decreasing in β and equals zero at $\beta = \frac{22}{9}$. Therefore, $\gamma_{\Psi=0} > \bar{\gamma}^{PC}$ if $\beta < \frac{22}{9}$, and vice versa.

[11] It is easy to verify that $\frac{4 - 3\beta}{3\beta} - \underline{\gamma}^{PC} = -\frac{\sqrt{241} - 7}{36\beta} < 0$ for $\beta > 0$, implying that $\frac{4 - 3\beta}{3\beta} < \underline{\gamma}^{PC}$. Because the second order condition requires that $\gamma > \underline{\gamma}^{PC}$, it must also be true that $\gamma > \frac{4 - 3\beta}{3\beta}$.

$\frac{10-9\beta+2\sqrt{7}}{9\beta} \equiv \gamma_{\left|\frac{\Psi}{\Omega}\right|=1} < \gamma < \overline{\gamma}^{PC}$ [12]. Note that $\gamma_{\left|\frac{\Psi}{\Omega}\right|=1} > 0$ if $\beta < \frac{2\sqrt{7}+10}{9}$; 1.699 and vice versa; therefore, the relevant range of γ is $\gamma_{\left|\frac{\Psi}{\Omega}\right|=1} < \gamma < \overline{\gamma}^{PC}$ for $\frac{9+\sqrt{7}}{9} < \beta < \frac{10+2\sqrt{7}}{9}$ and $0 < \gamma < \overline{\gamma}^{PC}$ for $\beta > \frac{10+2\sqrt{7}}{9}$

Substituting (19) into (15) and (17), we obtain the R&D investments and outputs under optimal subsidy s^{PC}

$$\begin{aligned} x_c^{PC} &= \frac{(5\beta+5\beta\gamma-4)\Theta(2A-2c-\delta)+4(\beta+\beta\gamma-1)(9\beta+9\beta\gamma-4)\delta}{2\Theta(9\beta+9\beta\gamma-4)(\beta+\beta\gamma-1)} \\ x_{nc}^{PC} &= \frac{(5\beta+5\beta\gamma-4)\Theta(2A-2c-\delta)-4(\beta+\beta\gamma-1)(9\beta+9\beta\gamma-4)\delta}{2(9\beta+9\beta\gamma-4)(\beta+\beta\gamma-1)\Theta}, \end{aligned} \tag{20}$$

and

$$\begin{aligned} q_{ii}^{PC} &= \frac{\beta(9\beta+9\beta\gamma-8)\Theta(2A-2c-\delta)+9\beta(\beta+\beta\gamma-1)(9\beta+9\beta\gamma-4)\delta}{6(9\beta+9\beta\gamma-4)(\beta+\beta\gamma-1)\Theta} \\ q_{ji}^{PC} &= \frac{\beta(9\beta+9\beta\gamma-8)\Theta(2A-2c-\delta)-9\beta(\beta+\beta\gamma-1)(9\beta+9\beta\gamma-4)\delta}{6(9\beta+9\beta\gamma-4)(\beta+\beta\gamma-1)\Theta}. \end{aligned} \tag{21}$$

From equations (15) and (17), it is clear that the optimal subsidy increases the firms' R&D investment as well as quantities; therefore, the lower bounds on A provided in Assumption 3 are sufficient to guarantee that the non-negativity conditions for R&D investments are satisfied. Regarding the non-negativity condition for post-R&D unit costs, it is necessary to modify the upper bound for A to ensure that $x_c^{PC} < c$ and $x_{nc}^{PC} < c+\delta$. Following the similar procedure to that of the case of no subsidy, we present the necessary assumption in the following[13].

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[12] Note that for $\Psi < 0$, $\left|\frac{\Psi}{\Omega}\right| < 1$, i.e., the optimal subsidy is stable, implies that $-\Psi < \Omega$, equivalently as $-\Psi-\Omega < 0$. It is easy to show that $(-\Psi-\Omega)$ is inverted U-shaped in γ with two roots: $\gamma_1 = \frac{10-9\beta-2\sqrt{7}}{9\beta}$ and $\gamma_2 = \frac{10-9\beta+2\sqrt{7}}{9\beta}$. Simple verification yields that $\gamma_1 < \underline{\gamma}^{PC}$, $\gamma_2 > \underline{\gamma}^{PC}$, and both γ_1 and γ_2 are smaller than $\gamma_{\Psi=0}$. In order for the equilibrium to be stable, we require that $\gamma < \gamma_1$ or $\gamma > \gamma_2$. Since $\gamma < \gamma_1 < \underline{\gamma}^{PC}$ violates the second order condition, we requires that $\gamma > \gamma^2 \equiv \gamma_{\left|\frac{\Psi}{\Omega}\right|=1}$. Note that $\gamma_{\left|\frac{\Psi}{\Omega}\right|=1} < \overline{\gamma}^{PC}$ if and only if $\beta > \frac{9+\sqrt{7}}{9} \approx 1.294$, we therefore need to modify our conditions on β and γ to the one such that $\beta > \frac{9+\sqrt{7}}{9}$ and $\gamma_{\left|\frac{\Psi}{\Omega}\right|=1} < \gamma < \overline{\gamma}^{PC}$.

