Appendices

Hyperbolas: Formulations, Types, Attributes, Calculations, and Graphs

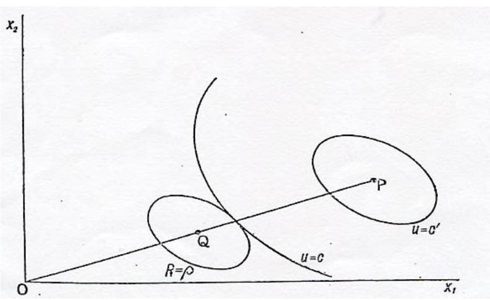
Appendices at the end of this book here include whole explanations of hyperbolas and twelve hyperbola graphs to 36 countries, KEWT 6.12, 1990/1960-2010. Short period data sources by country, sector, and year and, over years, 1990-2010, were presented at the end of related chapters to meet each aspect. Long period basic data sources such as seven endogenous parameters and structural ratios, 1960-2010, were presented in a few related chapters; data and figures (Chapter 6, using KEWT 7.13, 1960-2011). KEWT 1.07, 1960-2005, is the first KEWT series. When readers need to investigate basic data resources of KEWT 1.07, see *Papers of the Research Society of Commerce and Economics* (*PRSCE*) 48 (Sep, 1): 139-235, 2007, and/or enter 'Hideyuki Kamiryo' at http://ci.nii.ac.jp/ .

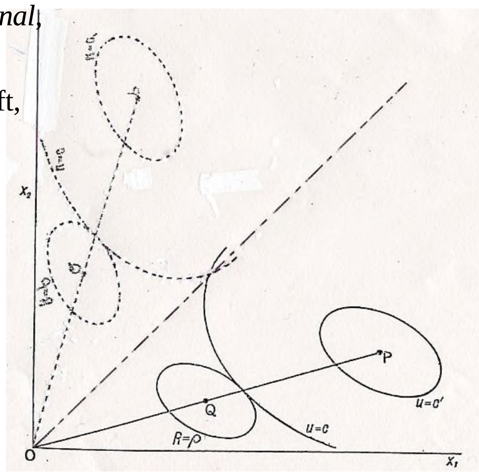
Recursive programming by country, 2010, was presented at the end of Chapter 16. Hyperbolas and recursive programming each show 36 countries so that readers are able to compare respective characteristics by country in the global economies. These Appendices are composed of four parts, **A**, **B**, **C**, and **D**: A. Circle behind hyperbola versus ellipse; B. 12 hyperbolas by type, with 5 attributes defined and calculated; C. Hyperbola graphs by country; and D. Endogenous equations and hyperbolas.

Appendix A. *Circle behind hyperbola versus ellipse*

BOX A-1 First appearance of ellipses by Ramsey, A. C.(53, 1927)

Left: Ramsey, A. C., page 53, *Economic Journal*, 37, 1927.
Right: The author's frame idea is added to Left, symmetrical to the diagonal of hyperbola.





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The author has investigated hyperbolas at libraries abroad for many years. The author even today finds no hyperbola in natural and social sciences, except for transmitting networks[1] in natural sciences and a partial use of hyperbola as a discount rate in social sciences (see several; References at the end). In Chapter 10 of EES, the author compared hyperbola with parabola. The author here introduces a unique article[2] that expresses ellipse intuitively and verbally, as shown on the LHS of the above Box. "*A Contribution to the Theory of Taxation*" by Ramsey, A. C. is similar to F. P. Ramsey's (1928) *A Mathematical Theory of Saving*. Both are based on demand and supply under the price-equilibrium.

The author gets a good inspiration from his image of ellipse drawn on the two dimensions. Hyperbola implicitly has its diagonal of 45^0 up to the right. If this line shifts up (e.g., a line of 65^0, through the origin) or down (e.g., a line of 25^0, through the origin), the circle behind its hyperbola will become the corresponding ellipse, as shown on the RHS of the above Box. Accordingly, the hyperbolic curve will be transformed to a different shape. Hyperbola or ellipse holds in two dimensions. The consistency between the above two dimensions and four dimensions including space and time simultaneously, was already discussed in Chapter 10.

Note: Topology between the circle and the ellipse: a supplement within scientific proofs.

There is a circle behind the hyperbola. On a point of 45^0 of the diagonal that crosses the origin, a point of hyperbola crosses a point of the circle. These points form an equilateral rectangular triangle. Contrarily, the ellipse differs from the hyperbola. Topologically, the ellipse locates above and below the 45^0 of the diagonal. The connector is a rectangular that forms the golden ratio, 3, 4, and 5, related to Einstein's great discovery. The golden rectangular prevails and is involved in imaginary and real numbers. Physics and element chemistry have already accepted both numbers within scientific proofs and try to prove theoretical proofs, further empirically in this world/zone. The imaginary numbers are indispensable to the spiritual zone coexisting behind the physical zone.

The author's Earth Endogenous System (EES) holds without introducing imaginary numbers. A reason is that the hyperbola, the circle, and the equilateral rectangular triangle exist, simultaneously with 'space and time' dimensionally as one. Ramsey, A. C (1927) correctly expressed his topology of ellipse, below the 45^0 of diagonal. To leave

[1] Nukiyama Heiichi and Nagai Kenzo. (March, 1928). A Hyperbolic theory of Transmitting Networks. *Journal of the Institute of Electrical Engineers of Japan*, Tohoku University, reprinted, 18p. The author found this article at National Library of Greece, Athens, on 24 March 2011.

[2] Ramsey, A. C. (1927). A Contribution to the Theory of Taxation. *Economic Journal* 37 (Mar., 145): 47-61.

this fact for the future, the author here clarified the relationship between the circle and the ellipse on the topology. The ellipse is now more definitely defined as a series of wave that presents one real number cycle and following half a cycle of imaginary number (abbreviate its empirical proof herein).

