Chapter 16

Recursive Programming to Reinforce the KEWT Data-Sets by Country

16.1 Introduction

This last Chapter *first* intentionally synthesizes the relationship between recursive programming and KEWT data-sets. The author shows related proofs deeply. Each chapter in the *EES* has presented each issue, rather focusing and narrowing the range of spread for simplicity. This chapter widely spreads the related issues and refers to other issues. This chapter compares each country's recursive programming and uses five types of combinations between parameters and variables. The five type combinations were selected among others so that characteristics by country are most effectively presented from various aspects. All the results of recursive programming are only compiled in this chapter. Readers are able to compare 36 countries in recursive programming by type. All the results of hyperbola graphs for 36 countries are compiled in Appendix at the end of the *EES*. Readers are able to compare each characteristic by country, comparing results of recursive programming and hyperbola graphs. This chapter, for simplicity, does not refer to hyperbola results.

Second, this Chapter is able to reply to some problems penetrated by Harcourt, G. C. (1972, 272p.) as the successor of Robinson, J. This is because Harcourt summarized the essence of UK Keynesians, comparing with Neo-classical theories, and showed hundred surprising diagrams; full of insight, yet without empirical results. This chapter does not wholly intend to comment or review his life-work. Yet, the author cites several diagrams of his and intends to bury the differences between UK and US (both) Keynesians. This challenge is hopeful, by using tight cooperation lying between the endogenous system and KEWT data-sets by country and, applying to one of his diagram the above five types of combinations obtained from recursive programming. For example, the relationship between the marginal productivity of labor and the average productivity of labor is solved using one of five types by country. Even his diagrams to double-switching and capital-reversing correspond with those of several countries shown in another of five types by country.

Harcourt (ibid., 35) refers to five assumptions set by Swan (1956): investment determined by saving, constant returns to scale, full employment, static expectations and perfect competition. Meade (1962) raises nine assumptions as the author discussed in earlier chapters. According to the author's viewpoint of purely endogenous, two assumptions of perfect competition and the price-equilibrium are decisively common to Keynesian and Neo-classical schools. The endogenous system totally decreased nine assumptions each by each although some assumptions were interrelated. Perfect assumption is shown by an endogenous fact that marginal productivity of labor (MPL) equal the wage rate and marginal productivity of capital (MPK) equals the rate of return, each in equilibrium. It implies that an average equals its marginal value. This fact is not realized when the price-equilibrium prevails in the global economies. Since a ratio such as the rate of return has no unit, capital must have a value but, this value is unknown under the price-equilibrium. Furthermore, as described by Harcourt (ibid., 5) 'Robinson argues that comparisons of equilibrium positions one with another are not the appropriate tools for the analysis of out-of-equilibrium processes or changes.' Under the endogenous-equilibrium, 'out-of-equilibrium processes' are exactly measured using the speed years and seven endogenous parameters in the endogenous system.

16.2 Theory and Practice between Recursive Programming and KEWT Data-sets

16.2.1 Relationship between recursive programming in the transitional path and KEWT data-sets

This section endogenously summarizes the relationship between the recursive programming in the transitional path and KEWT data-sets. Since theory and practice are united at the endogenous system, this relationship means to express the processes in recursive programming consistently with KEWT data-sets. KEWT data-sets hold without the help of recursive programming in the transitional path. Why, then, do we need to measure the recursive programming in the transitional path? KEWT data-sets only show all the parameters and variables at a moderate equilibrium, which is measured by the speed years for convergence in endogenous equilibrium. For example, suppose the speed years of a country are 48 years. KEWT data-sets are unable to show all the parameters and variables by year during 48 years. Recursive programming is solely able to show all the parameters and variables by year during 48 years. At the endogenous system, seven endogenous parameters control the whole system by country and by sector but, here the author presents, for simplicity, the processes at the total economy and also the processes directly related to 1) the quantitative net investment coefficient, β^*, and the diminishing returns to capital coefficient, δ_o.

In a fiscal year, the speed years for convergence in endogenous equilibrium (hereafter, the speed years) are each determined by country and by sector, using the recursive programming in the transitional path (hereafter, recursive programming).

In recursive programming, first of all, two determinants, β^* and δ_o, must be measured. If β^* and δ_0 are measured consistently, then, recursive programming and KEWT data-sets are all consistent each other. What guarantees and justifies this consistency between recursive programming and KEWT data-sets? The author justifies the mutual consistency by maintaining the equal relationship between the productivity of stock and the productivity of flow. The productivity of stock is presented by total factor productivity (*TFP*) as shown in the literature. The productivity of flow is presented by the rate of technological progress as shown in the endogenous system. There is no article that proves that *TFP* is equal to the rate of technological progress. This is natural since the rate of technological progress is not purely endogenous but essentially exogenous in the literature that uses the Cobb-Douglass production function in the constant returns to scale.

The author in this section proves the equal relationship between *TFP* and the rate of technological progress, $g_A^* = i(1-\beta^*)$, thoroughly limiting to the direct relationship.

Let the author follow the literature as much as possible and compare the discrete case with the continuous case. The discrete case of *TFP* is shown by stock; $g_A(t) = (A(t) - A(t-1))/A(t-1)$, where $TFP = A$. The continuous case of productivity as in growth accounting is shown by flow; $g_A(t) = g_y(t) - \alpha \cdot g_k(t)$, where each per capita. The continuous Cobb-Douglas production function in the literature, however, cannot synthesize discrete and continuous. The discrete Cobb-Douglas production function only synthesizes discrete and continuous. The author here indicates that Samuelson's lifework for welfare economy is full of insights yet based on the continuous Cobb-Douglas production function. Samuelson and Modigliani (see, Figure 1; 323, 1966) tried to get to a common destination with Keynesians such as Pasinetti and Kaldor. Why is it difficult to synthesize discrete and continuous? The author finds the answer from the assertion of Robinson's (157-166, 1959). A model needs the measurement of capital and its rate of return at the same time. The endogenous system simultaneously measures capital (physical/fixed assets or capital stock) and the rate of return at KEWT data-sets and its transitional path by year: K and $r^* = \Pi/K$ (see Chapter 6). As a result, $g_A(t) = (A(t) - A(t-1))/A(t-1) = g_y(t) - \alpha \cdot g_k(t)$ is endogenously synthesized and proved empirically.

At the initial/current year in the transitional path, the diminishing returns to capital coefficient, δ_0, is formulated and holds. At the convergence year at the steady state or the balanced growth state, δ_o reduces to the relative share of capital, α, where $\delta_0 = \alpha$ holds. This is proved using endogenous equations and also using the recursive programming in the transitional path. The ratio of net investment to output, $i = I/Y$, and the ratio of saving to output $s = S/Y$, are fixed in the transitional path. But, the quantitative net investment coefficient, β^* or $1 - \beta^*$, changes in the transitional path, similarly to δ_0, as formulated below.

16.2.2 Proofs of relationship between the rate of technological progress and the growth rate per capita output

In this section, the rate of technological progress is measured and proved, starting with the transitional path by time/year, t. The rate of technological progress, $g_{A(t)} = i_t(1 - \beta_t)$, presents the primary base for the endogenous model and its data-sets and further leads to related endogenous variables by t.

$\beta(t) = \beta(0)(1 + g_\beta)^t$ and $\delta(t) = \delta(0)(1 + g_\delta)^t$, where $\beta(t) \to \beta^*$ and $\delta(t) \to \alpha$, each at convergence, $t \to t^*$.

$i(t) = i \cdot y(t)$, where $TFP(0) = \frac{k(0)^{1-\alpha}}{\Omega(0)}$ and $y(0) = TFP(0) \cdot k(0)^\alpha$.

To simplify, notation A is used for total factor productivity, *TFP*. $L(t) = L(0)(1 + n)^t$ is set to clarify the capital-labor ratio, $k(t)$, and per capita output, $y(t)$. To simplify, relative statistics population is used at the initial year; $L(0)$=1.0000. The growth rate of statistics population is $n = (L_t - L_{t-1})/L_{t-1}$. The rate of change in population in equilibrium is designated by n_E. KEWT 6.12, 1990-2010, presumably sets a moderate equilibrium under full employment; $n_E = n$ while KEWT 5.11, 1990-2009, under $n_E \neq n$ to save some countries that fall into close-to-disequilibrium. To simplify, n is used in this section.

Using the above three values, basic numerical values by time are arranged.

Setting $i_K(t) = i(t) \cdot \beta(t)$, $k(t) = k(t-1) + i_K(t)$ holds.
Setting $i_A(t) = i(t)(1 - \beta(t))/k(t)^{\delta(t)}$, $A(t) = A(t-1) + i_A(t)$ holds.
$i(t) \neq i_K(t) + i_A(t)$ holds, because of the introduction of $k(t)^{\delta(t)}$ into $i_A(t)$.

Each variable of $g_A(t), g_k(t),$ and $g_y(t)$, is calculated using each difference of $A(t)$ and $A(t-1)$, $k(t)$ and $k(t-1)$, and $y(t)$ and $y(t-1)$: e.g., $g_{A(STOCK)}(t) = (A(t) - A(t-1))/A(t-1)$.

At convergence, the above $i_A(t) = i(t)(1 - \beta(t))/k(t)^{\delta(t)}$ reduces to $i_A^* = i(1 - \beta^*)$ and $g_A^* = i_A^*$ holds.

As a result, the discrete case is transformed and finalized:

$$g_A(t) = i_A(t) \cdot k(t)^{\alpha - \delta(t)} = \frac{i_A(t) \cdot y(t)}{A(t) \cdot k(t)^{\delta(t)}} = \frac{A(t+1) - A(t)}{A(t)}.$$

Or, $\quad g_A^* = i(1 - \beta^*)$ at convergence $\quad$ (1)

At convergence, $g_A(t) = g_A^*$ holds with CRC. Eq.1 reduces to $g_A^* = i_A^*$ since $k^{*\alpha-\alpha} = 1$. This is equivalent to $g_A^* = i(1-\beta^*)$, as shown in 1.1 above. Also, $g_y^* = g_k^*$ holds. Then, $g_A(t) = g_y(t) - \alpha \cdot g_k(t)$ reduces to $g_A^* = (1-\alpha)g_y^*$.

$$g_y^* = g_k^* = \frac{i_A^*}{1-\alpha}. \tag{2}$$

Eq.2 corresponds with Solow's exogenous equation (after correction[1]; 94, in 1.4, 1969). Therefore, regardless of whether the rate of technological progress is exogenous or endogenous, Eq. 2 holds as long as the Cobb-Douglas production is used. Then, how is the quantitative net investment coefficient, β^*, calculated? The following two steps are required to simultaneously formulate the capital-output ratio, Ω^*, and the quantitative coefficient, β^*.

16.2.3 Proof of the capital-output ratio and the quantitative net investment coefficient

The continuous case starts with $\Delta k(t) = \frac{i_K(t)\cdot y(t) - n\cdot k(t)}{1+n}$, from

$k(t+1) = \frac{k(t)+i_K(t)\cdot y(t)}{1+n} = \frac{K(t)+i_K(t)\cdot Y(t)}{(1+n)\cdot L(t)} = \frac{K(t)+\Delta K(t)}{(1+n)L(t)} = \frac{K(t+1)}{L(t+1)}$. Then,

$$g_k(t) = \frac{1}{1+n}\left(i_K(t)\cdot A(t)\cdot k(t)^{\alpha-1} - n\right) = \frac{i_K(t)\cdot y(t) - nk(t)}{(1+n)k(t)} \tag{3}$$

Accordingly, at convergence,

$$g_k^* = \frac{1}{1+n}\left(i_K^*\cdot A^*\cdot k^{*\alpha-1} - n\right) \tag{4}$$

Inserting $\frac{1}{\Omega^*} = \frac{k^{*1-\alpha}k^{\alpha-1}}{\Omega^*} = A^* k^{*\alpha-1}$ into Eq.4, we obtain

$$g_k^* = \frac{1}{1+n}\left(\frac{i_K^*}{\Omega^*} - n\right) \tag{5}$$

Since Eq.5 is equivalent to Eq.2 (by connecting these two cases), $\frac{i_A^*}{1-\alpha} = \frac{1}{1+n}\left(\frac{i_K^*}{\Omega^*} - n\right)$ is derived, where $i_A^* = i(1-\beta^*)$ and $i_k^* = i\cdot\beta^*$hold at convergence.

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[1] The author is grateful to Dr. Solow, R. M. for his direct reply to my question on 9 March 1998: "The answer to your question is that the statement on page 86 of my 1956 article is a mistake. I do not know how such a simple error of arithmetic occurred; but I discovered it very soon after the article was published. As you say, steady-state *K/Y* is constant. Once in a while someone notices the error and writes to me, as you did. The first person to write, probably in 1957, was T.N. Srinivasan, then a graduate student at Yale, and now a professor there. Thank you for your letter, and good luck with your book."

As a result, $\frac{i(1-\beta^*)}{1-\alpha} = \frac{1}{1+n}\left(\frac{i \cdot \beta^*}{\Omega^*} - n\right)$ or $i(1-\beta^*)(1+n) = (1-\alpha)\left(\frac{i \cdot \beta^*}{\Omega^*} - n\right)$ is derived.

Therefore, the capital-output ratio equation is obtained:

$$\Omega^* = \frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)(1+n)+n(1-\alpha)} \tag{6}$$

Or, differently, the quantitative net investment coefficient equation is obtained, when the capital-output ratio $\Omega^*$is given:

$$\beta^* = \frac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{i\left((1-\alpha)+\Omega^*(1+n)\right)} \tag{7}$$

It apparently seems that the relationship between Ω^* and β^* brings about tautology. There is no tautology if the condition of $\Omega^* = \Omega_0$ is used to avoid tautology. Avoiding tautology will be fully justified when we wholly step into endogenous equilibrium, as below.

16.2.4 Justify two conditions of $\Omega^* = \Omega_0$ and $r^* = r_0$

$\Omega^* = \Omega_0$ shows that the capital-output ratio in the initial/current situation is equal to that at convergence realized in the transition path. Similarly, $r^* = r_0$ shows that the rate of return at the initial/current situation is equal to that at convergence realized in the transition path. The above two conditions were explained by the author's earlier notion in Feb 2004, but without fully connecting this notion numerically with the endogenous-equilibrium. One of the author's today's excuses is that the author paid attention to the difference between the author's convergence using the transitional path and the exogenous convergence in the literature. The other excuses of the author today are that the transitional path holds after equilibrium holds, regardless of whether the equilibrium is price-oriented or endogenous-oriented. Later, the author succeeded in measuring the endogenous-equilibrium at the real assets (see Chapter 7). This section summarizes the justification of the two conditions of $\Omega^* = \Omega_0$ and $r^* = r_0$, verbally comparing the price-equilibrium in the literature with the endogenous-equilibrium in the endogenous model, since the price-equilibrium does not wholly contradict with the endogenous-equilibrium. The next section numerically clarifies the endogenous-equilibrium.

From the policy-oriented viewpoint, the endogenous model sets a parallel march of the current actual situation and the current endogenous situation at convergence (i.e., at the balanced state in the literature). Both situations are consistent with the condition of $\Omega^* = \Omega_0$ at the transitional path of the endogenous system. The relationship between the current actual situation and the current endogenous situation differs due to the difference of capital stock lying between statistics-data and endogenous-data. Actual capital is estimated based on perpetual inventory method, helped by the market data, while endogenous capital is accurately measured 'by sector' in the endogenous system. The neutrality of the financial/market assets to the real assets was earlier proved in Chapter 2. The neutrality proves, for example, that ten year market debt yield equals the rate of return at convergence when the situation holds in endogenous equilibrium measured by the speed years by country and by sector.

The condition of $\Omega^* = \Omega_0$ is only justified with the condition of $r^* = r_0$ and with the assumption of a fixed relative share of capital (or labor) throughout the transitional path.[2] A fixed relative share of capital solely holds in endogenous equilibrium. Upon revealing the mechanics of the endogenous-equilibrium, the endogenous model integrates 'at convergence' with 'in equilibrium' consistently with the price-equilibrium in the literature. The endogenous situation at convergence corresponds with the balanced state in the literature. The difference of the two equilibriums is specified as follows: For the endogenous-equilibrium, 'the situation at convergence' is precisely measured in equilibrium (free from correlation analysis) by country and by sector. For the price-equilibrium, 'the balanced state' is estimated using time-series analysis and/or cross country analysis, based on panel actual-data, as shown by Barro and Sala-i-Martin (36-39, 80-92, 1995) and Ark, Bart, and Nicholas Crafts (1-26; 271-326, 1996).