[13] The proof will be provided by the author upon request.

Assumption 4 *It is assume that* $c > \frac{(171\beta - 148)}{4(18\beta - 17)}\delta$ *for* $\frac{9+\sqrt{7}}{9} < \beta < \frac{4}{3}$ *and* $\beta > 1.987$*, and* $c > \frac{(9\beta - 8)}{2(\beta - 1)(3\beta - 4)}\delta$ *for* $\frac{4}{3} < \beta < 1.987$. *Define* $\bar{\gamma} = \frac{-2c - 9\delta + \sqrt{m}}{12c\beta}$*, where* $m = 4(6\beta - 7)^2 c^2 - 12\delta(18\beta - 19)c + 81\delta^2$*, and* $\underline{\gamma} = \frac{-2c + 7\delta + \sqrt{n}}{12\beta(c+\delta)}$*, where*

$n = 4(6\beta - 7)^2 c^2 + 4(-114\beta + 72\beta^2 + 41)c\delta + (-120\beta + 144\beta^2 + 49)\delta^2$.

For $\max(\gamma_{\left|\frac{\Psi}{\Omega}\right|=1}, 0) < \gamma < \bar{\gamma}$*, we assume that*

$$c + \frac{6\beta + 3\beta\gamma - 4}{\Theta}\delta < A < \frac{\beta(1+\gamma)(9\beta + 9\beta\gamma - 8)}{(5\beta + 5\beta\gamma - 4)}c - \frac{\beta(3(1+\gamma)(17\gamma + 7)\beta - 4(11\gamma + 5))}{2(5\beta + 5\beta\gamma - 4)\Theta}\delta.$$

For $\underline{\gamma} < \gamma < \bar{\gamma}^{PC}$*, we assume that*

$$c - \frac{3\beta(2\gamma + 1)}{\Theta}\delta < A < \frac{\beta(1+\gamma)(9\beta + 9\beta\gamma - 8)}{5\beta + 5\beta\gamma - 4}c + \frac{\beta K}{2(5\beta + 5\beta\gamma - 4)\Theta}\delta,$$

where $K = 54(1-\gamma)(\gamma + 1)^2\beta^2 - 9(1+\gamma)(11 - 3\gamma)\beta + 4(5\gamma + 11)$.

We summarize the results obtained in policy stage below:

Proposition 3 *Considering that Assumption 4 is satisfied,*

(i) The governments' policies are strategic substitutes for $\beta < \frac{22}{9} \approx 2.444$*, or* $\frac{22}{9} < \beta < 4$ *and* $\gamma < \gamma_{\Psi=0}$*. By contrast, the government's policies are strategic complements if* $\frac{22}{9} < \beta < 4$ *and* $\gamma > \gamma_{\Psi=0}$*, or* $\beta > 4$.

(ii) The governments have a unilateral incentive to subsidize the firms' R&D investments, and the symmetric optimal policy is always a subsidy,

$$s^{PC} = \frac{\beta}{9(\beta + \beta\gamma - 1)}(2A - 2c - \delta).$$

(iii) The optimal subsidy is increasing in the market size, and decreasing in the production cost, the initial cost difference of core and non-core products, the R&D cost parameter β*, and the degree of product substitutability.*

(iv) The multi-product firms invest more in their core(non-core) products for

$$\max\left(\gamma_{\left|\frac{\Psi}{\Omega}\right|=1}, 0\right) < \gamma < \bar{\gamma},\ (\underline{\gamma} < \gamma < \bar{\gamma}^{PC})$$ *with optimal subsidy* s^{PC}.

5. R&D Policy Cooperation

We demonstrate in the previous section that the governments non-cooperatively choose to subsidize domestic multi-product firms. However, it is well known that policy competition typically leads to results that are suboptimal from a global perspective. At least two reasons exist to explain this result. First, a government does not account for the other country's welfare when choosing its level of subsidy. Second, the business-stealing effect between two firms tends to induce the governments to over-subsidize their domestic firms which leads to a familiar prisoner's dilemma. Therefore, we also discuss the case where two governments cooperate in their subsidies to maximize joint welfare.