Appendix B. *12 hyperbolas by type, with 5 attributes defined and calculated*

It is essential for policy-oriented endogenous parameters in the endogenous model to use the rectangular hyperbola. Let the author start with the rectangular hyperbola *y*=*h*(*x*), by setting *h*(*x*)=(*cx*+*d*)/(*ax*+*b*). *y*=*h*(*x*) is now shown by $y = \frac{c}{a} + \frac{d - \frac{b \cdot c}{a}}{ax+b} = \frac{c}{a} + \frac{f}{ax+b}$, $f = d - \frac{b \cdot c}{a}$, and $\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$. There are six types in this form, starting with the standard type that elements $a\ b\ c\ d$ are all not zero. When *a*=0, the hyperbola reduces to a linear exceptionally, as shown at $r^*(n)$ in Chapter 15. Six types are classified, using the rate of net investment to output, $i = I/Y$, and the rate of change in population, *n*, each as an independent variable. And, thirteen functions are distributed to each respective type as follows:

BOX B-1 Cases when each of elements, *a*, *b*, *c*, *d*, is zero

i) *a*=0, the linear type: $\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{b}}$ or $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{b}}\mathrm{x} + \frac{\mathrm{d}}{\mathrm{b}}$. To $r^*(n)$.

ii) *b*=0: $\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d}}{\mathrm{ax}}$. To $r^*(i)$; $\beta^*(i)$.

iii) *c*=0 and d=1: $\mathrm{y} = \frac{1}{\mathrm{ax+b}}$. To $speed(i)$ and $speed(n)$.

iv) *d*=0: $\mathrm{y} = \frac{\mathrm{cx}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{-\frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$. To $n(i)$ or $i(n)$; $\Omega^*(i)$ and $\Omega^*(n)$; $\Omega^*(\beta^*)$.

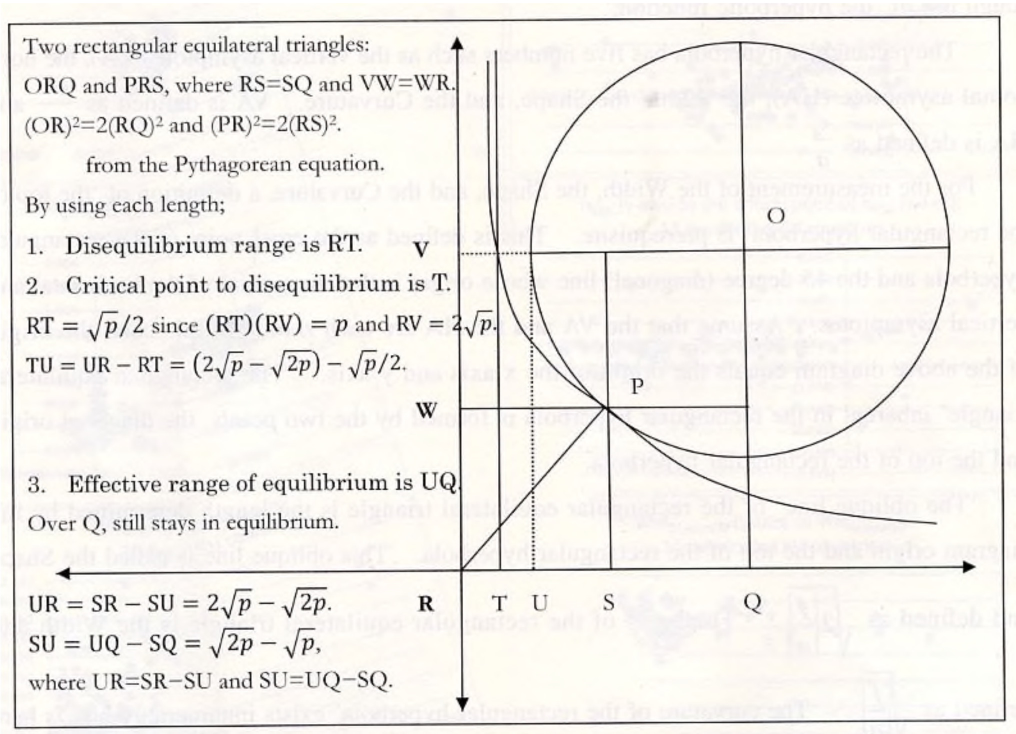
v) *c*=0: $\mathrm{y} = \frac{\mathrm{d}}{\mathrm{ax+b}}$. To $\Omega^*(n)$.

vi) No zero, the standard type: $\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d} - \frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$. To $\beta^*(n)$; $\alpha(i)$ and $\alpha(n)$.

Hyperbolas are each shown as a function of an independent parameter or variable to dependent parameter or variable assuming all the others are fixed.

The rectangular hyperbola has *five attributes* such as the vertical asymptote (VA), the horizontal asymptote (HA), the Width, the Shape, and the Curvature. VA is defined as $\frac{-\mathrm{b}}{\mathrm{a}}$ and, HA is defined as $\frac{\mathrm{c}}{\mathrm{a}}$. For the measurement of the Width, the Shape, and the Curvature, a definition of ‘the top of the rectangular hyperbola’ is prerequisite. This is defined as the cross point of the rectangular hyperbola and the 45 degree (diagonal) line whose origin is the cross point of the horizontal and vertical asymptotes. Assume that the VA and the HA are each zero. In this case, the origin of the above diagram equals the origin of the *x* axis and *y* axis. *The rectangular equilateral triangle* inherent in the rectangular hyperbola is formed by the two points, the diagonal origin and the top of the rectangular hyperbola.