As a result, the actual long-term market rate is compared with the current rate of return or the rate of return at convergence, in equilibrium. The above notion is traced back to von Neumann's turnpike theory, where turnpike is a short cut of the transitional path. Von Neumann (1-9, 1945-46) estimates the matrix for the price-equilibrium using actual statistics-data while the endogenous system measures endogenous-data in equilibrium. The capital-output ratio is by nature difficult to treat in the Cobb-Douglas production function. Nevertheless, Samuelson (1477-79, 1970) proves the constancy of the capital-output ratio in von Neumann turnpike theory and states that the constant capital-output ratio is the reciprocal of the von Neumann interest rate. Conditions of $\Omega^* = \Omega_0$ and $r^* = r_0$ are consistent with Samuelson's Law of Conservation of the Capital-Output Ratio using turnpike theory.

[2] $\alpha = \Omega^* \cdot r^*$ is a policy-oriented core in the endogenous model. In the transitional path, both the capital-output ratio Ω^* and the rate of return r^* each in equilibrium change under a fixed relative share of capital. The author presumes that the transitional path between the current/initial and at convergence is a sort of non-turnpike by time/year. Interesting to say, after convergence, Ω^* and r^* change inversely (from DRC to IRC and rarely from IRC to DRC). This fact is not clarified in the literature due to the use of the capital-labor ratio.

16.2.5 Diminishing returns to capital coefficient, $\boldsymbol{\delta_0}$, and the speed year coefficient, $\boldsymbol{\lambda^*}$

This section proves the relationship between the diminishing returns to capital coefficient, δ_0, and the speed year coefficient, λ^*. The endogenous-equilibrium is determined by the two speed year hyperbolas of $i = I/Y$ and n. Interestingly, n and δ_0 are involved in each vertical asymptote (see Appendix at the end of the *EES*.

First, δ_0 is obtained in the transitional path by setting a fact that the initial/ current δ_0 becomes equal to the relative share of capital at convergence, α. The discrete Cobb-Douglas production function holds at convergence with the minimum requirement of δ_0. A decisive idea is that the quantitative net investment coefficient, β^*, is connected with the capital-output ratio, *Omega*. Total productivity factor *A=TPF* as a stock in the C-D production function is, then, replaced by $B^* = (1-\beta^*)/\beta^*$ as a flow. And, define B^*_{TFP} as $B^{*1-\delta_0}$: $B^*_{TFP} \equiv B^{*1-\delta_0}$.

$\Omega = \frac{k^{1-\alpha}}{A}$ is an accounting identity in the C-D production function. This capital-output ratio is expressed as $\Omega = \frac{k^{1-\alpha}}{B_{TFP} \cdot k^{1-\delta_0}}$ using the above $B^*_{TFP} \equiv B^{*1-\delta_0}$. Define $TFP_B \equiv B_{TFP} \cdot k^{1-\delta_0}$. Then, $\Omega = \frac{k^{\delta_0-\alpha}}{B_{TFP}}$ holds. At convergence, $\alpha = \delta_0$ holds with $1 = k^{\delta_0-\alpha}$. Then, $\Omega^* = \frac{1}{B^*_{TFP}}$ or $\Omega^* = \frac{1}{B^{*1-\delta_0}}$ holds, resulting in $B^{*1-\delta_0} = \frac{1}{\Omega^*}$ or $1 = \Omega^* \cdot B^{*1-\delta_0}$. Therefore, for the DRC coefficient, δ_0, the following equation is proved.

$$\delta_0 = 1 - \frac{LN(1/\Omega^*)}{LN(B^*)}, \text{ or } \delta_0 = 1 + \frac{LN(\Omega^*)}{LN(B^*)} \tag{8}$$

$y = A \cdot k^\alpha$ is, however, not consistently connected with $B_{TFP} \equiv B^{1-\delta_0}$ in the transitional path, except for 'at convergence.' The use of B^*_{TFP} is only justified when the value of δ_0 is measured. The measurement of δ_0 connects Neo-classicists with Keynesians in the C-D production function. [3]

Second, the speed years for convergence in equilibrium are measured using the (endogenous) speed year coefficient, λ^*. The author assumes that the qualitative coefficient, β^*, and the DRC coefficient, δ_0, 'linearly' each change in

[3] The form of $y = B_{TFP} \cdot k$ is another expression of *Y=AK* model in Keynesian model (e.g., Thirlwall, A. P., 427-435, 2002). Thirlwall's model does not use the C-D production function, similarly to all the Keynesians, Neo- and New-. For discussions, see *JES* 11 (Feb, 1), 2008.

the discrete transitional path. As a result, the author does not use the exponential function, e^{-x}, differently of the literature. The convergence coefficient in the literature corresponds with the speed year coefficient. The convergence coefficient in the literature uses two exogenous ratios, instead of δ_0 and $g_A^* = i(1-\delta_0)$, [4] The speed years, *speed*, are the inverse number of the speed year coefficient, λ^*: $speed = 1/\lambda^*$. This equation is an accounting identity.

$$\lambda^* = (1-\alpha)n + (1-\delta_0)g_A^* \tag{9}$$

Then,

$$speed = \frac{1}{(1-\alpha)n+(1-\delta_0)i(1-\beta^*)} = \frac{1}{\lambda^*} \tag{10}$$

The author defines the speed year coefficient as a weighted average growth rate of the population and the endogenous rate of technological progress in equilibrium. This growth rate is per year so that the speed years are the inverse number of the speed year coefficient.[5]

The author happily finds a base common to the equation of the literature and the author's equation. In detail: suppose that 1) δ_0 equals *alpha* and 2) the endogenous rate of technological progress equals the exogenous rate of technological progress. Then, the convergence coefficient in the literature is expressed as $(1-\alpha)(n+g_{EXOGENOUS})$ under the price-equilibrium. In other words, the literature[6] has expressed a similar notion using panel data for an infinite period and exogenously in the price-equilibrium.

In the case of the endogenous model, the speed year coefficient is applied to before and after convergence. For example, if diminishing returns to capital (DRC) prevail before convergence, the DRC turns to increasing returns to capital (IRC) after convergence, and vice versa.

In recursive programming, $\beta(t)$ and $\delta_0(t)$ work each using $\beta(t) = \beta(0)(1+r_\beta)^t$ and, $\delta(t) = \delta(0)(1+r_\delta)^t$ by time/year. Here, r_β and r_{δ_0} are

[4] The author is grateful to Dr. Toshimi Fujimoto who has advised me in many respects. The author defines the speed year coefficient as the growth rate 'per year' so that the inverse number of λ^* is the speed years as an accounting identity.

[5] Using accounting identity, '1=turnover periods × turnover ratio' holds. The turnover periods correspond with the speed years and the turnover ratio corresponds with the above growth rate.

[6] Barro, Robert, J., and Xavier Sala-i-Martin. (1995). *Economic Growth,* 36-39, 80-92. New York and London: McGraw-Hill (1st ed.). And, Javier, Andres, Rafael, Doménech and César, Molinas, "Growth and convergence in OECD countries: a closer look," pp.347-387, In "*Quantitative Aspects of Post-War European Economic Growth*," edited by van Ark, Bart, and Nicholas Crafts, Cambridge: Cambridge University Press, 442p, 1996.

denoted each as the discount rate. Furthermore, using $LN(1+r_\beta) \fallingdotseq r_\beta$ abbreviated under Maclaurin's series, $\mathrm{LN}(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$, $r_\beta = \frac{LN(\beta^*) - LN(\beta_0)}{1/\lambda^*}$ and $r_{\delta_o} = \frac{LN(\delta_0) - LN(\alpha)}{1/\lambda^*}$ hold (see 158, *PRSCE*: 49 (Sep, 1), 2008).[7]

Note that the above equations with LN cannot be calculated when any of $\beta^*, \beta_0, \delta_0,$ or α are minus. A minus β^* implies a minus rate of technological progress, since $i > 0$ is a required condition in equilibrium. Disequilibrium occurs when the situation falls into $n_E < 0$ and $\beta^* < 0$. Then, recursive programming does not work.

Without finding the diminishing returns to capital coefficient, δ_0, the mechanics of endogenous equilibrium in the transitional path was not revealed. The transitional path, as von Neumann and Samuelson pursued, is a turnpike and the above devices are accepted for safety in the turnpike. In disequilibrium, the turnpike and the non-turnpike by time/year are shut down.[8]

Recursive programming has its own programming, similarly to KEWT data-sets. When a country is close to disequilibrium or meets an abnormal value, a special device is needed. For example, suppose $\beta^* = 1.05192$. In this case, the diminishing returns to capital (DRC) coefficient δ_0 is not calculated in recursive programming. The operator must be 'ABS' (absolute) in the corresponding Excel equation (see, Philippines 2010).

16.3 Reply to Harcourt, G. C. (1972): Synthesizing Keynesian and Neo-Classical Models

16.3.1 From unsolved to solved

In this section, the author selects four typical diagrams/figures in Harcourt (ibid., 70, 156, 223, 247) and cites four diagrams each as BOXES 16-1, 16-2, 16-3, and 16-4. These four figures show several implicit characteristics common to economics in the literature, in addition to two definite assumptions of perfect

[7] In the continuous case, for example, the same $r_\beta = (LN(\beta^*) - LN(\beta_0))/1/\lambda^*$ holds; processing from $\beta^* = \beta_0 e^{r_\beta(1/\lambda^*)}$ to $LN(\beta^*) = LN(\beta_0) + r_\beta(1/\lambda^*)$.

[8] Equations are formed without using LN: $(1+r_\beta)^{1/\lambda^*} \fallingdotseq 1 + (1/\lambda^*)r_\beta$ holds using another Maclaurin's series, $(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \cdots$, $\frac{\beta^*}{\beta_0} \fallingdotseq 1 + (1/\lambda^*)r_\beta$. Thus, $r_\beta = \frac{\beta^* - \beta_0}{\beta_0(1/\lambda^*)}$ holds and similarly, $r_{\delta_0} = \frac{\alpha - \delta_0}{\delta_0(1/\lambda^*)}$ holds.

competition and the price-equilibrium. Several implicit characteristics are: i) Heterogeneous capital, 2-3; ii)Micro-oriented, 9; iii) Diminishing returns and, increasing returns or learning by doing, 79 and 249; iv) Maximum per capita consumption, 240-243; v) The relative share of capital and the changes between the rate of return and the wage rate, 158-159; and vi) Double-switching and capital-reversing, 8. These implicit characteristics are interrelated and also explicitly connected with common assumptions.

Let the author briefly interpret these implicit characteristics from the viewpoint of the endogenous system and then, next sub-section, comment the above BOXES 16-1, 16-2, 16-3, and 16-4.

Heterogeneous capital is correct. Similarly, heterogeneous population or labor is correct. Quantity and quality are united at capital and labor by country. For capital, flow of capital is net investment after capital consumption. Capital flow is measured qualitatively. Then, the rate of technological progress is measured first of all. Labor flow is qualitative and measured by the rate of change in population. When the speed years fall in a moderate range of the endogenous-equilibrium, the growth rate of population equals the rate of change in population. This is called no unemployment or such that the rate of unemployment is zero. Thus, full employment is guaranteed in the endogenous system.

Micro-oriented or the use of an aggregated production function (Harcourt, ibid. 50) is a compromised expression. Micro-oriented prevails in any aspect in economics. An original point is Koopmans's diagram (Harcourt, ibid. 241n) for per capita consumption. Pasinetti (Harcourt, ibid. 9) forms an equation of $r = g/s_C$, based on corporate saving and neglecting the government sector. The endogenous system reduces this equation to $g = s_{SP/Y} = \alpha \cdot s_C$. It implies that the ratio of corporate undistributed profits to output equals the growth rate. Utility is individual-oriented and, everywhere from micro to macro is natural. In the endogenous system, macro-oriented and denies micro-oriented; reversely, from macro-oriented to micro-oriented. Otherwise, three equality of income = expenditures=output does not hold in the endogenous system.

For diminishing returns and increasing returns, the endogenous system clarifies dynamic movements at the ratio of net investment to output and the rate of return by using hyperbolic equation and its graph. Increasing returns diagrammed by Harcourt (ibid. 249) belongs to the rate of technological progress in the endogenous system; for example, learning by doing is a strategy and support the qualitative net investment coefficient. The rate of return always expresses diminishing returns to capital (DRC), before the convergence point of time in the transitional path. The endogenous system is unique in that it expresses DRC despite constant returns to scale (CRS). The literature aims at maximum per capita consumption as diagrammed by Harcourt (ibid. 79, 297-307). The endogenous system aims at maximum rate of return with minimum ratio of net investment to output. A goal of maximum per capita consumption is consistent with a goal of minimum ratio of net investment to output. Two goals show the same differently.

For the relative share of capital, Harcourt (ibid. 158-159) indicates the inconsistency between the relative share and MPK. It is natural. In the endogenous system, the relative share of capital is fixed in the transitional path as shown by recursive programming. And, average equals marginal each at capital and labor. Therefore, perfect competition assumption must be deleted, as indicated in the previous sub-section.

Finally, as a result, double-switching and capital-reversing occur at some countries and in some years. These results are shown in recursive programming. These results are explained in the next sub-section, comparing Harcourt's diagram with corresponding figure by country (for 36 countries, 2010, see Figures at the end of this chapter).

16.3.2 Comment to Harcourt's four diagrams

This sub-section takes four diagrams among hundreds of serious diagrams. The author does not deny the market principle under the price-equilibrium. Also, the following comments are not for Harcourt (1972) but for Keynesian and Neo-classical both schools. Or, essentially, comments are against the current economics and macroeconomics. The author, however, is not against Keynesian and Ne-classical researchers. They have executed every effort. Time has come so as to accept 'purely endogenous system.' In fact, the author has widely and historically absorbed the accumulated performances in the literature hitherto and, without these invaluable property and fortune, the endogenous system would not have been born.

The author takes four diagrams up that express Harcourt's scrupulous accumulations in his life, each by each as follows:

1) Harcourt (ibid. 70), see **BOX 16-1**: A reason why do MPL≠APL and MPK≠APK hold in Fig. 2.5a (Solow's embodied, malleable model. productivity view) in Harcourt (ibid., 70) is that the relationship between marginal productivity and average productivity follows Solow's cost view, as shown in Fig. 2.5b. 'Productivity view' and 'cost view' each reversely show the same relationship between marginal and average. Marginal parabolic curve is sharper than average parabolic curve. At the bottom point of average parabolic curve, the marginal parabolic curve crosses. Cost view diagram is shown more commonly than productivity view in textbooks, macro and micro.

Under both diagrams, it is impossible to have MPL=APL and MPK=APK realized. Or, at the macro level, it is unrealistic to assume MPL=APL and MPK=APK.

BOX 16-1 Harcourt's (70, 1972) diagram to Solow's (1960) embodied, malleable model, productivity view

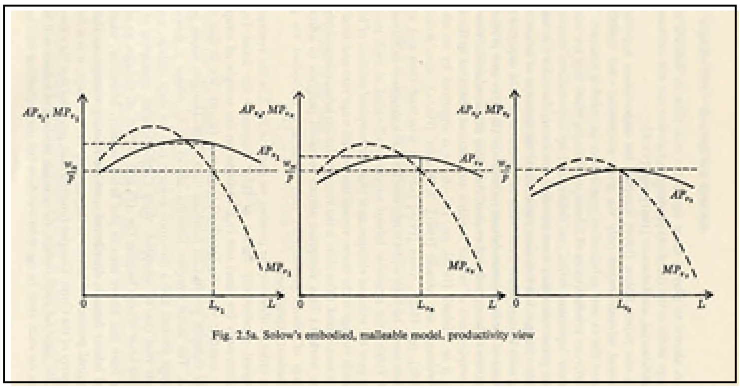

Fig. 2.5a. Solow's embodied, malleable model, productivity view

2) Harcourt (ibid. 156), see **BOX 16-2**: Fig. 4.14b shows Joan Robinson's pseudo-production function with double-switching. It is told that double-switching is one of key differences between Keynesian and Neo-classical researchers. Researchers, nevertheless, have not shown empirical proofs. To the author's understanding, double-switching is interpreted as a common phenomenon between two growth rates. The endogenous system presents the empirical proofs as shown in BOX 16-2.