The policy cooperation considered in this study is closer to the notion of policy harmonization in which two governments coordinate a uniform subsidy (or tax). It is possible to improve welfare even further if one allows for asymmetric policies between two governments; however, because we consider only the stable symmetric equilibrium in this study, we focus on the policy-harmonization type of policy cooperation as also done in Haaland and Kind (2010). Moreover, we compare the optimal subsidies between cases of monopoly and duopoly.

Suppose that the governments harmonize their R&D subsidies at a common level, such as $s_i = s_j = s$, and this common subsidy is chosen to maximize the joint welfare $W = W_1 + W_2$. By using (2), (3), (5), (6), (7), (15), (17), the joint welfare W is given by

$$W = W_1 + W_2 = \frac{4\beta(2A-2c-\delta)^2\Theta^2 + 2\beta(9\beta+9\beta\gamma-4)(9\beta-9\beta\gamma-8)\delta^2 - 9(\gamma+1)\Theta^2 s^2}{(9\beta+9\beta\gamma-4)\Theta^2}.$$

It is evident that $W > 0$ for $s = 0$, and $\frac{\partial W}{\partial s} < 0$ for all $s \neq 0$. Therefore, we obtain the following result:

Proposition 4 *Suppose that the governments cooperate with each other to maximize the joint welfare by granting a common level of subsidy to their domestic multi-product firms. It is optimal for them to set* $s_i = s_j = 0$.

To understand this result, recall that the difference between the investment level for two products is independent of governments' subsidies, as can be seen from (16). Although the government can use subsidies (taxes) to increase (decrease) the firm's investments for R&D for two goods, the firm always invests in such a way that $x_c - x_{nc} = \frac{4}{\Theta}\delta$. Therefore, the problem of choosing common s to maximize the joint profit is equivalent to that of choosing x_c and x_{nc} to maximize the joint welfare subject to $x_c - x_{nc} = \frac{4}{\Theta}\delta$. Because the countries are symmetric to each other, we only have to solve this problem for one country. Assuming that $s_i = s_j = 0$ and given the constraint $x_{nc} = x_c - \frac{4}{\Theta}\delta$, the government chooses x_c to maximize the joint welfare as follows:

$$W = \frac{4\Theta^2\left(2A-(c-x_c)-(c+\delta-\left(x_c-\frac{4}{\Theta}\delta\right))\right)^2-18\beta(\gamma+1)\left(2\Theta^2x_c^2-8\Theta\delta x_c-\delta^2(9\beta-9\beta\gamma-16)\right)}{9(\gamma+1)\Theta^2}.$$

The first order condition is

$$\frac{\partial W}{\partial x_c} = \frac{-8(9\beta+9\beta\gamma-4)\Theta x_c+16\Theta(2A-2c-\delta)+16(9\beta+9\beta\gamma-4)\delta}{9(\gamma+1)\Theta} = 0,$$

and the second order condition is

$$\frac{\partial^2 W}{\partial x_c^2} = -\frac{8(9\beta+9\beta\gamma-4)x}{9(\gamma+1)} < 0.$$

It is clear that the second order condition is satisfied, and solving the first order condition yields the harmonized R&D investment for the core product

$$x_c^H = \frac{2\Theta(2A-2c-\delta)+2(9\beta+9\beta\gamma-4)\delta}{(9\beta+9\beta\gamma-4)\Theta}.$$

From Equation (15), one can verify that x_c^H equals $x_c^*(s_i = s_j = 0)$; therefore, if the government chooses the R&D investment levels for two products subject to $x_c - x_{nc} = \frac{4}{\Theta}\delta$, it will choose the levels of R&D investments such that they are equal to those chosen by the multi-product firms under no-subsidy case.

To compare the optimal subsidies between monopoly and duopoly cases, subtracting s^M from s^{PC} yields

$$s^{PC} - s^M = -\frac{\beta\Psi}{9(\beta+\beta\gamma-1)(8\beta+8\beta\gamma-5)}(2A-2c-\delta).$$

As shown previously that the governments' policies are strategic substitutes (complements) if $\Psi < (>)\ 0$. Therefore, s^{PC} is greater than s^M if and only if $\Psi < 0$, and vice versa. We thus obtain the following result:

Proposition 5 *The governments grant (lower) higher subsidies to their domestic firms in a duopoly than in a monopoly if and only if the governments' policies are strategic substitutes (complements) under policy competition.*

If the governments' policies are strategic substitutes (i.e., $\Psi < 0$), a government has an incentive to grant a higher subsidy because it lowers the subsidy granted by the foreign government which then helps the domestic firm to steal more profit from the foreign firm. As a result, both governments choose a higher level of subsidy in a duopoly than that in a monopoly (no competition), which results in a familiar prisoner's dilemma situation.