'The oblique line' of the rectangular equilateral triangle is the length determined by the diagram origin and the top of the rectangular hyperbola. This oblique line is called the Shape and defined as $\sqrt{2\left|\frac{f}{a}\right|}$. 'The base' of the rectangular equilateral triangle is the Width and defined as $\sqrt{\left|\frac{f}{a}\right|}$. 'The curvature of the rectangular hyperbola' exists innumerably but, is here measured at 'the top of the rectangular hyperbola,' and accordingly, as the inverse number of the square root of the Shape: $1/\sqrt{2\left|\frac{f}{a}\right|}$. The Shape is upward to the right when $\frac{f}{a} > 0$, and the hyperbola spreads in the 1st and 2nd quadrants. The Shape is downward to the right when $\frac{f}{a} < 0$, and the hyperbola spreads in the 3rd and 4th quadrants. Main quadrants of the various hyperbolas locate in the 1st and, exceptionally in the 4th quadrants. Regardless of the sign of *a/f*, the Width, the Shape, and the Curvature remain each unchanged; when *a/f* is minus, its absolute value is used.



Note: The three hyperbolas and respective quadrant (A reduced linear in $(r^*)(i)$ is excluded here.)

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$(1/\lambda^*)(i)$ stays in the 1st quadrant but, $(1/\lambda^*)(n_E)$ in the 1st and 2nd quadrants. Each vertical asymptote shifts to the 2nd quadrant yet, the ranges of equilibrium are measured in the 1st quadrant in the case of $(1/\lambda^*)(i)$. This is because $i = I/Y$ on the x axis only shows a plus value in the case of equilibrium.

$(r^*)(i)$ has its horizontal asymptote and stays at the 1st quadrant in the case of inflation and at the 4th quadrant in the case of deflation. Mathematically, $\sqrt{|p|}$ holds in the 4th quadrant, instead of $\sqrt{p}$.

$speed(i)$ and $speed(n_E)$ are measured as a base for equilibrium. Here, 'speed' years is $1/\lambda^*$ but n_E is shown by n, as a case of full-employment in equilibrium, where $n = n_E$.

1-1 $\boldsymbol{speed(i)}$:

$speed\ (i)= \frac{1}{(1-\beta^*)(1-\delta_0)i+n(1-\alpha)}$.

$y = \frac{1}{ax+b}$, where $y = \frac{1}{\lambda^*}$, $x = i$, c=0, d=1, a $= (1-\beta^*)(1-\delta_0)$, and b $= n(1-\alpha)$.

$VA_{speed(i)} = -\frac{n(1-\alpha)}{(1-\beta^*)(1-\delta_0)}$. $\quad HA_{speed(i)} = 0$.

$Width_{speed(i)} = \sqrt{\frac{1}{(1-\beta^*)(1-\delta_0)}}$. $\quad Shape_{speed(i)} = \sqrt{\frac{2}{(1-\beta^*)(1-\delta_0)}}$.

$Curvature_{speed(i)} = 1/\sqrt{\frac{2}{(1-\beta^*)(1-\delta_0)}}$.

$\left(\mathrm{y}-\frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x}+\frac{\mathrm{b}}{\mathrm{a}}\right)=\frac{\mathrm{f}}{\mathrm{a}}$. For $speed(i)$: $(\mathrm{y}-0)(\mathrm{x}+0.05557852) = 6.56530$.

1.2 $\boldsymbol{i(speed)}$:

$i(speed) = \frac{-n(1-\alpha)}{(1-\beta^*)(1-\delta_0)} + \frac{1}{(1-\beta^*)(1-\delta_0)speed}$.

$y = \frac{Cx+1}{Ax}$. $\ y = \frac{C}{A} + \frac{1}{Ax}$, where A $= (1-\beta^*)(1-\delta_0)$, B $= -n(1-\alpha)$, D =1.0.

$VA_{i(speed)} = 0$. $\ HA_{i(speed)} = \frac{-n(1-\alpha)}{(1-\beta^*)(1-\delta_0)}$. $\ Width_{i(speed)} = \sqrt{\frac{1}{(1-\beta^*)(1-\delta_0)}}$.

$Shape_{i(speed)} = \sqrt{\frac{2}{(1-\beta^*)(1-\delta_0)}}$. $\ Curvature_{i(speed)} = 1/\sqrt{\frac{2}{(1-\beta^*)(1-\delta_0)}}$.

$\left(\mathrm{y}-\frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x}+\frac{\mathrm{b}}{\mathrm{a}}\right)=\frac{\mathrm{f}}{\mathrm{a}}$. For $i(speed)$: $(\mathrm{y}+0.05557852)(\mathrm{x}+0) = 6.56530$.

1.3 $\boldsymbol{speed(n)}$:

$speed(n) = \frac{1}{(1-\alpha)n+i(1-\beta^*)(1-\delta_0)}$. $\quad y = \frac{1}{ax+b}$.

$VA_{speed\ (n)} = -\frac{i(1-\beta^*)(1-\delta_0)}{(1-\alpha)}$. $HA_{speed\ (n)} = 0$. $Width_{speed\ (n)} = \sqrt{\frac{1}{1-\alpha}}$.

$Shape_{speed\ (n)} = \sqrt{\frac{2}{1-\alpha}}$. $Curvature_{speed\ (n)} = 1/\sqrt{\frac{2}{1-\alpha}}$.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $speed(n)$: $(\mathrm{y} - 0)(\mathrm{x} + 0.017366348) = 1.1481903$.

1.4 $n(speed)$:

$n(speed) = \frac{-i(1-\beta^*)(1-\delta_0)speed+1.0}{(1-\alpha)speed}$.

$\mathrm{y} = \frac{\mathrm{Cx}+1}{\mathrm{Ax}}$ and $\mathrm{y} = \frac{\mathrm{C}}{\mathrm{A}} + \frac{1}{\mathrm{Ax}}$, where $\mathrm{A} = 1 - \alpha$, $\mathrm{C} = -i(1 - \beta^*)(1 - \delta_0)$, and F=D=1.0.

$VA_{n(speed)} = 0$. $HA_{n(speed)} = \frac{-i(1-\beta^*)(1-\delta_0)}{1-\alpha}$.