BOX 16-2 Harcourt's (156, 1972) double-switching vs. Author's $g_y(t)$ and $g_k(t)$, using Germany, 1990-2010, speed 86 years

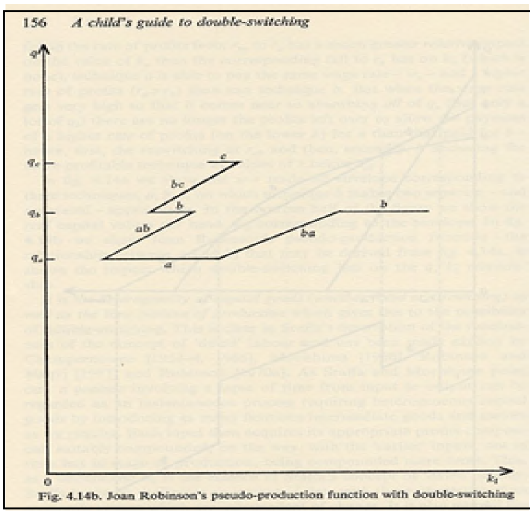

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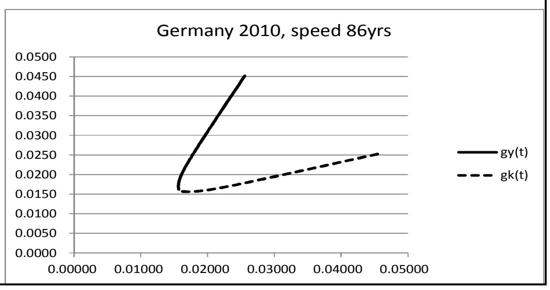
Fig. 4.14b. Joan Robinson's pseudo-production function with double-switching

$g_y(t)$, the Y axis, and $g_k(t)$, the X axis:
shown by a line.
$g_k(t)$ the Y axis, to $g_y(t)$, the X axis:
shown by a plotted line.



Chapter 16

The LHS of BOX 16-2 is Harcourt's imaginary diagram while the RHS is an endogenous example differently from double-switching at KEWT 6.12 data-sets. On the Y axis, we are able to take $g_y(t)$ or $g_k(t)$, where each sub-figure reversely shows the same relationship between $g_y(t)$ or $g_k(t)$.

3) Harcourt (ibid. 223): Both Keynesian and Neo-classical researchers have used an inverse of the capital-output ratio as shown in **BOX 16-3**. On the Y axis, Y/K is used while on the X axis the rate of return, $r = \Pi/K$, is used. The author is not against the use of $1/\Omega = Y/K$. Yet, the author thinks that the product of the Y axis and the X axis should be meaningful. For example, $\alpha = \Omega \cdot r$ is a meaningful product c=a × b, since without $\alpha = \Omega \cdot r$, the relationship between DRC and IRC is not clarified numerically, as discussed below in iv).

BOX 16-3 Harcourt's (223, 1972) diagram to Meade (162-164, 1966) and Harcourt's (247, 1972) diagram to choice of technique: selected by the author

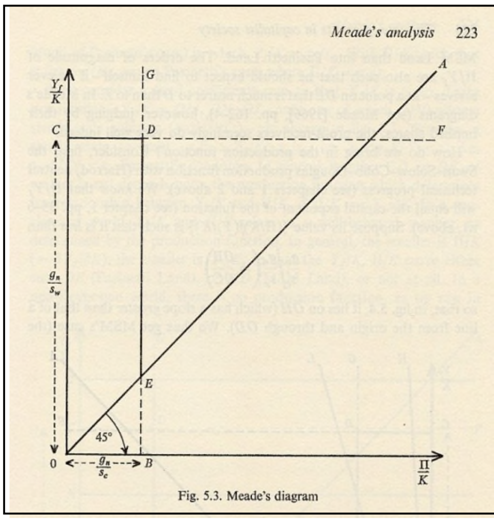


Fig. 5.3. Meade's diagram

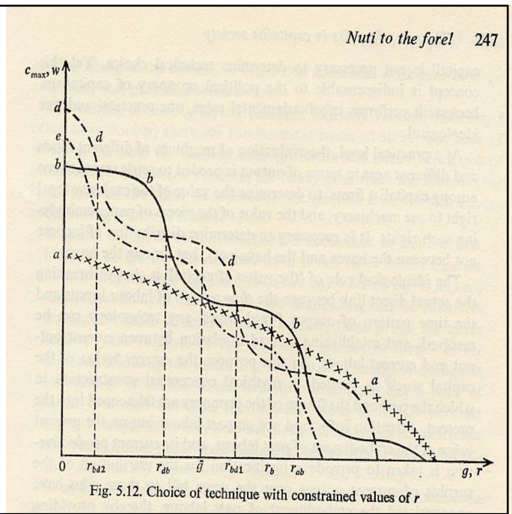


Fig. 5.12. Choice of technique with constrained values of r

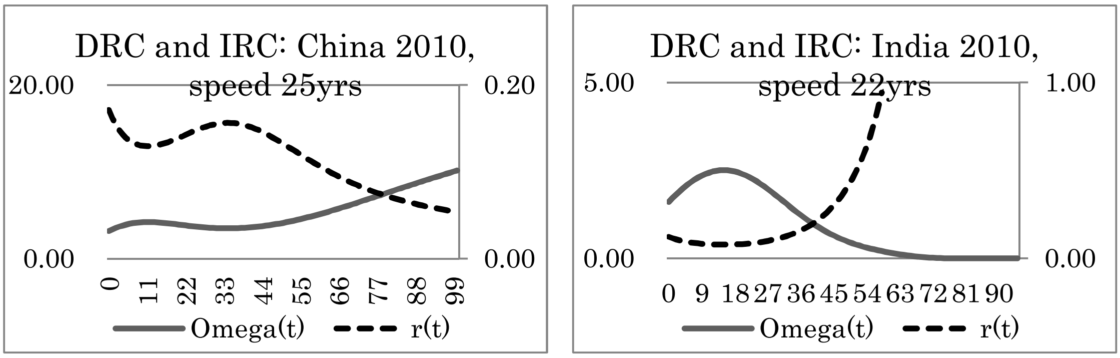
For the diagram use of product — Double-switching and capital-reversing

4) Harcourt (ibid. 247), see **BOX 16-4**: Double-switching and capital-reversing are differently expressed by the relationship between DRC, IRC, and CRC, reinforced by the above meaningful product, $\alpha = \Omega \cdot r$. Under a fixed capital share α or labor share $1 - \alpha$, the rate of return is expressed by either DRC or IRC.

Harcourt (ibid. 8), defines double-switching such a possibility that the same technique may be the most profitable of all possible techniques at two or more separated values of the rate of profits even though other techniques have been the most profitable at rates of profits in between. Also, capital-reversing is defined as the possibility of a positive relationship between the value of capital and the rate of profits. These notions are against an empirical fact that along with the increase in capital stock, the rate of return decreases.

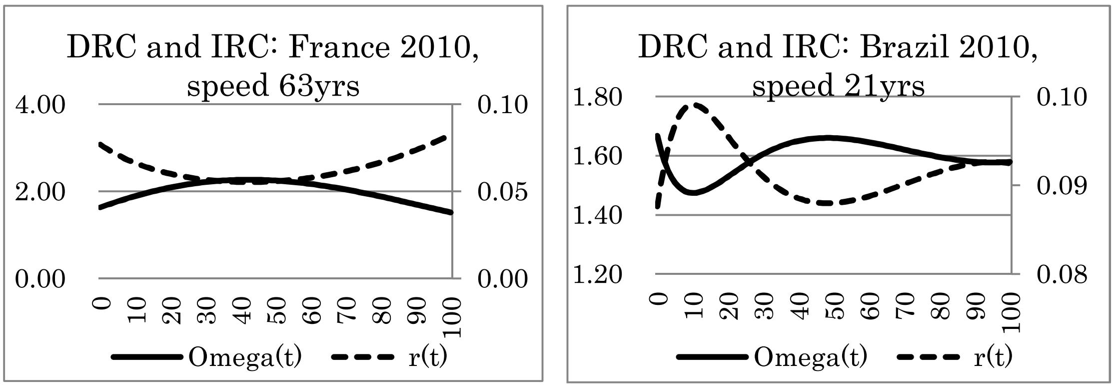
The endogenous system or KEWT data-sets for 36 countries clarify the possibility of double-switching and capital-reversing (see **Figures D4**, D**5**, and D**6** at the end of this chapter). If the endogenous-equilibrium is unstable due to huge deficit, double-switching and capital-reversing seldom occur, as mostly observed at developed countries. Do developing countries then have more possibility of double-switching and capital-reversing than developed countries? Compare China and India, 2010 at **BOX 16-4**. India is unstable partly due to deficit and as a result, India seldom has the possibility of capital-reversing in the transitional path.

BOX 16-4 DRC and IRC: China versus India



Under these circumstances, the endogenous system does not concretely distinguish one technique with another technique. The rate of technological progress is, rather vaguely and wholly at the macro level, measured by using qualitative net investment coefficient, β^*. In this sense, double-switching and capital-reversing are the same or, double-switching is absorbed into capital-reversing. Capital-reversing indicates that an economy is robust and realizes maximum rate of return, repeatedly as shown by e.g., Brazil (see **BOX 16-5**).

BOX 16-5 DRC and IRC: France versus Brazil



In short, capital reversing indicates that the endogenous-equilibrium recovers in a short run. In a sense, integrated policies are well controlled with accumulation of experiences in the past. Leaning by dong is a strategy to improve seven endogenous parameters at the macro-level. Leaning by dong implicitly works for policy combinations and its integration.

16.4 Results of Recursive Programming

This section examines and clarifies the results of recursive programming and focuses *two points*. *The first point* is the relationship between the rate of technological progress as flow, $g_{A(FLOW)}(t)$, and the growth rate of total factor productivity (*TFP*) as stock, $g_{TFP(STOCK)}(t)$, with the growth rate of $k = K/L$, $g_k(t)$ in the transitional path. *The second point* is the relationship between diminishing returns to capital (DRC), the constant returns to capital (CRC), and the increasing returns to capital (IRC) in the transitional path. Both points are interrelated each other. The author proves two points in recursive programming.

For the above proofs, the author uses KEWT 6.12, 1990-2010, at the total economy level. 36 countries are selected among 81 countries. 36 countries are divided into three groups; i) developed countries versus BRICs, ii) European countries excluding Euro currency countries, and iii) Asian countries. The first group is the same as the author used for hyperbola graphs in Chapters 14 and 15.

i) The US, Japan, Australia, France, Germany, and the UK.
China, India, Brazil, Mexico, Russia, and South Africa.
ii) Denmark, Finland, Netherlands, Norway, Sweden, and Canada.
Greece, Iceland, Ireland, Italy, Portugal, and Spain.
iii) Indonesia, Korea, Malaysia, Philippines, Singapore, and Thailand.
Bangladesh, Pakistan, Saudi Arabia, Sri Lanka, Czech Rep, Poland.

Figures T1, T2, and **T3** each show 12 countries for the rate of technological progress. For *the first point*, the author selects $g_{A(FLOW)}(t)$, $g_{TFP(STOCK)}(t)$, and $g_k(t)$. This is because at convergence time of the transitional path, $t^* = t$, $g^*_{A(FLOW)} = g_{A(FLOW)}(t)$, is equal to $g^*_{A(STOCK)} = g_{A(STOCK)}(t)$, by denoting $A(STOCK) = TFP$. In the endogenous equilibrium, $g^*_{A(FLOW)} = g^*_{A(STOCK)}$, without exception by country (among 81 countries). This fact is one of proper attributes of the endogenous system. Then, why did the author select the growth rate of $g_k(t)$? There are two primary growth rates of output and per capita output, $g_Y(t)$ and $g_y(t)$,which are derived from the rate of technological progress, $g_{A(FLOW)}(t) =$

$i(1-\beta^{*}(t))$, as shown repeatedly in the *EES*. $g_y(t) = g_A(t)/(1-\alpha(t))$ is common to the equation of the literature and the equation at the endogenous system, where only difference is whether each equation is exogenous or purely endogenous. Then, why didn't the author include the rate of return in the above relationship? This is because the rate of return is more properly related to the second point.

For the second point, the rate of return in equilibrium shows either diminishing returns to capital (DRC) or the increasing returns to capital (IRC). And, at convergence of the transitional path, the constant returns to capital (CRC) are shown. Only if the conditions of the rate of return by time, $r(t) = \Pi(t)/K(t)$, is close to the CRC by time, the DRC or the IRC becomes close to the CRC. When $r(t) = \Pi(t)/K(t)$ by time shows a close-to-parabolic convex curve upwards to the right, the situation indicates the IRC before the convergence and, the DRC after the convergence. Adversely, when $r(t) = \Pi(t)/K(t)$ by time shows a close-to-parabolic concave curve downwards to the right, the situation indicates the DRC before the convergence and, the IRC after the convergence.

For the relationship to connect the rate of return with the growth rate of output, the endogenous Phelps coefficient, $x = (\alpha/i \cdot \beta^{*})$, is used. The $x = (\alpha/i \cdot \beta^{*})$ influences each of $g_{A(FLOW)}(t)$, $g_{TFP(STOCK)}(t)$, and $g_k(t)$ and reflects the results of the DRC and the IRC at the rate of return in the transitional path. As shown by Figures D4, D5, and D6, most of 36 countries each indicate the DRC before the convergence and, the IRC after the convergence.

Watch each of sixteen Figures by country. Each country has its own results and reflects policy-oriented causes and effects. It implies that each country maintains its national taste and culture in cooperation with the global standard. When policy-oriented results are not well controlled in the endogenous system in the short run, the situation falls into the close-to-disequilibrium or disequilibrium by year and accordingly, in the transitional path. Each of $g_{A(FLOW)}(t)$, $g_{TFP(STOCK)}(t)$, and $g_k(t)$ shows different curve by country. The closer to disequilibrium in the short run, the more abnormal the situation is. This fact is directly shown by the speed years inserted by country title. If the speed years are more than 100 yrs. or less than five yrs. or minus yrs, as shown in the case of Russia, each graph becomes typically abnormal. Also, we realize much differences between developed and developing countries. Robust sustainable and weak unstable countries similarly show low net investment to output, but we concretely confirm significant differences between robust and weak by each curve.

16.5 Conclusions

This chapter is the last one and basic data are wholly used. These data are KEWT 6.12 and commonly used to other chapters. This chapter focuses on recursive programming (RP) with fundamental RP graphs, as shown at the end (See For readers' convenience: contents of Tables and Figures on the next page). For hyperbolic graphs, Chapter 14 used $i = I/Y$ for business cycle, and Chapter 15 used the rate of change in population for growth and stop-macro inequality. For hyperbolic graphs, earlier step by step, Chapter 5 used the speed years and $r^*(i)$; Chapter 7, the speed years for structural analysis; Chapter 8, hyperbola of $\Omega(\beta^*)$ for policy-potential to widen various real-asset policies; and Chapter 10, the essence of endogenous model and system and its geometrical philosophy, theoretically.

This chapter, by using recursive programming, proves that the rate of technological progress equals the growth rate of total factor productivity, or flow technology equal stock technology. This chapter also proves the relationship between the diminishing returns to capital, the constant returns to capital, and the increasing returns to capital, each in the transitional path. These results and facts were shown using sixteen Figures.

All of these facts or proofs were not realized in the literature. This is because statistic actual researches have not been executed wholly as a system but partially, widely, and independently, and with various assumptions. The endogenous system contrarily is based on the discrete Cobb-Douglas production function and starts with seven endogenous parameters that control all the parameters and variables as a whole and consistently by year and over years. Endogenous equations, related hyperbolas, and related recursive programming graphs are all consistently connected with each other. There is no assumption in these results. The author is grateful to the efforts of researchers, in particular, Meade and Stone for the conceptions and frameworks they established, and for rigid arrangements of nine basic assumptions.

Economics, apart from econometrics, eventually needs a system, where all the values and ratios are consistent over years. Typically Chapter 6 and Chapter 16 prove the essence of a system. As a result, surprisingly scientific discoveries accumulated in the economic literature are all and ever harmonized.

The following Appendix is final explanations. Mathematical proof is most ridged and strict among sciences, natural and social. The author understands mathematical spirit and the *EES* was thankfully written so as to satisfy mathematical proofs. Wait: Any partial holds in mathematics. Mathematics needs no empirical proof while economics needs empirical proofs. When theory and practice are one, proofs hold, as wholly shown in this chapter.

Acknowledgements: The author got Permission Invoice P12845, 30th Oct 2012, from Svetlana Shadrina, Assistant Permissions Controller; Cambridge University Press, with the postal address of Prof Emeritus G. C. Harcourt. Two conditions were executed happily. The author is grateful to Dr. Harcourt, one of teachers, and Cambridge University Press. On the 22nd to 25th of March, 2012, I attended Royal Economic Society Conference, Cambridge, staying at Robinson College. I recollected past days with Dr. Harcourt and Marshall Library, autumn of 1996.