6. Concluding Remarks

We studied the optimal process R&D portfolios for multi-product firms, each producing two goods with initially differential unit costs, in a reciprocal dumping model. The governments were allowed to subsidize (or tax) their domestic firms' R&D investments. First, we showed that the firms do not necessarily invest more in its core product than in its non-core product. In fact, the investment level for the core product is higher (lower) than that for the non-core product if the degree of product substitutability is higher (lower) than a particular level. Moreover, provided that the products are sufficiently similar, the firms not only invest more in their non-core products but also do so to the extent that the post-R&D unit cost of the non-core product becomes lower than that of the core product; the firms turn their non-core products into core products after R&D investment.

We studied policy competition between two governments where each could subsidize or tax the R&D investment to maximize its welfare, including the consumer surplus and the domestic firm's net profit from subsidy payment (or tax). We found that whether the governments' policies are strategic substitutes or complements depends on the degree of product substitutability and the R&D cost parameter. In fact, if the value of the R&D cost parameter exceeds some threshold (i.e., R&D investments are sufficiently costly), two governments' policies are strategic complements. For other cases, the policies are strategic complements (substitutes) if the degree of product substitutability is higher (lower) than some threshold, depending on the value of the R&D cost parameter; the policies are more likely to be strategic complements (substitutes) if two goods are similar (differentiated) to each other. We also showed that the government has a unilateral incentive to subsidize the domestic firm, and the symmetric equilibrium is always a subsidy despite the strategic complementarity. Furthermore, the optimal subsidy decreases as two products become more similar to each other.

Lastly, we considered policy cooperation between two governments where they jointly choose a common level of subsidy to maximize the joint welfare. We observed that laissez-faire is optimal in this case. The joint welfare would be maximized if the governments choose $s_i = 0$, $i = 1,2$. Moreover, we showed that whether the optimal subsidy in a duopoly is higher (or lower) than that in a monopoly depends on whether the governments' policies are strategic substitutes or strategic complements. If the policies are strategic substitutes, it will induce each government to choose a level of subsidy in a duopoly higher than that in a monopoly. By contrast, if the policies are strategic complements, the optimal subsidy under policy competition is lower than that for a multi-product monopolist.

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Appendix

A.1 *Non-negativity conditions of R&D investments and post-R&D costs with optimal subsidy under Monopoly:*

The second order condition is $8\beta + 8\beta\gamma - 5 > 0$ or $\gamma > \underline{\gamma}^M = \frac{5-8\beta}{8\beta}$. If $\beta > \frac{5}{8}$, then $\underline{\gamma}^M < 0$, and second order condition satisfies for all γ. For $\frac{1}{2} < \beta < \frac{5}{8}$, in which, $\underline{\gamma}^M > 0$, we require that $\bar{\gamma}^M > \underline{\gamma}^M$; otherwise, it admits no value of γ such that the second order conditions in both R&D and policy stages are satisfied. A simple verification shows that $\bar{\gamma}^M > \underline{\gamma}^M$ if and only if $\beta > \frac{9}{16}$. Therefore, we assume that $\beta > \frac{9}{16}$. Moreover, we assume that $\underline{\gamma}^M < \gamma < \bar{\gamma}^M$ for $\frac{9}{16} < \beta < \frac{5}{8}$ and $0 < \gamma < \bar{\gamma}^M$ for $\beta > \frac{5}{8}$.

The non-negativity condition for the post-R&D production cost (i.e., $(c - x_{11}) > 0$) is

$$A < A_U \equiv \frac{8\beta(\gamma+1)}{5}c - \frac{\beta(9\gamma-1)}{5(2\beta-2\beta\gamma-1)}\delta.$$

The non-negativity condition for R&D investment (i.e., $x_{21} > 0$) is

$$A > A_L = c + \frac{(2\beta-1)}{(2\beta(1-\gamma)-1)}\delta.$$

We require that $A_U > A_L$, and it is true if and only if

$$c > \frac{(9\beta+9\beta\gamma-5)}{(8\beta+8\beta\gamma-5)(2\beta-2\beta\gamma-1)}\delta.$$

A.2 *Non-negativity conditions of R&D investments and post-R&D costs without subsidy under Duopoly:*