$Width_{n(speed)} = \sqrt{\frac{1}{1-\alpha}}$. $Shape_{n(speed)} = \sqrt{\frac{2}{1-\alpha}}$. $Curvature_{\ n(speed)} = 1/\sqrt{\frac{2}{1-\alpha}}$.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $n(speed)$: $(\mathrm{y} + 0.0173663480)(\mathrm{x} + 0) = 1.1481903$.

2-1 $r^*(i)$:

$r^*(i) = \frac{\alpha \cdot i\ (1-\beta^*)(1+n)+\alpha \cdot n(1-\alpha)}{\beta^*(1-\alpha)i}$, which is derived from

$r^* = \frac{\alpha}{\Omega^*}$ and $\Omega^* = \left(\frac{i \cdot \beta^*(1-\alpha)}{i\ (1-\beta^*)(1+n)+n(1-\alpha)}\right)$.

$y = \frac{c}{a} + \frac{d}{ax} = \frac{cx+d}{ax}$, where $a = \beta^*(1-\alpha)$, $b = 0$, $\mathrm{c} = \alpha(1-\beta^*)(1+n)$, $\mathrm{f} = \mathrm{d} = \alpha \cdot$

$n(1-\alpha)$, $\mathrm{e} = \frac{\mathrm{c}}{\mathrm{a}}$, and $\frac{\mathrm{f}}{\mathrm{a}} = \frac{\alpha \cdot n}{\beta^*} = \frac{\alpha \cdot n(1-\alpha)}{\beta^*(1-\alpha)}$.

$r^*(i) = \frac{\alpha(1-\beta^*)(1+n)}{\beta^*(1-\alpha)} + \frac{\alpha \cdot n(1-\alpha)}{\beta^*(1-\alpha) \cdot i}$: $VA_{r^*(i)} = 0 = -\frac{b}{a}$. $HA_{r^*(i)} = \frac{\alpha(1-\beta^*)(1+n)}{\beta^*(1-\alpha)}$.

$Width_{r^*(i)} = \sqrt{\left|\frac{\alpha \cdot n}{\beta^*}\right|} = \sqrt{\left|\frac{f}{a}\right|}$. $Shape_{r^*(i)} = \sqrt{2\left|\frac{\alpha \cdot n}{\beta^*}\right|}$. $Curvature_{r^*(i)} = 1/\sqrt{2\left|\frac{\alpha \cdot n}{\beta^*}\right|}$.

$\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$. For $r^*(i)$, $(y - 0.0516375)(x + 0) = 0.00168776$.

2-2 $i(r^*)$:

$i(r^*) = \frac{\alpha \cdot n(1-\alpha)}{\beta^*(1-\alpha)r^* - \alpha(1-\beta^*)(1+n)}$.

$\mathrm{y} = \frac{\mathrm{D}}{\mathrm{Ax}+\mathrm{B}}$, where $r^*{}_{VA(i)} = \frac{\mathrm{B}}{\mathrm{A}}$, $r^*{}_{Width(i)} = \sqrt{\left|\frac{F}{A}\right|}$, and $\frac{F}{A} = \frac{\alpha \cdot n}{\beta^*}$.

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$VA_{i(r^*)} = \frac{\alpha(1-\beta^*)(1+n)}{\beta^*(1-\alpha)}$. $HA_{i(r^*)} = 0$.

$Width_{i(r^*)} = \sqrt{\left|\frac{\alpha\cdot n}{\beta^*}\right|}$. $Shape_{i(r^*)} = \sqrt{2\left|\frac{\alpha\cdot n}{\beta^*}\right|}$. $Curvature_{i(r^*)} = 1/\sqrt{2\left|\frac{\alpha\cdot n}{\beta^*}\right|}$.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $i(r^*)$, $(\mathrm{y} - 0)(\mathrm{x} - 0.051647739) = 0.00168776$.

2-3 $r^*(n)$: from hyperbola to linear

$r^*(n) = \frac{\{i\cdot\alpha(1-\beta^*)+\alpha(1-\alpha)\}n+i\cdot\alpha(1-\beta^*)}{i\cdot\beta^*(1-\alpha)}$.

$\mathrm{y} = \frac{\mathrm{C}}{\mathrm{B}}\mathrm{x} + \frac{\mathrm{D}}{\mathrm{B}} = \frac{\mathrm{Cx+D}}{\mathrm{B}}$. $\mathrm{B} = i\cdot\beta^*(1-\alpha)$. $\mathrm{C} = i\cdot\alpha(1-\beta^*) + \alpha(1-\alpha)$.

$\mathrm{D} = i\cdot\alpha(1-\beta^*)$. $\frac{\mathrm{C}}{\mathrm{B}} = \frac{i\cdot\alpha(1-\beta^*)+\alpha(1-\alpha)}{i\cdot\beta^*(1-\alpha)}$. $\frac{\mathrm{D}}{\mathrm{B}} = \frac{\alpha(1-\beta^*)}{\beta^*(1-\alpha)}$.

$r^*(n) = \left(\frac{i\cdot\alpha(1-\beta^*)+\alpha(1-\alpha)}{i\cdot\beta^*(1-\alpha)}\right)n + \frac{\alpha(1-\beta^*)}{\beta^*(1-\alpha)}$.

$Gradient_{r^*(n)} = \frac{\alpha\{i(1-\beta^*)+(1-\alpha)\}}{i\cdot\beta^*(1-\alpha)}$. $Intercept_{r^*(n)} = \frac{\alpha(1-\beta^*)}{\beta^*(1-\alpha)}$.

B=0.064298155. C=0.115721425. D=0.003288235.

For $r^*(n)$: $\mathrm{y} = 1.79976276\mathrm{x} + 0.051140425$, where y=0.0686 when x=0.00972.

2-4 $n(r^*)$:

$n(r^*) = \frac{i\cdot\beta^*(1-\alpha)r^*-i\cdot\alpha(1-\beta^*)}{\alpha(1-\alpha)+i\cdot\alpha(1-\beta^*)}$.