For readers' convenience: contents of Tables and Figures hereunder

Table 1 Resource data by country 2012: for 36 countries

i)	rho/r	r_{DEBT}	I	C	Y	W	L	K	k=K/L
1. the US	1.1551	0.0279	723	13710	14116	11869	317.51	26926	85
2. Japan	1.0346	0.0084	16766	387427	418764	374469	127.25	3760633	29553
3. Australi	0.9811	0.0338	272	1060	1310	1080	23	3581	155
4. France	1.0345	0.0254	142	1670	1805	1614	64	3152	49
5. Germany	0.9620	0.0150	116	2036	2355	2116	83	4539	55
6. the UK	1.2522	0.0191	39	1354	1356	1081	62.78	1321	21
7. China	0.9666	0.0600	18828	26183	46465	27088	1377	129384	94
8. India	0.9912	0.1060	24645	59974	79702	60505	1241	186055	150
9. Brazil	1.2180	0.3664	0.78	37	37	30	199	55	0.28
10. Mexic	0.9592	0.0560	2452	11841	13953	12344	121	25102	208
11. Russia	1.0456	0.0550	6941	39064	50782	37361	143.17	44482	311
12. S.Afric	1.0258	0.0790	392	2614	2840	2549	52	2718	52
ii)	rho/r	r_{DEBT}	I	C	Y	W	L	K	k=K/L
1. Denmark	0.9682	0.0311	59	1419	1620	1466	5.60	3095	553
2. Finland	1.0101	0.0188	17	158	173	156	5	324	60
3. Netherla	0.9698	0.0193	38	439	534	452	17	1107	66
4. Norway	0.9521	0.0157	2458	1794	2587	1885	5	9099	1823
5. Sweden	0.9597	0.0159	196	2674	3170	2787	10	4929	518
6. Canada	0.9609	0.0321	328	1383	1643	1440	34.84	4594	132
7. Greece	1.3807	0.2250	4	177	172	128	11	370	46
8. Iceland	0.9797	0.0228	321	1352	1520	1380	0	3995	12105
9. Ireland	1.1684	0.0960	30	107	147	91	5	634	138
10. Italy	1.1185	0.0451	51	1271	1394	1136	61	2496	41
11. Portuga	1.1800	0.1055	4	144	147	122	11	381	36
12. Spain	1.1220	0.0585	23	829	935	739	47	1606	34
iii)	rho/r	r_{DEBT}	I	C	Y	W	L	K	k=K/L
1. Indones	1.0538	0.1180	2556	5229	7418	4962	246.86	13008	53
2. Korea	1.0103	0.0343	191	882	1120	873	49	3382	69
3. Malaysi	0.9523	0.0325	370	587	844	616	29	2329	80
4. Philippi	1.0653	0.0568	(430)	8933	9512	8386	97	2663	28
5. Singapo	0.9195	0.0146	62	169	304	184	5	821	155
6. Thailan	0.9711	0.0353	2724	7838	10238	8071	66.79	36968	553
7. Banglad	0.9766	0.1300	993	7376	8324	7553	155	8162	53
8. Pakistan	1.5613	0.1173	(504)	19753	18588	12652	179	6678	37
9. Saudi Ar	0.9511	0.0000	209	861	1258	906	26	2542	99
10. Sri La	1.0295	0.1328	1713	6296	6824	6115	21.10	10359	491
11. Czech	0.9771	0.0389	678	2664	3276	2727	10	10874	1045
12. Poland	0.9823	0.0578	149	1134	1272	1154	38	1714	45

Data source: KEWT 6.12 of 81 countries by sector, 1990-2012, whose ten original data for the real assets come from *International Financial Statistics Yearbook*, IMF.

Table 2 Calculated parameter data by country 2012: for 36 countries

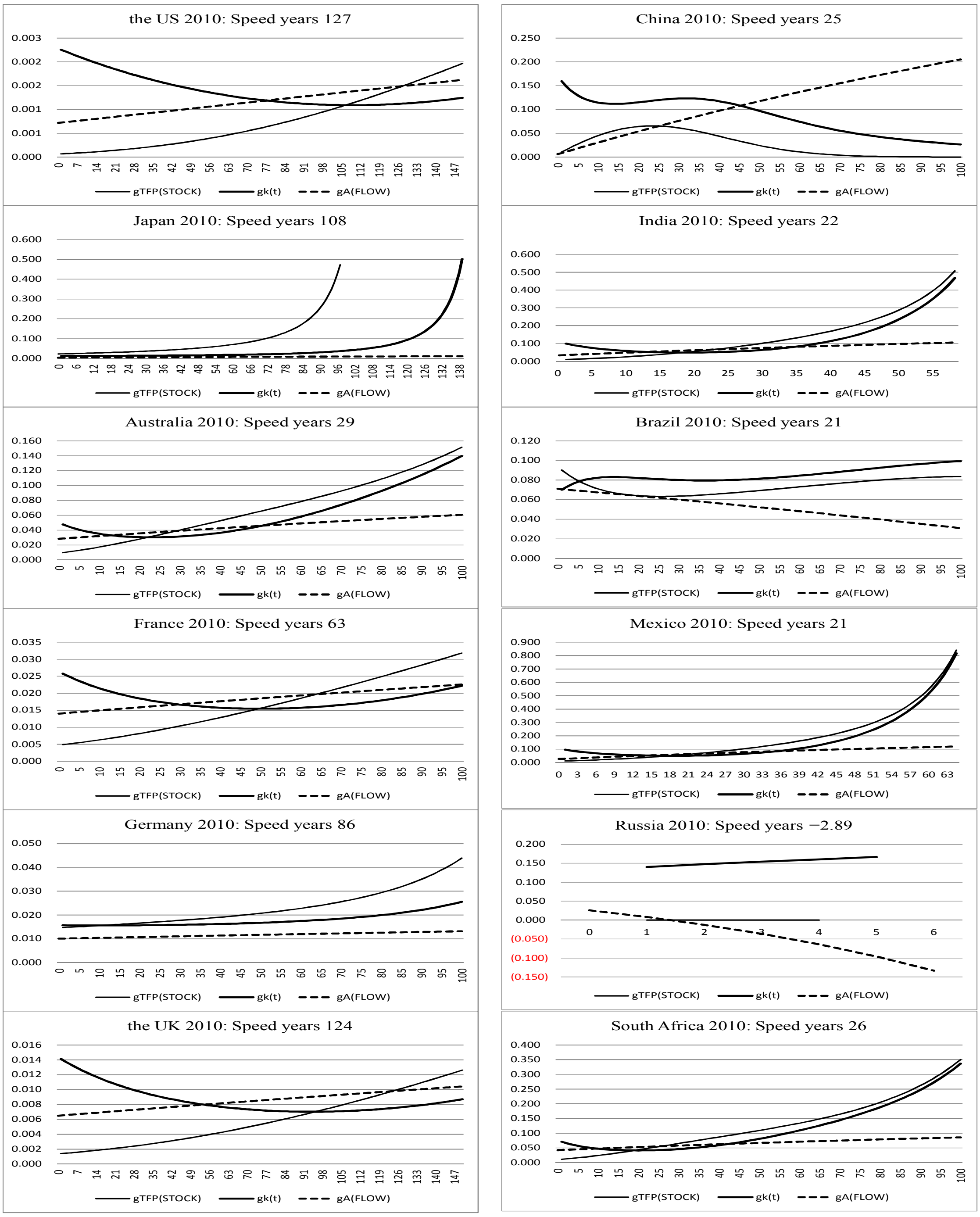
i)	i=I/Y	α	n	k	Ω	β*	B*	δ_0
1. the US	0.0512	0.1592	0.01412	84.80	1.9074	0.8563	0.1678	0.6382
2. Japan	0.0400	0.1058	(0.00079)	29553.11	8.9803	0.8934	0.1193	0.1799
3. Australi	0.2079	0.1751	0.01363	155.34	2.7338	0.8117	0.2320	0.3117
4. France	0.0785	0.1057	0.01123	49.29	1.7464	0.7478	0.3373	0.4870
5. Germany	0.0494	0.1014	(0.00265)	54.82	1.9276	0.6485	0.5420	(0.0714)
6. the UK	0.0290	0.2028	0.01144	21.05	0.9741	0.7245	0.3802	1.0271
7. China	0.4052	0.4170	0.00636	93.95	2.7845	0.8353	0.1972	0.3693
8. India	0.3092	0.2409	0.00000	149.86	2.3344	0.7546	0.3252	0.2453
9. Brazil	0.0209	0.1877	0.0267	0.2766	1.4735	1.3081	(0.2355)	0.7319
10. Mexic	0.1757	0.1153	0.01248	207.71	1.7990	0.7149	0.3989	0.3611
11. Russia	0.1367	0.2643	(0.00188)	310.69	0.8759	0.5375	0.8604	1.8809
12. S.Afric	0.1379	0.1025	0.0295	51.8783	0.9571	0.6208	0.6107	1.0889
ii)	i=I/Y	α	n	k	Ω	β*	B*	δ_0
1. Denmark	0.0365	0.0953	0.00358	552.60	1.9103	0.7396	0.3521	0.3799
2. Finland	0.0959	0.0974	0.00371	59.83	1.8703	0.6988	0.4310	0.2560
3. Netherla	0.0713	0.1529	0.00240	66.25	2.0741	0.7307	0.3685	0.2692
4. Norway	0.9501	0.2714	0.01012	1823.45	3.5171	0.8362	0.1959	0.2286
5. Sweden	0.0618	0.1209	0.01386	518.27	1.5548	0.7667	0.3042	0.6291
6. Canada	0.1995	0.1236	0.01015	131.86	2.7953	0.7968	0.2550	0.2478
7. Greece	0.0243	0.2555	0.00117	45.73	2.9506	0.8273	0.2088	0.3093
8. Iceland	0.2109	0.0924	0.01227	12105.23	2.6276	0.7845	0.2747	0.2522
9. Ireland	0.2040	0.3800	0.01104	138.35	4.3035	0.9043	0.1058	0.3502
10. Italy	0.0383	0.1423	0.00611	41.00	1.8843	0.7823	0.2783	0.5047
11. Portuga	0.0280	0.1706	0.00046	35.91	2.5892	0.7678	0.3024	0.2047
12. Spain	0.0268	0.1439	0.01234	34.34	1.8603	0.9556	0.0465	0.7977
iii)	i=I/Y	α	n	k	Ω	β*	B*	δ_0
1. Indones	0.3446	0.3311	0.01255	52.69	1.7536	0.7438	0.3444	0.4731
2. Korea	0.1708	0.2201	0.00554	69.03	3.0206	0.8157	0.2259	0.2569
3. Malaysi	0.4381	0.2696	0.01669	79.66	2.7604	0.8152	0.2267	0.3158
4. Philippi	(0.0452)	0.1184	0.01746	27.53	0.2799	0.1625	5.1534	0.2235
5. Singapo	0.2055	0.3959	0.02119	154.92	2.7001	0.8703	0.1490	0.4783
6. Thailan	0.2661	0.2117	0.00315	553.50	3.6110	0.8289	0.2064	0.1863
7. Banglad	0.1193	0.0927	0.01204	52.76	0.9804	0.5696	0.7556	1.0706
8. Pakistan	(0.0271)	0.3194	0.0170	37.2759	0.3593	0.2030	3.9256	0.2514
9. Saudi Ar	0.1662	0.2799	0.0000	98.8337	0.0000	0.7373	0.3563	0.3181
10. Sri Lai	0.2510	0.1039	0.00812	490.93	1.5179	0.6488	0.5413	0.3201
11. Czech I	0.2069	0.1677	0.00000	1044.61	3.3194	0.7995	0.2507	0.1327
12. Poland	0.1171	0.0924	0.00000	45.06	1.3482	0.5976	0.6732	0.2450

Data source: KEWT 6.12 of 81 countries by sector, 1990-2012, whose ten original data for the real assets come from *International Financial Statistics Yearbook*, IMF.

Table 3 Calculated variable data by country 2012: for 36 countries

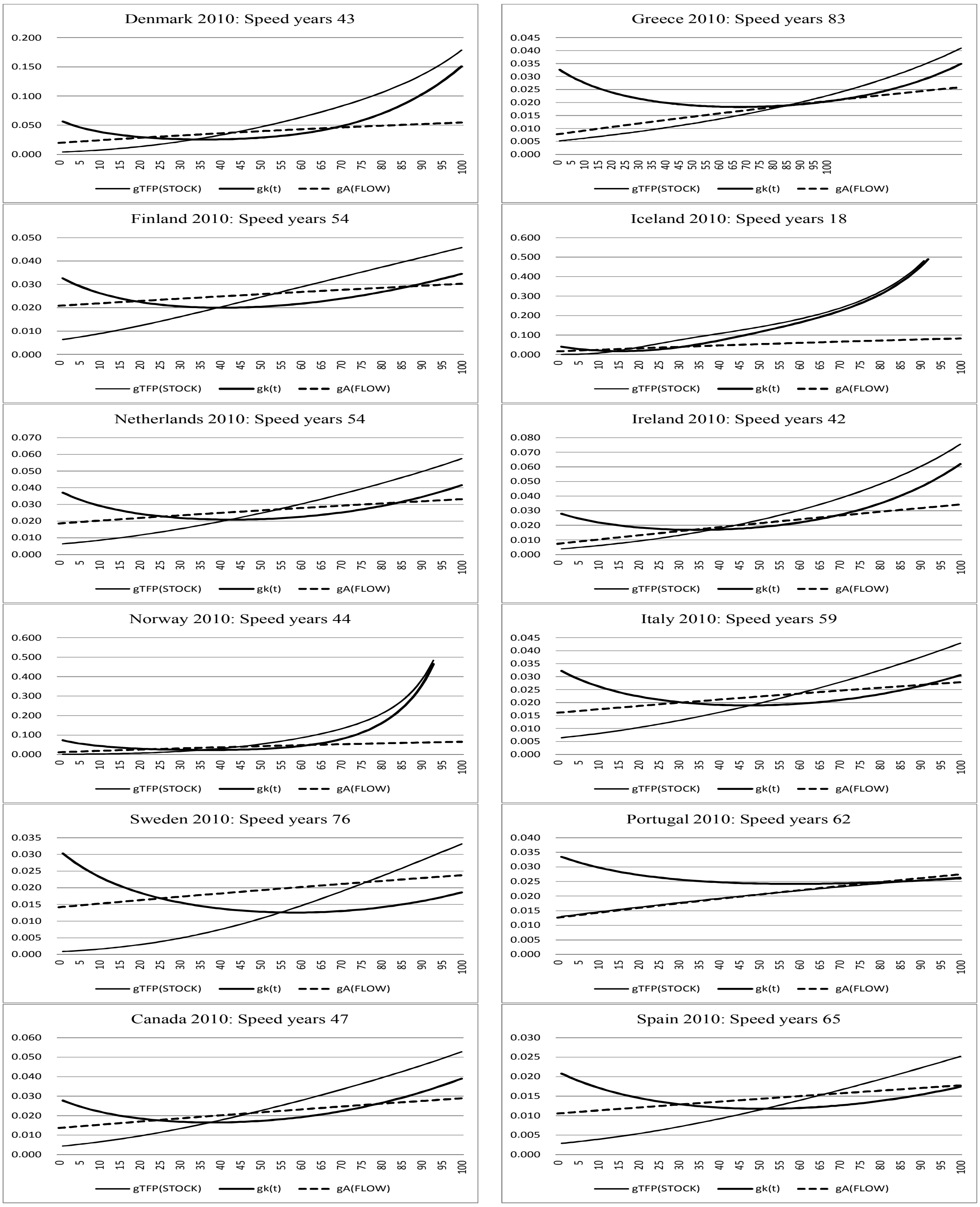
i)	g_A^*	r^*	$x=\alpha/(i\cdot\beta^*)$	g_Y^*	g_y^*	$r^*-g_Y^*$	v^*	speed coeff	speed yrs
1. the US	0.0074	0.0834	3.629	0.0230	0.0088	0.0605	1.380	0.0145	68.805
2. Japan	0.0043	0.0118	2.957	0.0040	0.0048	0.0078	1.511	0.0028	357.470
3. Australi	0.0392	0.0640	1.037	0.0617	0.0475	0.0023	27.848	0.0382	26.183
4. France	0.0198	0.0605	1.800	0.0336	0.0222	0.0269	2.249	0.0202	49.495
5. Germany	0.0174	0.0526	3.166	0.0166	0.0193	0.0360	1.462	0.0162	61.671
6. the UK	0.0080	0.2081	9.646	0.0216	0.0100	0.1866	1.116	0.0089	112.322
7. China	0.0667	0.1498	1.232	0.1216	0.1145	0.0282	5.309	0.0458	21.837
8. India	0.0759	0.1032	1.032	0.1000	0.1000	0.0032	32.015	0.0573	17.462
9. Brazil	0.0072	0.1274	6.859	0.0186	(0.0079)	0.1088	1.171	0.0200	50.060
10. Mexic	0.0501	0.0641	0.918	0.0698	0.0566	(0.0057)	(11.184)	0.0431	23.228
11. Russia	0.0632	0.3017	3.597	0.0839	0.0859	0.2178	1.385	0.0571	17.524
12. S.Afric	0.0523	0.1071	1.197	0.0895	0.0583	0.0176	6.069	0.0218	45.859
ii)	g_A^*	r^*	$x=\alpha/(i\cdot\beta^*)$	g_Y^*	g_y^*	$r^*-g_Y^*$	v^*	speed coeff	speed yrs
1. Denmark	0.0095	0.0499	3.535	0.0141	0.0105	0.0358	1.395	0.0091	109.516
2. Finland	0.0289	0.0521	1.453	0.0358	0.0320	0.0162	3.208	0.0248	40.258
3. Netherla	0.0192	0.0737	2.936	0.0251	0.0227	0.0486	1.517	0.0161	62.284
4. Norway	0.1556	0.0772	0.342	0.2259	0.2136	(0.1487)	(0.519)	0.1274	7.847
5. Sweden	0.0144	0.0778	2.551	0.0305	0.0164	0.0473	1.645	0.0175	57.033
6. Canada	0.0222	0.0442	0.778	0.0569	0.0462	(0.0126)	(3.498)	0.0394	25.396
7. Greece	0.0042	0.0866	12.700	0.0068	0.0056	0.0798	1.085	0.0038	265.066
8. Iceland	0.0325	0.0352	0.559	0.0630	0.0501	(0.0278)	(1.265)	0.0451	22.157
9. Ireland	0.0195	0.0883	2.060	0.0429	0.0315	0.0454	1.944	0.0195	51.214
10. Italy	0.0083	0.0755	4.755	0.0159	0.0097	0.0597	1.266	0.0094	106.719
11. Portuga	0.0065	0.0659	7.939	0.0083	0.0078	0.0576	1.144	0.0056	180.157
12. Spain	0.0012	0.0774	5.627	0.0137	0.0014	0.0636	1.216	0.0360	27.803
iii)	g_A^*	r^*	$x=\alpha/(i\cdot\beta^*)$	g_Y^*	g_y^*	$r^*-g_Y^*$	v^*	speed coeff	speed yrs
1. Indones	0.0883	0.1888	1.291	0.1462	0.1320	0.0426	4.431	0.0549	18.211
2. Korea	0.0315	0.0729	1.580	0.0461	0.0404	0.0268	2.724	0.0277	36.086
3. Malaysi	0.0810	0.0977	0.755	0.1294	0.1108	(0.0317)	(3.078)	0.0676	14.797
4. Philippi	(0.0379)	0.4228	(16.099)	(0.0263)	(0.0430)	0.4491	0.942	0.0140	71.303
5. Singapo	0.0266	0.1466	2.214	0.0662	0.0441	0.0804	1.824	0.0140	71.303
6. Thailan	0.0455	0.0586	0.960	0.0611	0.0577	(0.0025)	(23.789)	0.0395	25.299
7. Banglad	0.0514	0.0946	1.364	0.0693	0.0566	0.0252	3.748	0.0073	137.034
8. Pakistan	(0.0216)	0.8890	(58.002)	(0.0153)	(0.0318)	0.9043	0.983	0.0046	216.016
9. Saudi Ar	0.0437	0.1385	2.284	0.0606	0.0606	0.0778	1.779	0.0298	33.586
10. Sri Lar	0.0882	0.0684	0.638	0.1073	0.0984	(0.0389)	(1.761)	0.0672	14.878
11. Czech I	0.0415	0.0505	1.014	0.0498	0.0498	0.0007	73.217	0.0360	27.800
12. Poland	0.0471	0.0685	1.320	0.0519	0.0519	0.0166	4.124	0.0356	28.115