For $\Theta > 0$, we require that $x^*_{nc}(s_i = s_j = 0) > 0$, which is true if and only if

$$A > \underline{A}_{s=0} \equiv c + \frac{6\beta+3\beta\gamma-4}{\Theta}\delta.$$

The lower bound of A is positive because $\Theta > 0$ implying that $\beta > \frac{4}{3}$ and $(6\beta+3\beta\gamma-4) > 0$. The non-negativity condition for the post-R&D unit cost of the core product (i.e., $(c - x^*_c) > 0$) is

$$A < \bar{A}_{s=0} \equiv \frac{9\beta(1+\gamma)}{4}c - \frac{3\beta(2\gamma+1)}{\Theta}\delta.$$

To ensure that $\underline{A}_{s=0} < \bar{A}_{s=0}$, we require that

$$\bar{A}_{s=0} - \underline{A}_{s=0} = \frac{(9\beta+9\beta\gamma-4)(\Theta c - 4\delta)}{4\Theta} > 0,$$

which is true if and only if $(\Theta c - 4\delta) > 0$ given that $\Theta > 0$. Rearranging terms, it is equivalent to

$$\gamma < \frac{(c(3\beta-4)-4\delta)}{(3c\beta)} = \left(\gamma_{\Theta} - \frac{4\delta}{3c\beta}\right).$$

Since γ can only take positive value, we need to assume that $\left(\gamma_{\Theta} - \frac{4\delta}{3c\beta}\right) > 0$, which is equivalent to $c > \frac{4}{(3\beta-4)}\delta$. Thus, we assume that $c > \frac{4}{(3\beta-4)}\delta$, $\gamma < \left(\gamma_{\Theta} - \frac{4\delta}{3c\beta}\right)$, and $\overline{A}_{s=0} < A < \underline{A}_{s=0}$ for $\Theta > 0$.

For the case $\Theta < 0$, we require that $x_c^*(s_i = s_j = 0) > 0$ and $\left(c + \delta - x_{nc}^*(s_i = s_j = 0)\right) > 0$. The non-negativity condition for R&D investment is

$$A > \underline{A}'_{s=0} \equiv c - \frac{3\beta(2\gamma+1)}{\Theta}\delta.$$

The non-negativity condition for post-R&D cost of the non core product is

$$A < \overline{A}'_{s=0} = \frac{9\beta(\gamma+1)}{4}c + \frac{3\beta\left(9\beta\left(1-\gamma^2\right)-(4\gamma+8)\right)}{4\Theta}\delta.$$

To ensure that $\underline{A}'_{s=0} < \overline{A}'_{s=0}$, we require that

$$\overline{A}'_{s=0} - \underline{A}'_{s=0} = -\frac{(9\beta+9\beta\gamma-4)(-\Theta c-3\beta(1-\gamma)\delta)}{4\Theta} > 0,$$

which is true if and only if $\left(-\Theta c - 3\beta(1-\gamma)\delta\right) > 0$ given that $\Theta < 0$. Rearranging terms, it is equivalent to

$$\gamma > \frac{((3\beta-4)(c+\delta)+4\delta)}{3\beta(c+\delta)} = \gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)}.$$

In addition, we require that $\gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)} < \overline{\gamma}^{PC} \equiv \frac{9\beta-8}{9\beta}$; otherwise, there exist no value of $\gamma \in (\gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)},\ \overline{\gamma}^{PC})$ such that both second order condition and non-negativity condition are satisfied. It can be verified that it is true if and only if $c > 2\delta.$ Therefore, we assume that $c > 2\delta$, $(\gamma_{\Theta=0} + \frac{4\delta}{3\beta(c+\delta)}) < \gamma < \overline{\gamma}^{PC}$, and $\underline{A}'_{s=0} < A < \overline{A}'_{s=0}$ for $\Theta < 0$.

Comparing the lower bound of c in the case of $\Theta > 0$ and that in the case of $\Theta < 0$, it is easy to see that $2\delta > \frac{4}{(3\beta-4)}\delta$ for $\frac{8}{9} < \beta < \frac{4}{3}$. Moreover, a simple verification shows that $\frac{4}{(3\beta-4)}\delta > 2\delta$ if and only if $\beta < 2$, and vice versa. Therefore, if $\frac{8}{9} < \beta < \frac{4}{3}$ or $\beta > 2$, we assume that $c > 2\delta$. By contrast, if $\frac{4}{3} < \beta < 2$, we assume that $c > \frac{4}{(3\beta-4)}\delta$.

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