$y = \frac{C}{B}x + \frac{D}{B} = \frac{Cx+D}{B}$. $B = \alpha(1-\alpha) + i\cdot\alpha(1-\beta^*)$. $\mathrm{C} = i\cdot\beta^*(1-\alpha)$.

$D = -i\cdot\alpha(1-\beta^*)$. $\frac{C}{B} = \frac{i\cdot\beta^*(1-\alpha)}{\alpha(1-\alpha)+i\cdot\alpha(1-\beta^*)}$. $\frac{D}{B} = \frac{-i\cdot\alpha(1-\beta^*)}{\alpha(1-\alpha)+i\cdot\alpha(1-\beta^*)}$.

$Gradient_{n(r^*)} = \frac{i\cdot\beta^*(1-\alpha)}{\alpha\{i\cdot(1-\beta^*)+(1-\alpha)\}}$.

$\mathrm{Intercept}_{n(r^*)} = \frac{-i\cdot\alpha(1-\beta^*)}{\alpha\{i\cdot(1-\beta^*)+(1-\alpha)\}}$.

B=0.115721425. C=0.064298155. D=−0.003288235.

For $n(r^*)$: $\mathrm{y} = 0.555628786\mathrm{x} - 0.028415092$, where y=0.00972 when x=0.0686.

3-1 $\Omega^*(i)$:

$\Omega^*(i) = \left(\frac{\beta^*(1-\alpha)\cdot i}{(1-\beta^*)(1+n)\cdot i+n(1-\alpha)}\right)$.

$\mathrm{y} = \frac{\mathrm{cx}}{\mathrm{ax+b}}$. $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{-\frac{\mathrm{b\cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$. $\mathrm{f} = -\frac{\mathrm{b\cdot c}}{\mathrm{a}}$. $\mathrm{f} = \frac{-\beta^*(1-\alpha)n(1-\alpha)}{(1-\beta^*)(1+n)}$.

$VA_{\Omega^*(i)} = \frac{-\mathrm{b}}{\mathrm{a}} = \frac{-n(1-\alpha)}{(1-\beta^*)(1+n)}$. $HA_{\Omega^*(i)} = \beta^*(1-\alpha)/(1-\beta^*)(1+n) = {}^{c}/_{a}$.

$Width_{\Omega^*(i)} = \sqrt{\left|\frac{-\beta^* \cdot n(1-\alpha)^2}{(1-\beta^*)^2(1+n)^2}\right|}$. $Shape_{\Omega^*(i)} = \sqrt{2\left|\frac{-\beta^* \cdot n(1-\alpha)^2}{(1-\beta^*)^2(1+n)^2}\right|}$.

$Curvature_{\Omega^*(i)} = 1/\sqrt{2|\{-\beta^* \cdot n(1-\alpha)^2\}/\{(1-\beta^*)^2(1+n)^2\}|}$.

f=–0.0211633888. f/a=–0.081716.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $\Omega^*(i)$: $(\mathrm{y} - 2.50012)(\mathrm{x} + 0.0327231) = -0.081716$.

3-2 $i(\Omega^*)$:

$i(\Omega^*) = \frac{-n(1-\alpha)\Omega^*}{(1-\beta^*)(1+n)\Omega^* - \beta^*(1-\alpha)}$.

$\mathrm{y} = \frac{\mathrm{Cx}}{\mathrm{Ax+B}}$ and $\mathrm{y} = \frac{\mathrm{C}}{\mathrm{A}} + \frac{-\frac{\mathrm{B \cdot C}}{\mathrm{A}}}{\mathrm{Ax+B}}$, where $i_{HA(\Omega^*)} = -\frac{\mathrm{C}}{\mathrm{A}}$, $\Omega^*_{VA(i)} = \frac{\mathrm{B}}{\mathrm{A}}$, and

$\mathrm{F} = \frac{-\beta^*(1-\alpha) \cdot n(1-\alpha)}{(1-\beta^*)(1+n)} = \frac{-\mathrm{B \cdot C}}{\mathrm{A}}$.

$i = \frac{-n(1-\alpha)}{(1-\beta^*)(1+n)} + \frac{-\beta^*(1-\alpha) \cdot n(1-\alpha)/(1-\beta^*)(1+n)}{(1-\beta^*)(1+n)\Omega^* - \beta^*(1-\alpha)}$.

$VA_{i(\Omega^*)} = \frac{\beta^*(1-\alpha)}{(1-\beta^*)(1+n)} = \frac{-\mathrm{B}}{\mathrm{A}}$. $HA_{i(\Omega^*)} = \frac{-n(1-\alpha)}{(1-\beta^*)(1+n)} = \frac{\mathrm{C}}{\mathrm{A}}$. $Width_{i(\Omega^*)} = \sqrt{\left|\frac{\mathrm{F}}{\mathrm{A}}\right|}$.

$Width_{i(\Omega^*)} = \sqrt{\left|\frac{-\beta^* \cdot n(1-\alpha)^2}{(1-\beta^*)^2(1+n)^2}\right|}$. $Shape_{i(\Omega^*)} = \sqrt{2\left|\frac{-\beta^* \cdot n(1-\alpha)^2}{(1-\beta^*)^2(1+n)^2}\right|}$.

$Curvature_{i(\Omega^*)} = 1/\sqrt{2\left|\frac{-\beta^* \cdot n(1-\alpha)^2}{(1-\beta^*)^2(1+n)^2}\right|}$.

F=–0.0211633888. F/A=–0.081716.

$\left(\mathrm{y} - \frac{\mathrm{C}}{\mathrm{A}}\right)\left(\mathrm{x} + \frac{\mathrm{B}}{\mathrm{A}}\right) = \frac{\mathrm{F}}{\mathrm{A}}$. For $i(\Omega^*)$: $(\mathrm{y} + 0.0327231)(\mathrm{x} - 2.50012) = -0.081716$.