Data source: KEWT 6.12 of 81 countries by sector, 1990-2012, whose ten original data for the real assets come from *International Financial Statistics Yearbook*, IMF.



Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure T1 The rate of tech. progress, $g_{A(FLOW)}(\mathrm{t})$, the growth rate of *TFP*, $g_{TFP(STOCK)}(t)$, and the growth rate of $\mathrm{k} = K/L$, $g_k(t)$: i) developed vs. BRICs countries

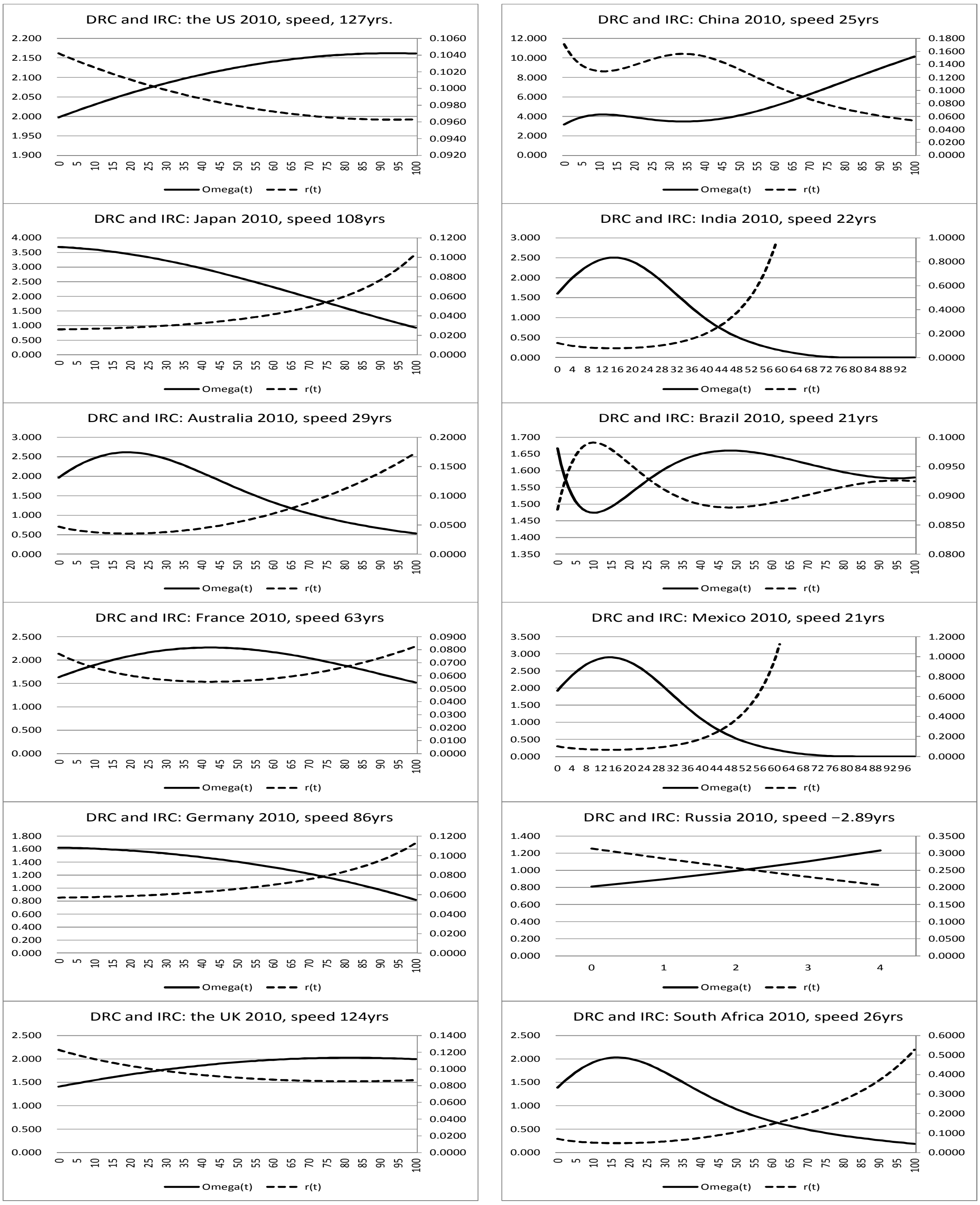


Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure T2 The rate of tech. progress, $g_{A(FLOW)}(t)$, the growth rate of *TFP*, $g_{TFP(STOCK)}(t)$, and the growth rate of $\mathrm{k} = K/L$, $g_k(t)$: ii) 12 European countries

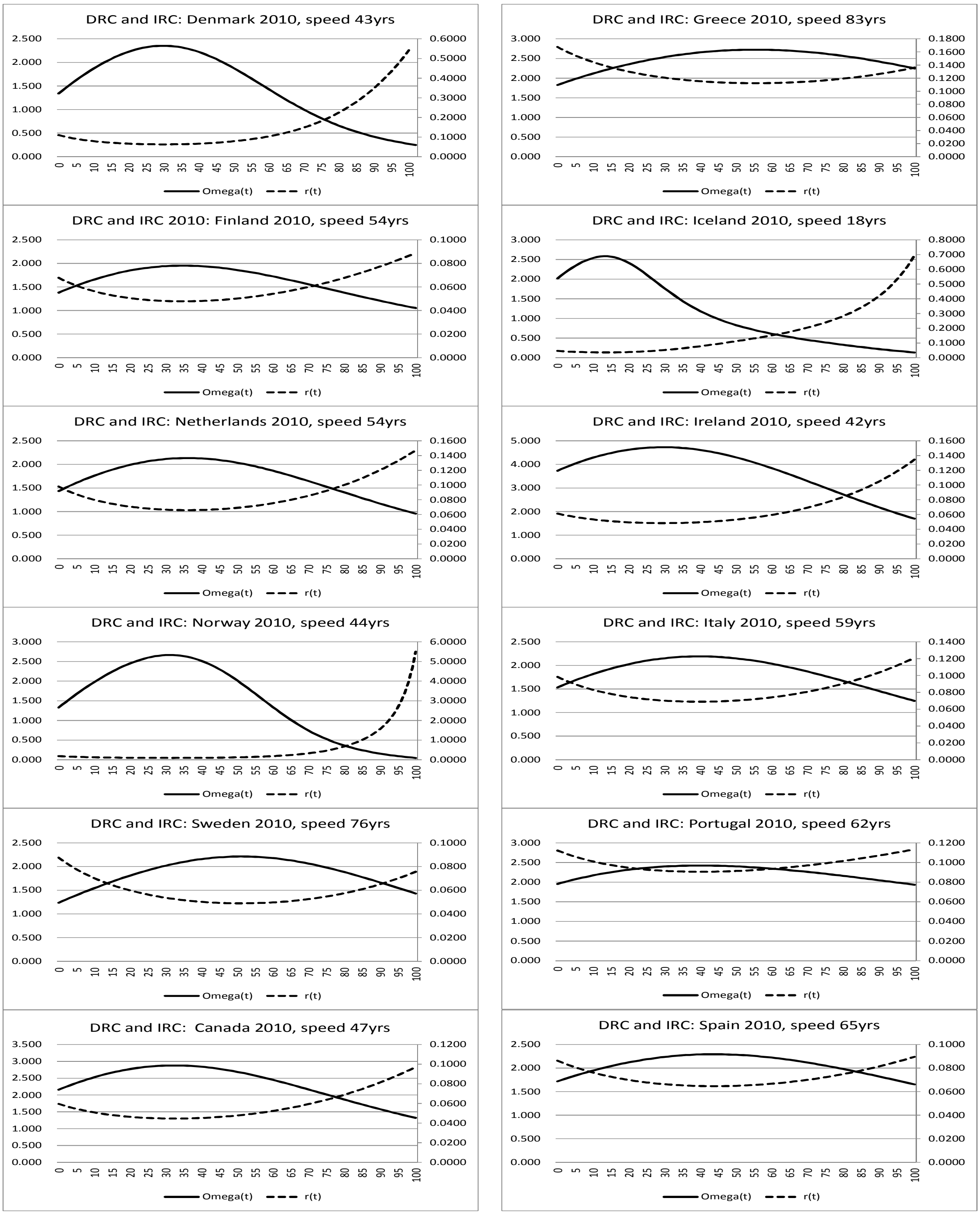
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure T3 The rate of tech. progress, $g_{A(FLOW)}(t)$, the growth rate of *TFP*, $g_{TFP(STOCK)}(t)$, and the growth rate of $k = K/L$, $g_k(t)$: iii) 12 Asian countries



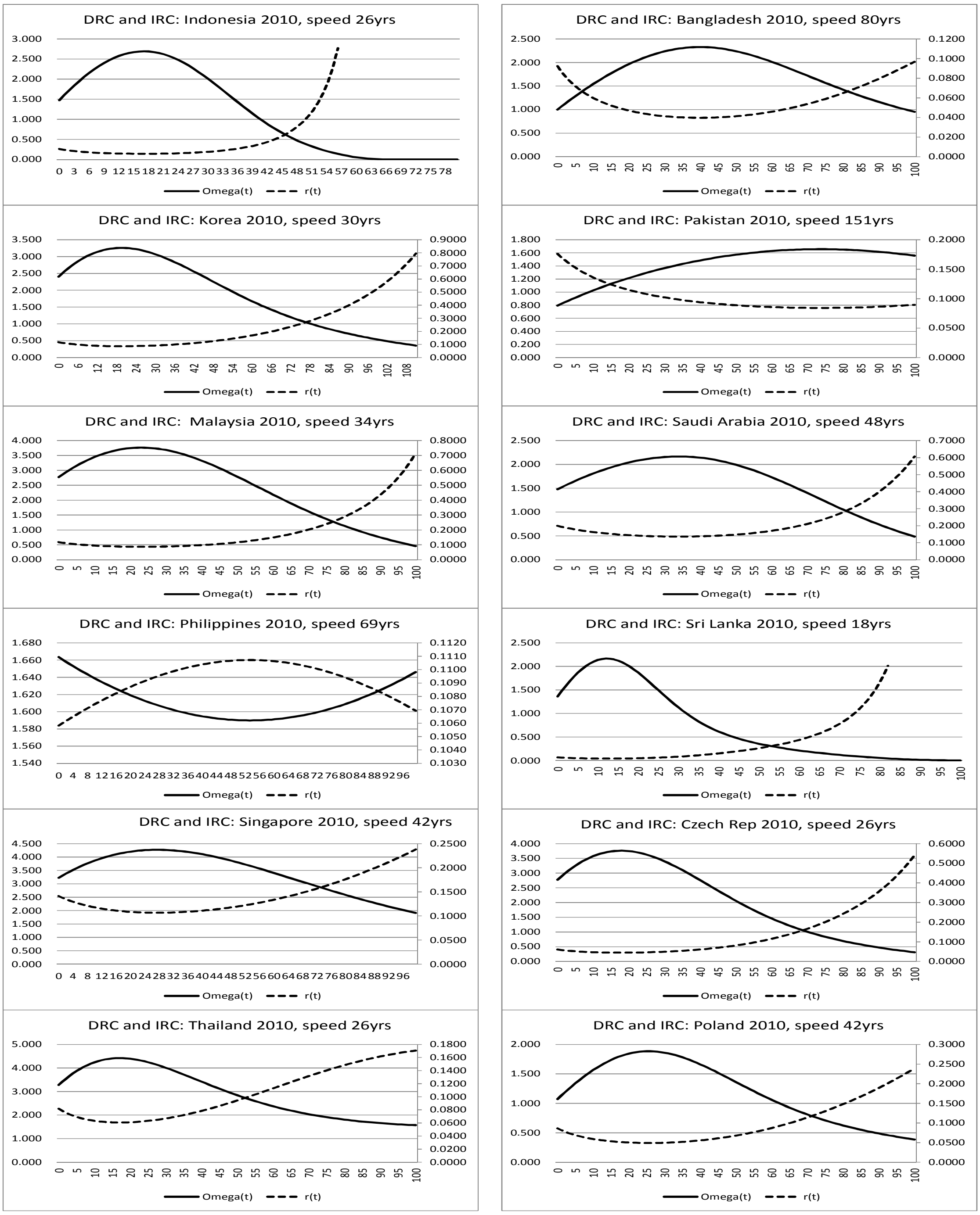
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure D4 The rate of return and the capital-output ratio in equilibrium for DRC and IRC: i) developed vs. BRICs countries