3-3 $\Omega^*(n)$:

$\Omega^*(n) = \frac{\beta^* \cdot i(1-\alpha)}{\{i(1-\beta^*)+(1-\alpha)\}n + i \cdot (1-\beta^*)}$. $\mathrm{y} = \frac{\mathrm{d}}{\mathrm{ax+b}}$, where $\mathrm{c} = 0$ and $\mathrm{e} = 0$.

$\mathrm{a} = i(1-\beta^*) + (1-\alpha)$. $\mathrm{b} = i \cdot (1-\beta^*)$. $\mathrm{d} = \beta^* \cdot i(1-\alpha)$. $\mathrm{f} = \mathrm{d}$.

$\frac{\mathrm{f}}{\mathrm{a}} = \frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}$. $Width_{\Omega^*(n)} = \sqrt{\left|\frac{\mathrm{f}}{\mathrm{a}}\right|}$ $\Omega^*(n = 0) = \frac{\beta^*(1-\alpha)}{(1-\beta^*)}$

$VA_{\Omega^*(n)} = \frac{-i \cdot (1-\beta^*)}{i(1-\beta^*)+(1-\alpha)} = \frac{-b}{a}$. $HA_{\Omega^*(n)} = 0$.

$Width_{\Omega^*(n)} = \sqrt{\left|\frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}\right|}$. $Shape_{\Omega^*(n)} = \sqrt{2\left|\frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}\right|}$.

$Curvature_{\Omega^*(n)} = 1/\sqrt{2\left|\frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}\right|}$.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $\Omega^*(n)$: $(\mathrm{y} - 0)(\mathrm{x} + 0.028415093) = 0.071735923$.

3-4 $\boldsymbol{n(\Omega^*)}$:

$n(\Omega^*)=\frac{-i(1-\beta^*)\Omega^*+\beta^*\cdot i(1-\alpha)}{\{i(1-\beta^*)+(1-\alpha)\}\Omega^*}$: $\quad y=\frac{Cx+D}{Ax}$. $\quad y=\frac{C}{A}+\frac{D}{Ax}$.

$\mathrm{A}=i(1-\beta^*)+(1-\alpha)$. $\;B=0$. $\;C=-i(1-\beta^*)$. $\;D=\beta^*\cdot i(1-\alpha)$.

$\mathrm{E}=\frac{\mathrm{C}}{\mathrm{A}}$. $\;\mathrm{F}=\mathrm{D}$. $\;\frac{\mathrm{F}}{\mathrm{A}}=\frac{\beta^*\cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}$ $\quad Width_{n(\Omega^*)}=\sqrt{\left|\frac{\mathrm{F}}{\mathrm{A}}\right|}$.

$VA_{n(\Omega^*)}=0=\frac{-B}{A}$. $\quad HA_{n(\Omega^*)}=\frac{-i(1-\beta^*)}{i(1-\beta^*)+(1-\alpha)}$.

$Width_{n(\Omega^*)}=\sqrt{\left|\frac{\beta^*\cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}\right|}$. $\;Shape_{n(\Omega^*)}=\sqrt{2\left|\frac{\beta^*\cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}\right|}$

$Curvature_{n(\Omega^*)}=1/\sqrt{2\left|\frac{\beta^*\cdot i(1-\alpha)}{i(1-\beta^*)+(1-\alpha)}\right|}$.

$\left(\mathrm{y}-\frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x}+\frac{\mathrm{b}}{\mathrm{a}}\right)=\frac{\mathrm{f}}{\mathrm{a}}$. $\quad$ For $n(\Omega^*)$: $(\mathrm{y}+0.028415093)(\mathrm{x}+0)=0.071735923$.

4-1 $\boldsymbol{i(n)}$:

$i(n)=\frac{-(1-\alpha)\Omega^*\cdot n}{(1-\beta^*)\Omega^*\cdot n+\beta^*(1-\alpha)-(1-\beta^*)\Omega^*}$.

Here starting with $\beta^*=\frac{(1+n)\Omega^*\cdot i+(1-\alpha)\Omega^*\cdot n}{i\left((1-\alpha)+\Omega^*(1+n)\right)}$ and using $\beta^* i\left((1-\alpha)+\Omega^*(1+n)\right)=$ $(1+n)\Omega^*\cdot i+(1-\alpha)\Omega^*\cdot n$.

$\mathrm{y}=\frac{\mathrm{cx}}{\mathrm{ax+b}}$ and $\mathrm{y}=\frac{\mathrm{c}}{\mathrm{a}}+\frac{-\frac{\mathrm{b\cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$.

$\mathrm{a}=(1-\beta^*)\Omega^*$, $\mathrm{b}=(1-\beta^*)\Omega^*-\beta^*(1-\alpha)$, and $\mathrm{c}=-(1-\alpha)\Omega^*$.

$\mathrm{f}=\frac{-(\beta^*(1-\alpha)-(1-\beta^*)\Omega^*)\cdot(1-\alpha)\Omega^*}{(1-\beta^*)\Omega^*}=\frac{\mathrm{b\cdot c}}{\mathrm{a}}$. $\quad\frac{\mathrm{f}}{\mathrm{a}}=\frac{-(\beta^*(1-\alpha)-(1-\beta^*)\Omega^*)\cdot(1-\alpha)\Omega^*}{((1-\beta^*)\Omega^*)^2}=\frac{\mathrm{b\cdot c}}{\mathrm{a}^2}$.

$VA_{i(n)}=\frac{(1-\beta^*)\Omega^*-\beta^*(1-\alpha)}{(1-\beta^*)\Omega^*}=\frac{-\mathrm{b}}{\mathrm{a}}$. $\;HA_{i(n)}=\frac{-(1-\alpha)}{(1-\beta^*)}=-\frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{i(n)}=\sqrt{\left|\frac{(\beta^*(1-\alpha)-(1-\beta^*)\Omega^*)\cdot(1-\alpha)\Omega^*}{-((1-\beta^*)\Omega^*)^2}\right|}$.

$Shape_{i(n)}=\sqrt{2\left|\frac{(\beta^*(1-\alpha)-(1-\beta^*)\Omega^*)\cdot(1-\alpha)\Omega^*}{-((1-\beta^*)\Omega^*)^2}\right|}$.

$Curvature_{i(n)}=1/\sqrt{2\left|\frac{(\beta^*(1-\alpha)-(1-\beta^*)\Omega^*)\cdot(1-\alpha)\Omega^*}{((1-\beta^*)\Omega^*)^2}\right|}$.

$i(n)=\frac{-1.63825n}{0.48250215n+0.165012}$, where a=0.48250215, b=0.165012, c=−1.63825, b·c=−0.270330909. $\;\mathrm{f}=-0.560268817=\frac{-\mathrm{bc}}{\mathrm{a}}$, and $\frac{\mathrm{f}}{\mathrm{a}}=-1.16117372$.

$VA_{i(n)}=-0.3419923=\frac{-\mathrm{b}}{\mathrm{a}}$. $\quad HA_{i(n)}=-3.395322=\frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{i(n)}=1.0775777=\sqrt{1.16117372}$. $\;Shape_{i(n)}=1.523925=\sqrt{2.32234744}$.

$Curvature_{i(n)} = 0.6562 = 1/\sqrt{2.32234744}$.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $i(n)$: $(y + 3.395322)(x - 0.3419923) = -1.161174$.

4-2 $\boldsymbol{n(i)}$:

$n(i) = \frac{-((1-\beta^*)\Omega^* - \beta^*(1-\alpha))i}{(1-\beta^*)\Omega^* \cdot i + \Omega^*(1-\alpha)}$.

Here starting with $\beta^* = \frac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{i((1-\alpha)+\Omega^*(1+n))}$ and using $\beta^* i\big((1-\alpha) + \Omega^*(1+n)\big) = (1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n$.

$\mathrm{y} = \frac{\mathrm{cx}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{-\frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$.

$\mathrm{a} = (1-\beta^*)\Omega^*$, $\mathrm{b} = \Omega^*(1-\alpha)$, $\mathrm{c} = -((1-\beta^*)\Omega^* - \beta^*(1-\alpha))$, $\frac{\mathrm{f}}{\mathrm{a}} = \frac{-\Omega^*(1-\alpha)\{(1-\beta^*)\Omega^* - \beta^*(1-\alpha)\}}{((1-\beta^*)\Omega^*)^2}$.

$VA_{n(i)} = \frac{-\Omega^*(1-\alpha)}{(1-\beta^*)\Omega^*} = \frac{-b}{a}$. $HA_{n(i)} = \frac{-((1-\beta^*)\Omega^* - \beta^*(1-\alpha))}{(1-\beta^*)\Omega^*} = \frac{c}{a}$.

$Width_{n(i)} = \sqrt{\left|\frac{-\Omega^*(1-\alpha)\{(1-\beta^*)\Omega^* - \beta^*(1-\alpha)\}}{((1-\beta^*)\Omega^*)^2}\right|}$.

$Shape_{n(i)} = \sqrt{2\left|\frac{-\Omega^*(1-\alpha)\{(1-\beta^*)\Omega^* - \beta^*(1-\alpha)\}}{((1-\beta^*)\Omega^*)^2}\right|}$.

$Curvature_{n(i)} = 1/\sqrt{2\left|\frac{-\Omega^*(1-\alpha)\{(1-\beta^*)\Omega^* - \beta^*(1-\alpha)\}}{((1-\beta^*)\Omega^*)^2}\right|}$.

$n(i) = \frac{-0.165012i}{0.48250215i + 1.63825}$, where a=0.48250215, b=1.63825, c=–0.165012, b·c=–0.270330909. $\mathrm{f} = -0.560268817 = \frac{-\mathrm{bc}}{\mathrm{a}}$, and $\frac{\mathrm{f}}{\mathrm{a}} = -1.16117372$.

$VA_{n(i)} = -3.395322 = \frac{-\mathrm{b}}{\mathrm{a}}$. $HA_{n(i)} = -0.341992258 = \frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{n(i)} = 1.0775777 = \sqrt{|-1.16117372|}$. $Shape_{n(i)} = 1.523925 = \sqrt{|-2.32234744|}$.

$Curvature_{n(i)} = 0.6562 = 1/\sqrt{|-2.32234744|}$.

$\left(\mathrm{y} + \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$.

For $n(i)$: $(y - 0.3419923)(x + 3.395322) = -1.161174$.

4-3 $\boldsymbol{\Omega^*(\beta^*)}$:

$\Omega^*(\beta^*) = \frac{-i(1-\alpha)\beta^*}{i(1+n)\beta^* - (i(1+n)+n(1-\alpha))}$, using. $\Omega^* = \frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)(1+n)+n(1-\alpha)}$.

$y = \frac{cx}{ax+b}$ and $y = \frac{c}{a} + \frac{-\frac{b \cdot c}{a}}{ax+b}$. $a = i(1+n)$. $b = -(i(1+n)+n(1-\alpha))$. $c = -i(1-\alpha)$.

$VA_{\Omega^*(\beta^*)} = \frac{i(1+n)+n(1-\alpha)}{i(1+n)}$. $HA_{\Omega^*(\beta^*)} = \frac{-i(1-\alpha)}{i(1+n)}$. $\frac{f}{a} = \frac{-(i(1+n)+n(1-\alpha)) \cdot i(1-\alpha)}{(i(1+n))^2} = \frac{-bc}{a^2}$.

$\Omega^* = \frac{-0.08648037\beta^*}{0.100265196\beta^* - 0.108730344}$. $\frac{f}{a} = -0.935336465$.

$VA_{\Omega^*(\beta^*)} = 1.084427581 = \frac{-b}{a}$. $HA_{\Omega^*(\beta^*)} = -0.862516341 = \frac{c}{a}$.

$Width_{\Omega^*(\beta^*)} = 0.967127946 = \sqrt{|-0.935336465|}$.