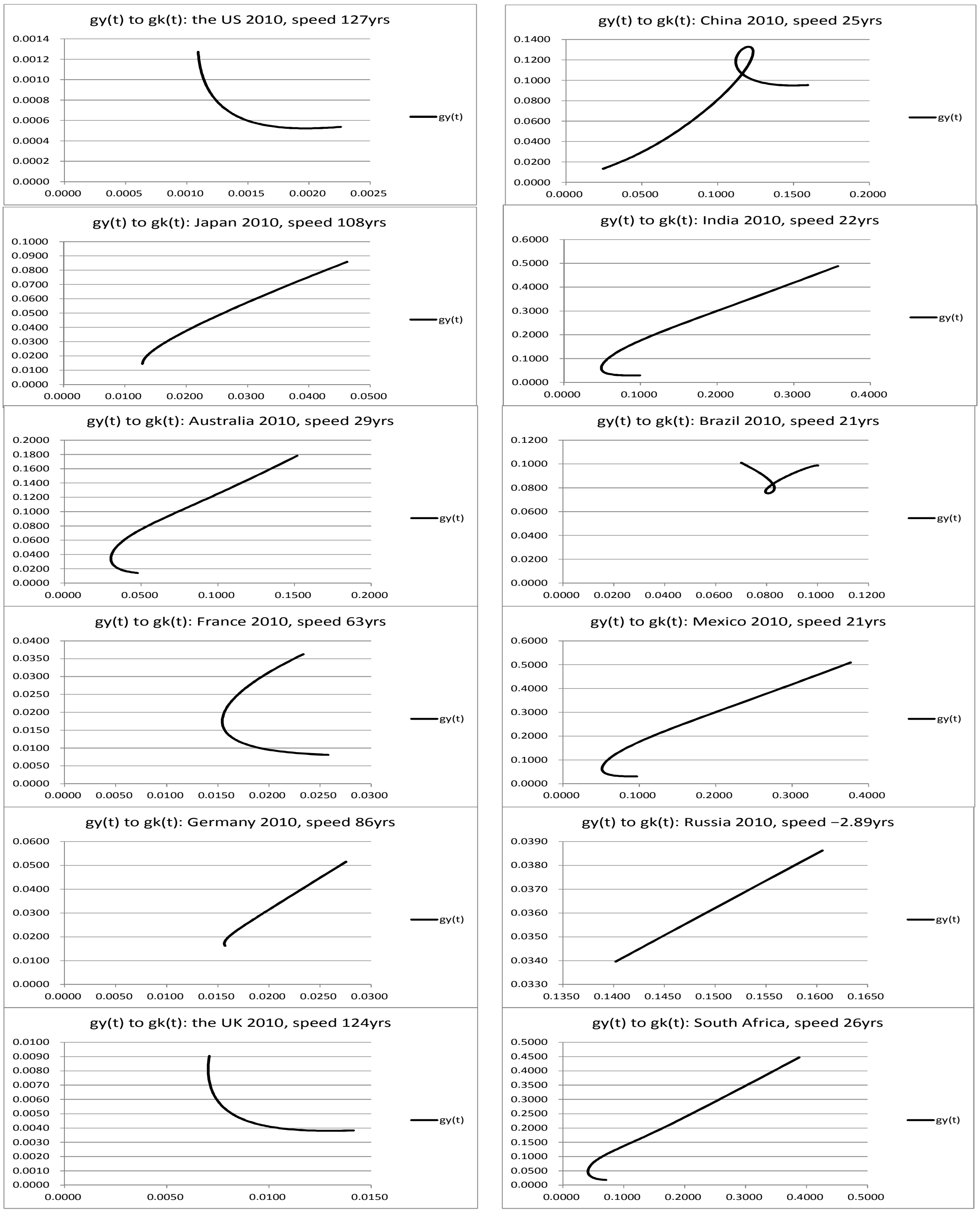
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure D5 The rate of return and the capital-output ratio in equilibrium for DRC and IRC: ii) 12 European countries



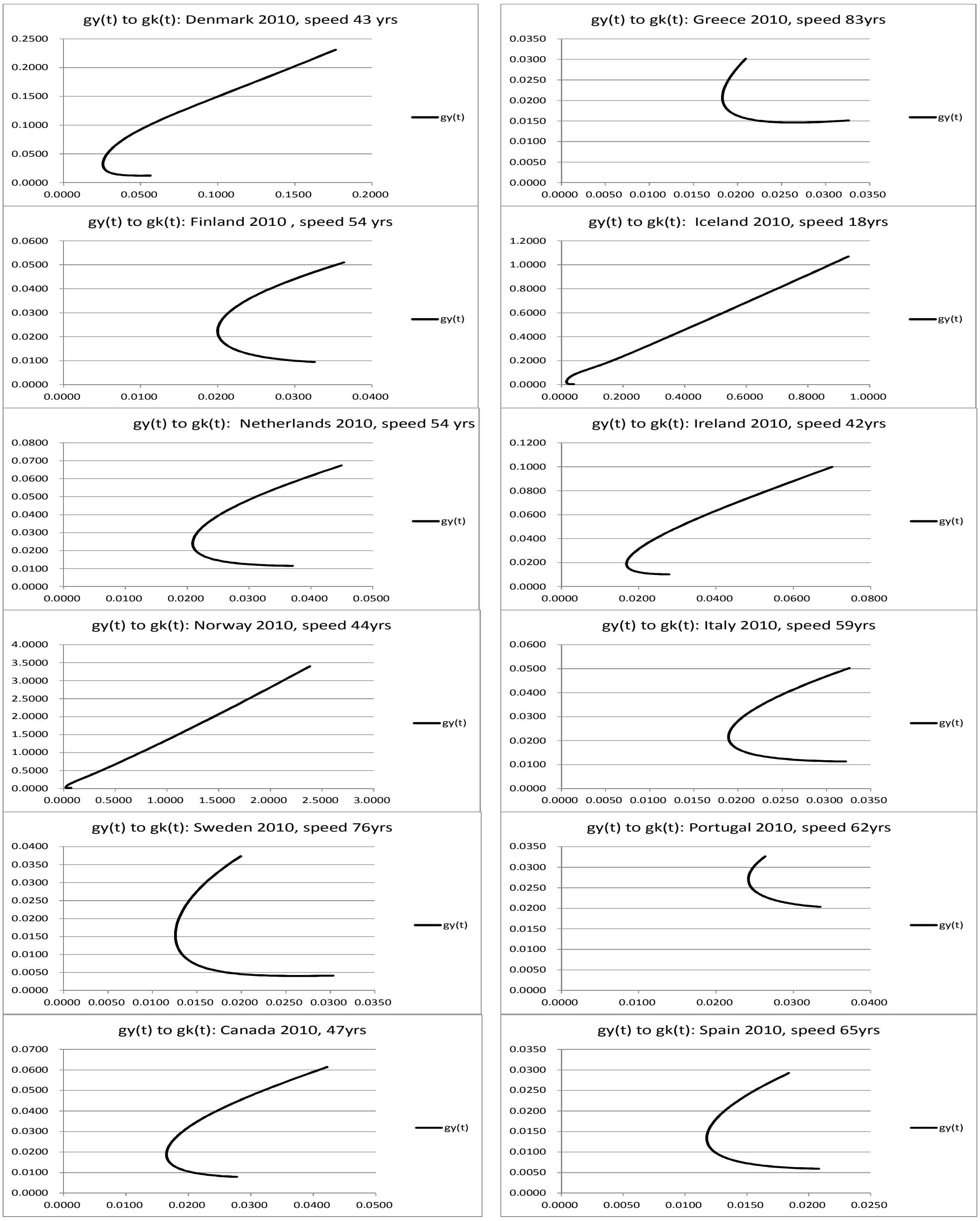
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure D6 The rate of return and the capital-output ratio in equilibrium for DRC and IRC: iii)12 Asian countries



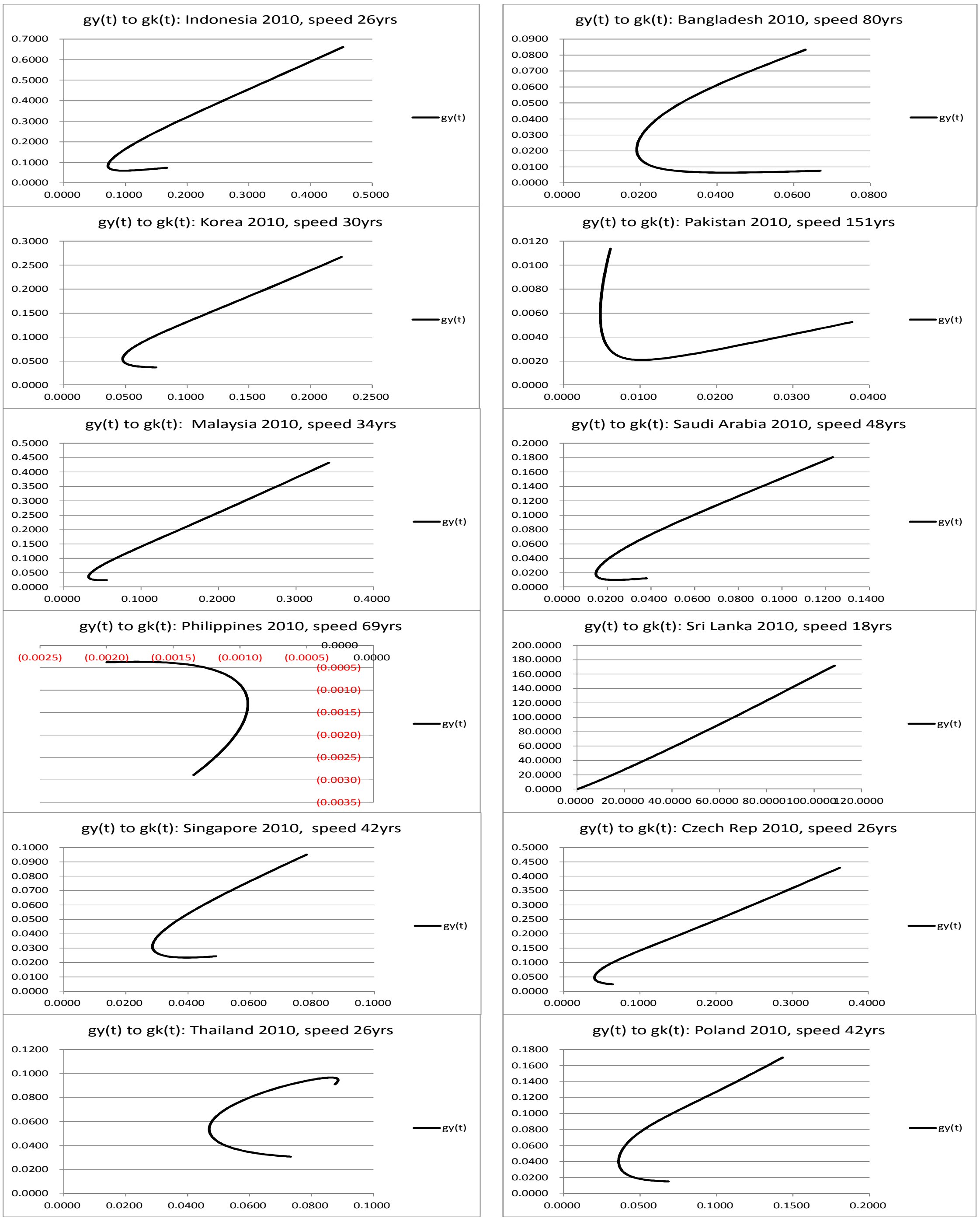
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure G7 The growth rate of output per capita to the growth rate of capital per capita: i) developed vs. BRICs countries



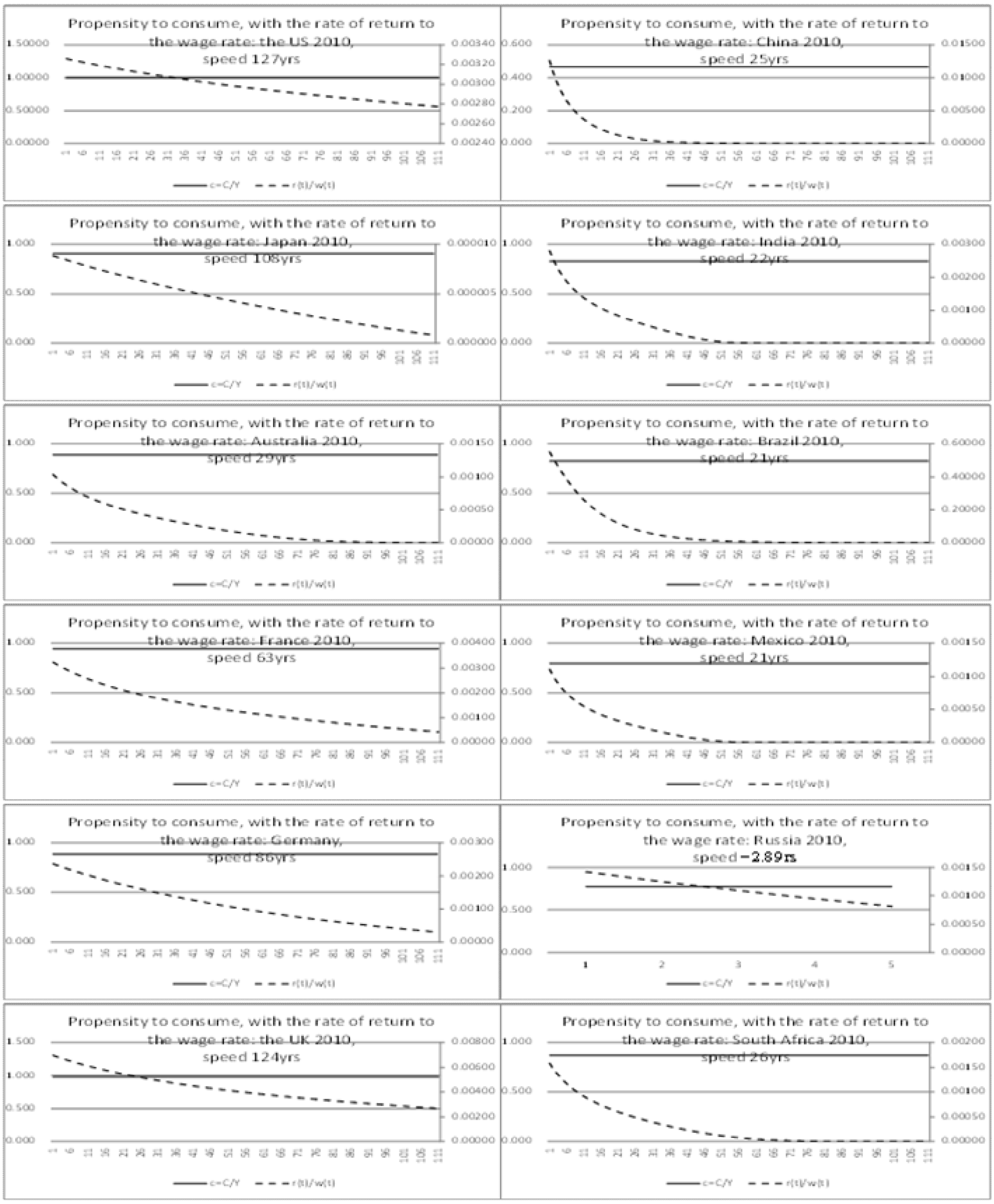
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure G8 The growth rate of output per capita to the growth rate of capital per capita: ii) 12 European countries



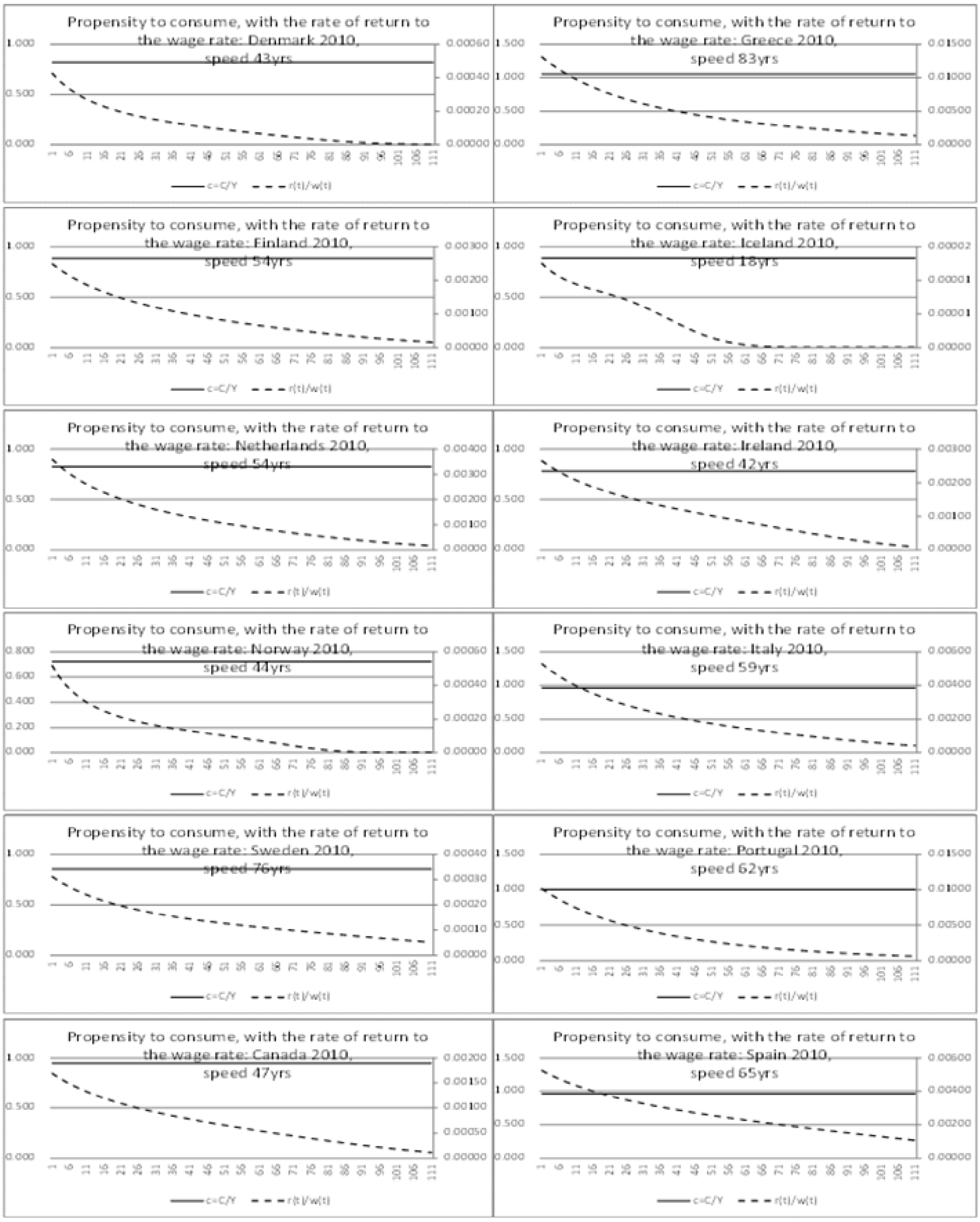
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure G9 The growth rate of output per capita to the growth rate of capital per capita: iii) 12 Asian countries



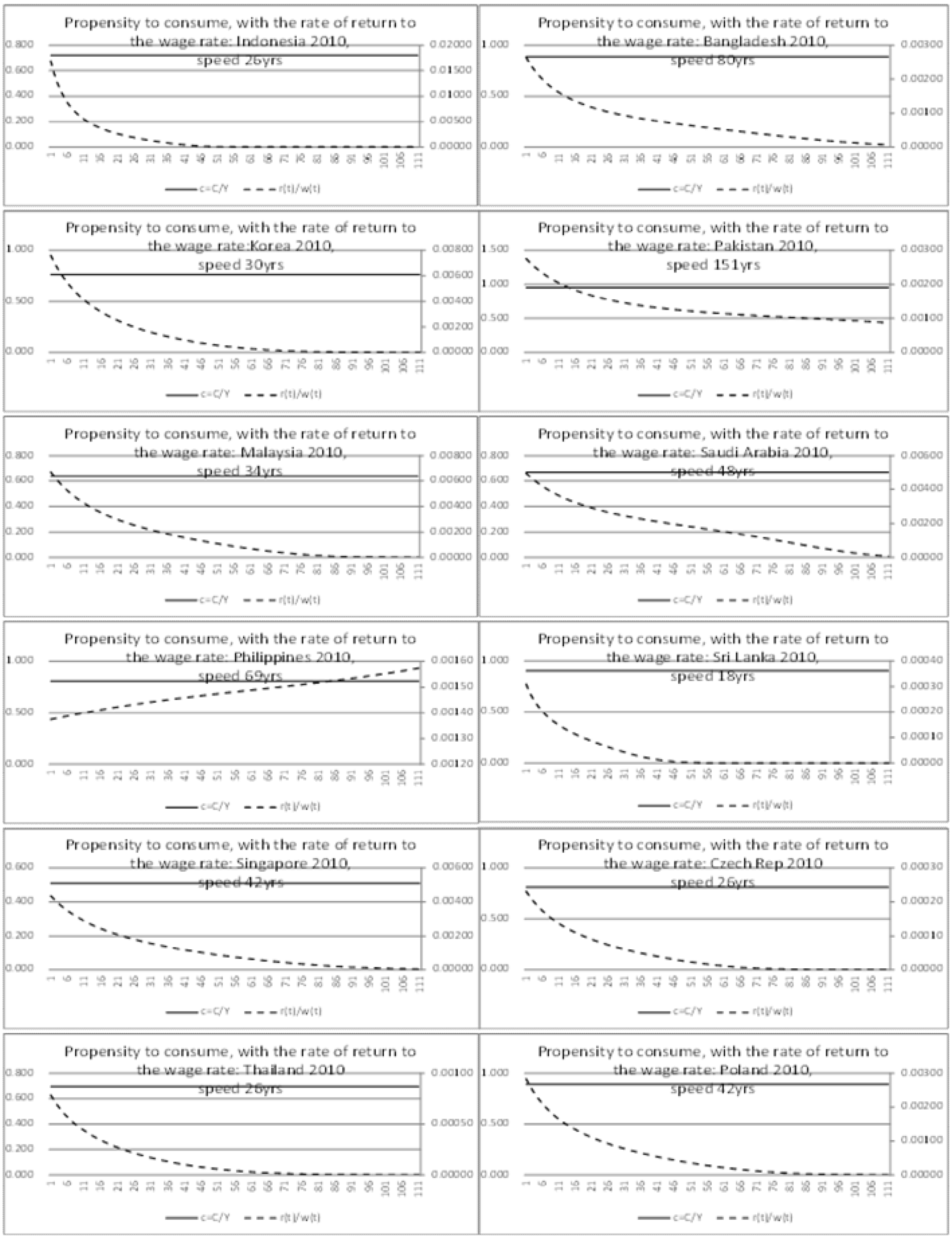
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure P10 Propensity to consume, c $= C/Y$, with the rate of return divided by the wage rate in equilibrium: i) developed vs. BRICs countries



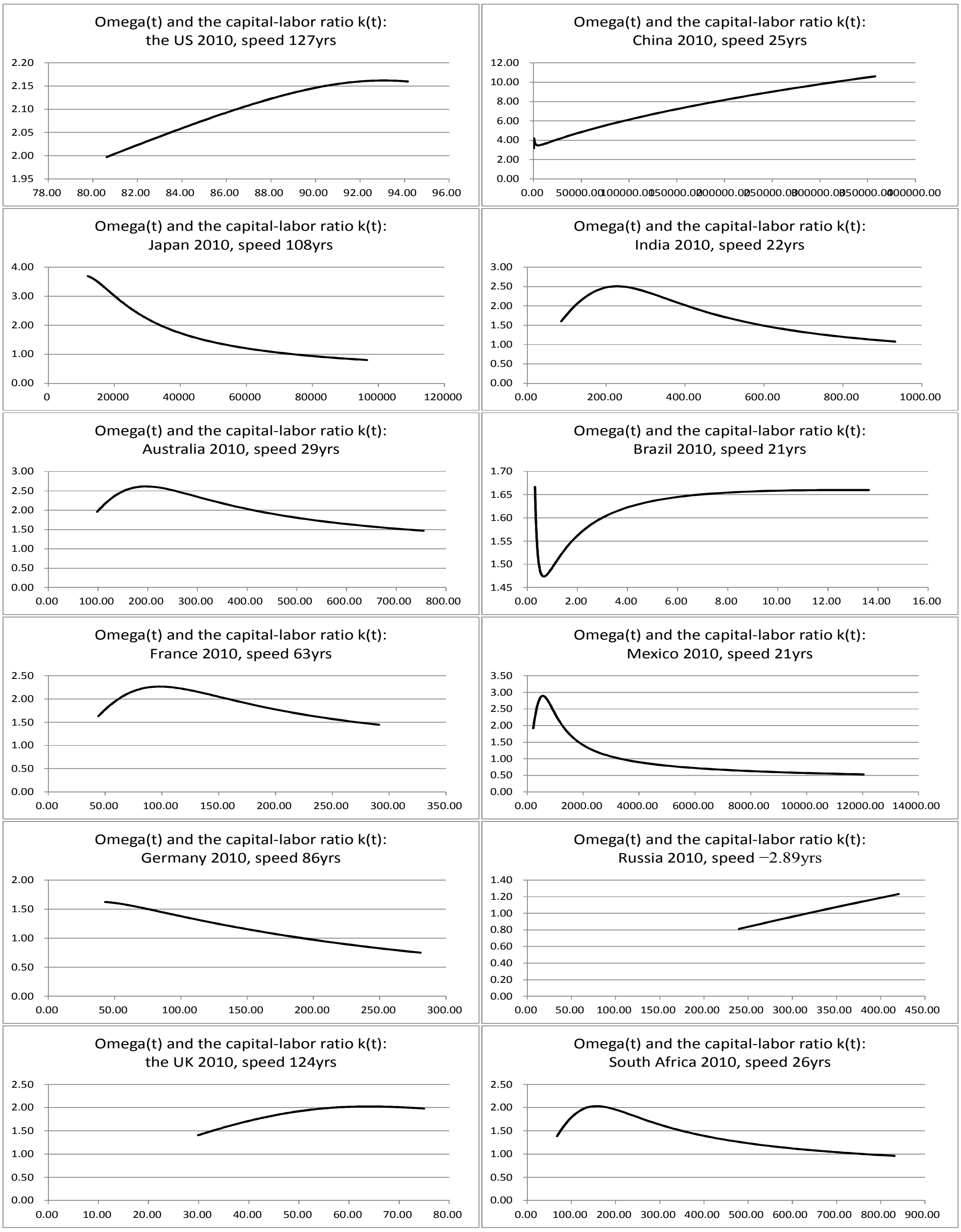
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure P11 Propensity to consume, c $= C/Y$, with the rate of return divided by the wage rate in equilibrium: ii) 12 European countries



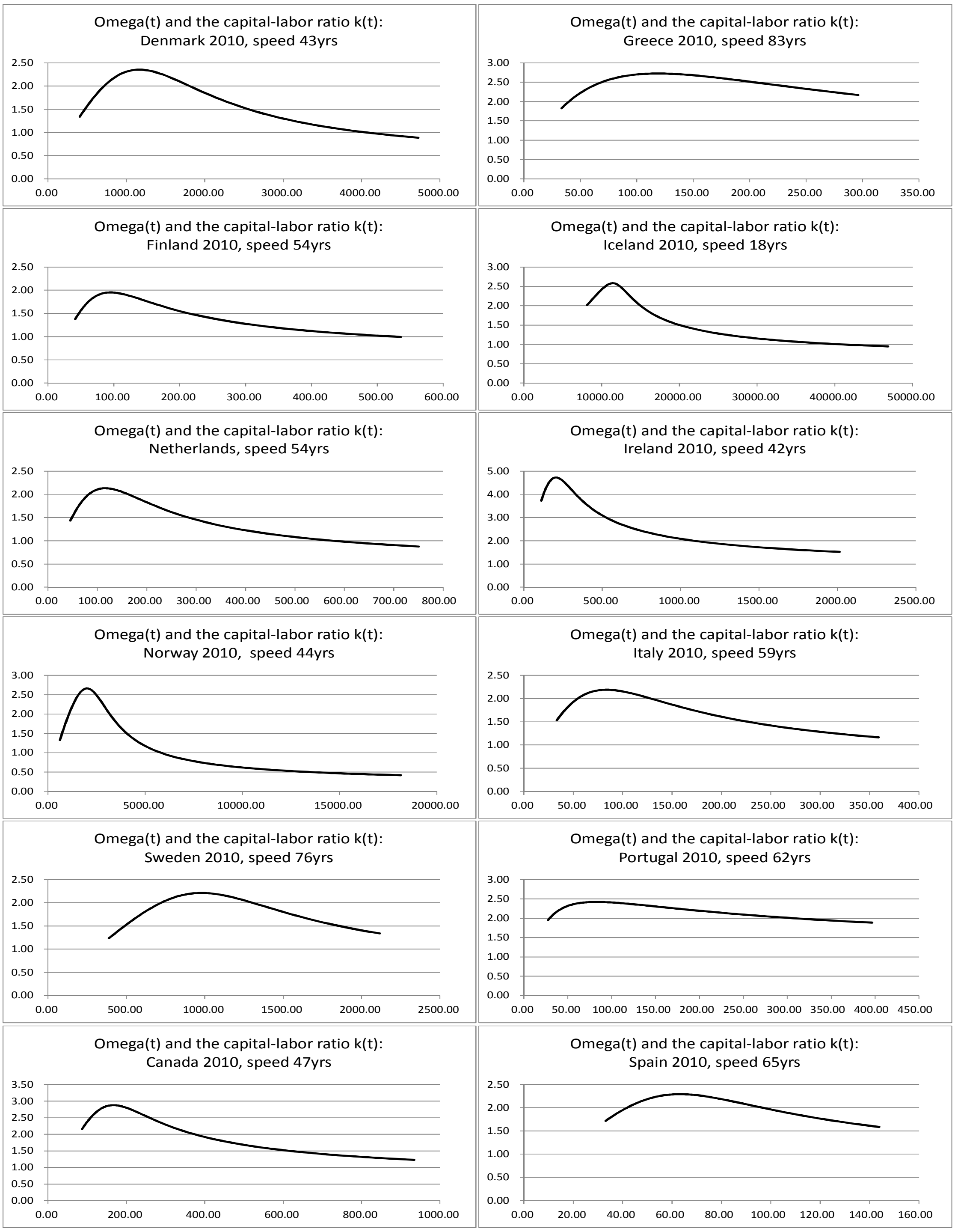
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure P12 Propensity to consume, c $= C/Y$, with the rate of return divided by the wage rate in equilibrium: iii) 12 Asian countries



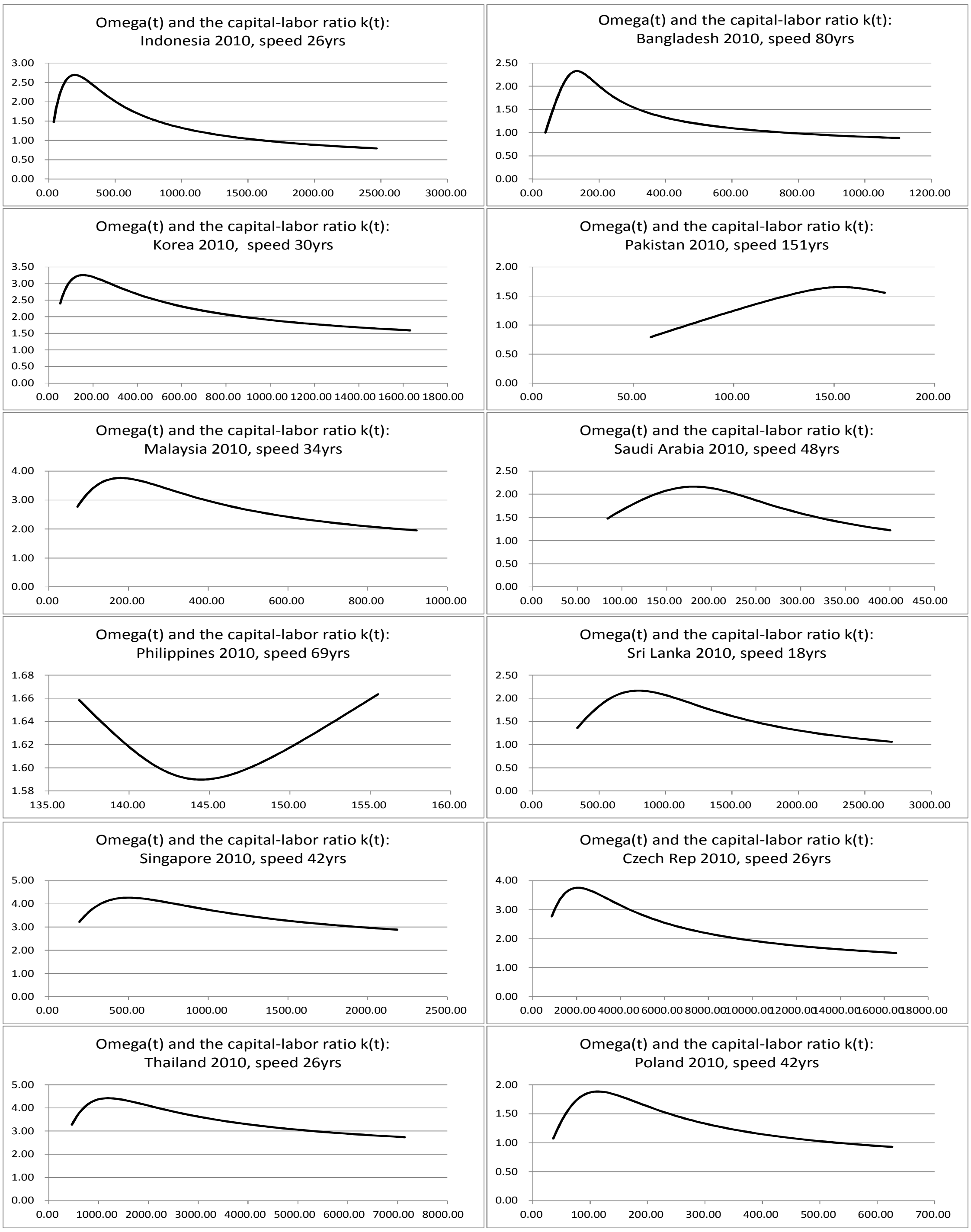
Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure C13 Capital-output ratio, $\Omega(t)$, to capital-labor ratio, $k(t)$: i) developed vs. BRICs countries



Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure C14 Capital-output ratio, $\Omega(t)$, to capital-labor ratio, $k(t)$: ii) 12 European countries



Data source: KEWT 6.12 of 81 countries by sector, 1990-2010, whose 10 original data from the real assets and 15 original data from the financial/market assets, each at *International Financial Statistics Yearbook*, IMF.

Figure C15 Capital-output ratio, $\Omega(t)$, to capital-labor ratio, $k(t)$:
iii) 12 Asian countries

Chapter 16

Appendix Problems to be examined in recursive programming

This Appendix first shows basic framework of recursive programming, secondly, procedure of recursive programming, and thirdly, revisits mechanics of the data-sets: endogenous versus actual. The basic framework is shown using nine endogenous parameters, $\lambda^*, \Omega^*, \alpha, \beta^*, \delta_0$ and, n, n_G, i, i_G, and several variables of growth rates and rates of return in equilibrium. The basic framework is summarized as follows:

1. Constant endogenous parameters in transitional path are: the ratio of net investment to output, $i = I/Y$, the growth rate of population, *n*, the relative share of capital, *alpha.*, and the speed years for convergence, $1/\lambda^*$.
2. Endogenous parameters that change by time/year are: the capital-output ratio, *Omega=K/Y*, the capital-labor ratio, *k=K/L*.
3. Two endogenous parameters, *beta*(*t*) and *delta*(*t*), by assumption, each change 'linearly' by time/year, using each constant discount rate of *beta* and *delta*.
4. Endogenous variables are: the level of technology or total factor productivity as stock, *A*(*t*)=*TFP*(*t*), the rate of technological progress, $g_A(t)$, the growth rate of per capita capital, $g_k(t)$, the growth rate of per capita output, $g_y(t)$, the growth rate of capital, $g_K(t)$, and the growth rate of output, $g_Y(t)$, the rate of return, *r*(*t*), and the wage rate, *w*(*t*).
5. The elasticity of substitution, *sigma*, and the relative price level, *p*, each maintain 1.0 by time/year in transitional path (note that KEWT shows *sigma*≠1 but *p*=1).

Secondly, procedure of recursive programming is shown step by step as follows:

1. $\beta(t) = \beta(0)(1 + r_\beta)^t$ and $\delta(t) = \delta(0)(1 + r_\delta)^t$, where r_β and r_{δ_0} are respectively the discount rate.[9] These discount rates are assumed to change compound by time/year during speed years for convergence in the discrete case; $\beta(t) \to \beta^*$ and $\delta(t) \to \alpha$.
2. $i(t) = i \cdot y(t)$, where $TFP(0) = \frac{k(0)^{1-\alpha}}{\Omega(0)}$ and $y(0) = TFP(0) \cdot k(0)^\alpha$. For convenience, *A* is used for *TFP*.
3. $L(t) = L(0)(1 + n)^t$ holds. However, (1) for the first following approach to clarify *k* and *y*, *L*(0)=1.0000 is used and (2) for the following second approach to clarify absolute values such as *K* and *Y*, $L_{POPU}(0)$, is used as actual population at the initial time/year.

[9] These discount rates are shown as: $r_\beta = \frac{LN(\beta^*)-LN(\beta_0)}{1/\lambda^*}$ and $r_{\delta_o} = \frac{LN(\alpha)-LN(\delta_0)}{1/\lambda^*}$ (see 158, *PRSCE*: 49 (Sep, 1), 2008), where $LN(1 + r_\beta) \fallingdotseq r_\beta$ holds using Maclaurin's series. The speed of convergence is derived using the growth rate in equilibrium: $speed = \frac{1}{(1-\alpha)n+(1-\delta_0)i(1-\beta^*)} = \frac{1}{\lambda^*}$.

For the first approach to clarify k and y:

1. Using $i_K(t) = i(t) \cdot \beta(t)$, $k(t) = k(t-1) + i_K(t)$ holds.
2. Using $i_A(t) = i(t)(1-\beta(t))/k(t)^{\delta(t)}$, $A(t) = A(t-1) + i_A(t)$ holds. Note that $i(t) \neq i_K(t) + i_A(t)$ holds, due to the introduction of $k(t)^{\delta(t)}$ into $i_A(t)$.
3. Each variable of $g_A(t), g_k(t)$, and $g_y(t)$ is calculated using each difference of $A(t)$ and $A(t\text{-}1)$, $k(t)$ and $k(t-1)$, and $y(t)$ and $y(t-1)$: e.g., $g_{A(STOCK)}(t) = (A(t) - A(t-1))/A(t-1)$.
4. $\Omega(t) = k(t)/y(t)$ is derived as an endogenous parameter.
5. $r(t) = \alpha/\Omega(t)$ is derived as an endogenous variable. $r(t) = \alpha/\Omega(t)$ reduces to $r(t) = \alpha \cdot A(t) \cdot k(t)^{\alpha-1}$.
6. $w(t) = (1-\alpha)y(t)$ is derived as an endogenous variable.
7. The growth rate of A as stock, $g_{A\,STOCK}(t)$, equals the growth rate of A as flow, $g_{A(FLOW)}(t)$. There are two methods to measure $g_{A\,FLOW}(t)$ in the transitional path: (1) Using $y(t) = A(t)k(t)^{\alpha}$ and $g_y(t) = g_A(t) + \alpha \cdot g_k(t)$, $g_{A\,FLOW}(t) = g_y(t) - \alpha \cdot g_k(t)$ is derived. (2) Using the weighted average of $r(t)$ and $w(t)$, $g_{A\,FLOW}(t) = \alpha \cdot g_r(t) + (1-\alpha)g_w(t)$ is derived.

For the second approach to clarify absolute values such as K and Y:

1. $Y(t)=y(t)\cdot L_{POPUL}(t)$, where $L_{POPUL}(t) = L_{POPUL}(0) \cdot L(t)$.
2. $K(t)=L_{POPUL}(t)\cdot k(t)$.
3. $Y(t) = A(t) \cdot K(t)^{\alpha} \cdot L_{POPUL}(t)^{1-\alpha}$, where $A(t)$ remain unchanged.
4. $W(t) = w(t) \cdot L_{POPUL}(t)$.
5. $\Pi(t) = Y(t) - W(t)$.
6. Elasticity of substitution, *sigma*:

$$\sigma = 1.0000 \text{ by time/year holds: } \sigma = \frac{-\Delta k/\left(\frac{k_0+k_1}{2}\right)}{\Delta(r/w)/\left(\frac{r_0+r_1}{2}/\frac{w_0+w_1}{2}\right)}.$$

7. Relative price level, p=1.0000 by time/year holds:

$$p(t) = (r(t)K(t) + w(t)L_{POPUL}(t))/Y(t).$$

For the approach to clarify absolute values at convergence such as K^* and Y^*:

1. $A^* = A_0(1+g_A^*)^{1/\lambda^*}$, where $1/\lambda^*$ is the speed years for convergence. The assumption of a constant rate of technological progress is required during the speed years for convergence.
2. $L^* = L_0(1+n)^{1/\lambda^*}$, where the rate of change in population, $n_E = n$, is constant.
3. $k^* = (A^* \cdot \Omega^*)^{\frac{1}{1-\alpha}}$, where the assumption of $\Omega^* = \Omega_0$ is required, as stated already above.
4. $y^* = A^* \cdot k^{*\alpha}$.

5. $K^* = k^* \cdot L^*$.
6. $Y^* = A^* \cdot K^{*\alpha} \cdot L^{*1-\alpha}$.

The above whole approach was realized by connecting the capital-labor ratio with the capital-output ratio. Up to date, there is no way to measure 'values at convergence,' except for the above approach.

A few problems hidden in recursive programming are reviewed in this Appendix. These are shown using Figures in Appendix at the end: (1) Time-series analysis of main variables, (2) the relationship between the capital-output ratio, $\Omega(t)$, and $1 = \Omega(t) \cdot B(t)^{1-\delta(t)}$, where $B(t) = (1 - \beta(t))/\beta(t)$, (3) the relationship between the capital-output ratio, $\Omega(t)$, and the growth rate of output per capita, $g_y(t)$, and (4) the capital-output ratio, $\Omega(t)$, and the capital-labor ratio, $k(t)$. There is no empirical research of the capital-output ratio in the literature. Neo-classicists have used the capital-labor ratio but no empirical work for capital after 1995, due to some problems, which the author confirmed directly from PWT researchers. The author clarifies the four problems as follows:

First, for time series analysis, the author erased the assumption of diminishing returns to capital (DRC) perceived in the literature. When the transitional path shows increasing returns to capital (IRC) at the initial time/year, the capital-output ratio first increases, and hits the maximum. This point of time corresponds with the capital-output ratio at convergence theoretically. In recursive programming by country, this matching does not precisely occur due to the assumption of $\Omega^* = \Omega_0$. When the transitional path shows DRC at the initial time/year, the capital-output ratio first decreases, and hits the minimum. This point of time corresponds with the capital-output ratio at convergence theoretically. In recursive programming by country, this matching does not precisely occur due to the assumption of $\Omega^* = \Omega_0$.[10] After convergence, DRC turns to IRC or the capital-output ratio turns towards zero in infinite time/year while IRC turns to DRC or the capital-output ratio rises up/diverges towards infinity.

Second, for $1 = \Omega(t) \cdot B(t)^{1-\delta(t)}$, there is some problem to be examined. In recursive programming, this condition does not hold by time/year. It is theoretically true that this condition holds only at convergence. The purpose of the condition is traced back to the endogenous measurement of $delta_0$ at the initial time/year.

Instead of using A as a stock, using $B^* = (1 - \beta^*)/\beta^*$ as a flow, first define B as $B^*_{TFP} \equiv (B^*)^{1-\delta_0}$. Since $\Omega = \frac{k^{1-\alpha}}{A}$ holds (as first proved in the author's PhD thesis (Note 19, 38, 2003)) using the C-D production function, this capital-output ratio is expressed as

[10] This assumption corresponds with the law of conservation of the capital-output ratio applied to von Neumann (1945-46) turnpike theory and proved by Samuelson (1477-79, 1970). 'The constant capital-output ratio was the reciprocal of the von Neumann interest rate or of the equivalent maximal rate of balanced growth.'

$\Omega = \frac{k^{1-\alpha}}{B_{TFP} \cdot k^{1-\delta_0}}$ or $\Omega = \frac{k^{\delta_0-\alpha}}{B_{TFP}}$.

At convergence, $\alpha = \delta_0$ holds under constant returns to capital (CRC), resulting in $1 = k^{\delta_0-\alpha}$. Then, $\Omega^* = \frac{1}{B^*_{TFP}}$ or $\Omega^* = \frac{1}{(B^*)^{1-\delta_0}}$ holds, resulting in $(B^*)^{1-\delta_0} = \frac{1}{\Omega^*}$.

Therefore, $1 = \Omega^* \cdot B^{*1-\delta_0}$ holds at convergence and $\delta_0 = 1 - \frac{LN(1/\Omega^*)}{LN(B^*)}$, or $\delta_0 = 1 + \frac{LN(\Omega^*)}{LN(B^*)}$ are derived. In other words, if $\Omega^* = 1/B(t)^{1-\delta(t)}$ holds, there is no problem at all.

In short, $y = A \cdot k^\alpha$ is not consistently connected with $B_{TFP} \equiv B^{1-\delta_0}$ in the transitional path over years, except for one point of time/year at convergence. The purpose of B_{TFP}: $TFP_B \equiv B_{TFP} \cdot k^{1-\delta_0}$ is to derive the value of *delta*$_0$. The capital-output ratio and, *delta*$_0$ or *beta* are tightly related. For this reason, the author (151, *JES*, Sep 2006, after revise) assumes that $\Omega^* = \Omega_0$ holds. Without *delta*$_0$, DRC, IRC, and CRC are not specified.

Third, for the relationship between $\Omega(t)$ and $g_y(t)$, the patterns differ by country. Nevertheless, it is true that the lower the $\Omega(t)$ the higher the $g_y(t)$. This evidence is important to interpret the results of deficit since the higher the deficit to government output the higher the $\Omega_G(t)$.

Fourth, for the relationship between $\Omega(t)$ and $k(t)$, the patterns differ by country. It is true that the capital-labor ratio cannot directly be connected with technology. The author finds that beyond some level of $k(t)$ remains roughly unchanged. This implies that we can take either $\Omega(t)$ or $k(t)$ after $\Omega(t)$ reaches a constant. Yet, when we observe more precisely, the relationship between $\Omega(t)$ or $k(t)$ is complicated. This implies that it may be impossible to directly formulate the equation of the capital-labor ratio. A fact remains unchanged that we cannot formulate the endogenous model without using the capital-output ratio.

Thirdly, for revisit mechanics of the data-sets: endogenous *vs*. actual

KEWT data-sets differ from one year recursive programming so that direct comparison is inappropriate, although both have 1.0 for the relative price level; *p*=1.0. KEWT measures variables at convergence by using the endogenous speed years between the initial/current period and at convergence. As a result, the current growth rate of the level of technology as a stock fluctuate over years in 1990-2011 while the endogenous rate of technology as a flow is measured steadily over years. In statistics, actual variables are published yet unstably by year. Endogenous theoretical variables are stable in recursive programming and accordingly in KEWT by year.

Over years (not by year), actual data and endogenous data march in parallel. As a result, actual data cannot be far apart from theoretical data over years. This is another reason why actual current data fluctuate by year. The fluctuation of actual data comes from the change in net investment by year while endogenous data are based on smooth change in net investment in endogenous equilibrium. Actual data result in business cycle. Endogenous data show sustainable robustness by year, smoothening business cycle. And, nine endogenous parameters change by year inconspicuously. Policy-makers must watch these changes underlying in actual data. If policy-makers do not pay attention to these changes of endogenous parameters, some of endogenous parameters such as $delta_0$ suddenly change and the current situation gets into disequilibrium.

For example, each range of g_A^*/g_Y^*, $g_{A(G)}^*/g_{Y(G)}^*$, and $g_{A(PRI)}^*/g_{Y(PRI)}^*$ by country and sector change over years. Yet, for a certain short periods, g_A^*/g_Y^*, $g_{A(G)}^*/g_{Y(G)}^*$, and $g_{A(PRI)}^*/g_{Y(PRI)}^*$ show abnormal values, reflecting sudden unstable speed years for convergence, and this is a signal to disequilibrium. Unstable speed years often occur due to fiscal policy failure. Fiscal policy exists as a core of real, financial, and market policies. (see www@riee.tv, www.megaegg.ne.jp/~kamiryo/, and http://ci.nii.ac.jp/).

References

The author separated Harcourt's (1972) references in this chapter to 2-1 of 'Specific References' on page 541 as follows:

1. Author's, including the first appearances;
2-1. Referring to Harcourt, G. C. (1972) at Chapter 16, numbering 1 to 22;
2-2.Translator, Kamiya Denzo's supplement to Keynesians, Neo- and New-, and Neo-classicists, numbering 1 to 20;
3. Historical References influential to author's endogenous system and discoveries.

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