$Shape_{\Omega^*(\beta^*)} = 1.367725458 = \sqrt{|-1.87067293|}$.

$Curvature_{\Omega^*(\beta^*)} = 0.731140883 = 1/\sqrt{|1.87067293|}$.

$\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$. For $\Omega^*(\beta^*)$: $(y + 0.862516341)(x - 1.084427581) = -0.935336465$.

4-4 $\boldsymbol{\beta^*(\Omega^*)}$:

$\beta^*(\Omega^*) = \frac{(i(1+n)+n(1-\alpha))\Omega^*}{i(1+n)\Omega^* + i(1-\alpha)}$, using $\beta^* = \frac{\Omega^*(i+i \cdot n+n(1-\alpha))}{i((1-\alpha)+\Omega^*(1+n))}$.

$y = \frac{cx}{ax+b}$ and $y = \frac{c}{a} + \frac{-\frac{b \cdot c}{a}}{ax+b}$. $a = i(1+n)$. $b = i(1-\alpha)$. $c = i(1+n) + n(1-\alpha)$.

$\frac{f}{a} = \frac{i(1-\alpha)(i(1+n)+n(1-\alpha))}{(i(1+n))^2} = \frac{-bc}{a^2}$.

$\beta^* = \frac{0.108730344\Omega^*}{0.100265196\Omega^* + 0.08648037}$. $\frac{f}{a} = -0.935336465$.

$VA_{\beta^*(\Omega^*)} = -0.862516341 = \frac{-b}{a}$. $HA_{\beta^*(\Omega^*)} = 1.084427581 = \frac{c}{a}$.

$Width_{\beta^*(\Omega^*)} = 0.967127946 = \sqrt{|-0.935336465|}$.

$Shape_{\beta^*(\Omega^*)} = 1.367725458 = \sqrt{|-1.87067293|}$.

$Curvature_{\beta^*(\Omega^*)} = 0.731140883 = 1/\sqrt{|-1.87067293|}$.

$\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$ For $\beta^*(\Omega^*)$: $(y - 1.084427581)(x + 0.862516341) = -0.935336465$.

5-1 $\boldsymbol{\beta^*(n)}$:

$\beta^*(n) = \frac{\Omega^* \cdot n(i+(1-\alpha))+\Omega^* \cdot i}{\Omega^* \cdot i \cdot n+i(1-\alpha+\Omega^*)}$ from $\beta^* = \frac{\Omega^*(i+i \cdot n+n(1-\alpha))}{i((1-\alpha)+\Omega^*(1+n))}$.

$y = \frac{cx+d}{ax+b}$ and $y = \frac{c}{a} + \frac{d-\frac{b \cdot c}{a}}{ax+b}$,

where $a = \Omega^* \cdot i$, $b = i(1-\alpha+\Omega^*)$, $c = \Omega^*(i+(1-\alpha))$, $d = \Omega^* \cdot i$,

$e = \frac{\Omega^*(i+(1-\alpha))}{\Omega^* \cdot i}$, and $\frac{f}{a} = \frac{1-\Omega^*(1-\alpha+i)i(1-\alpha+\Omega^*)}{(\Omega^* \cdot i)^2} = \frac{1-bc}{a^2}$,

where the Width as a base of the rectangle equilateral triangle is $\sqrt{\left|\frac{f}{a}\right|}$.

$VA_{\beta^*(n)} = \frac{-b}{a} = \frac{-i(1-\alpha+\Omega^*)}{\Omega^* \cdot i}$. $HA_{\beta^*(n)} = \frac{c}{a} = \frac{\Omega^*(i+(1-\alpha))}{\Omega^* \cdot i}$.

$Width_{\beta^*(n)} = \sqrt{|(\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1+\Omega^*+i))|}$.

$Shape_{\beta^*(n)} = \sqrt{2|(\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1+\Omega^*+i))|}$.

$Curvature_{\beta^*(n)} = 1/\sqrt{2|(\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1+\Omega^*+i))|}$.

$\beta^*(n) = \frac{1.8251n+0.18679}{0.18679n+0.27327}$.

E.g., a=0.18679323=1.8811×0.0993, b=0.2732736=0.0993×2.752, c=1.8251=1.8811×(0.0993+0.8709), d=0.18679323=1.8811×0.0993, where x=n=0.00972,

$f = -2.48328 = 0.18679323 - \frac{0.2732736 \times 1.8251}{0.18679323} = d - \frac{b \cdot c}{a}$.

$\frac{f}{a} = -13.2938285 = \frac{-2.483197165}{0.18679323}$.

c·n+d=0.2045332=1.8251×0.00972+0.18679323 and

a·n+b=0.27508923=0.18679323×0.00972+0.2732736, and thus,

*beta**=y=0.7435=0.2045332÷0.27508923.

−2.483197=0.18679323×(1.8811×0.1291+0.0993×0.1291−(2.8811+0.0993)) when

$f = (\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1+\Omega^*+i))$ is used.

$VA_{\beta^*(n)} = -1.46297379 = \frac{-0.2732736}{0.18679323} = \frac{-b}{a}$. $HA_{\beta^*(n)} = 9.77039275 = \frac{1.8251}{0.18679323} = \frac{c}{a}$.

$Width_{\beta^*(n)} = 3.64607 = \sqrt{|-13.2942732|}$.

$\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$. For $\beta^*(n)$: $(y - 9.77039275)(x + 1.46297379) = -13.2938285$.

5-2 $\boldsymbol{n(\beta^*)}$:

$n(\beta^*) = \frac{-i(1-\alpha+\Omega^*)\beta^*+\Omega^*\cdot i}{\Omega^*\cdot i\cdot\beta^*-\Omega^*(1-\alpha+i)}$ from $n = \frac{\beta^*\cdot i(1-\alpha+\Omega^*)-\Omega^*\cdot i}{\Omega^*(1-\alpha+i-\beta^*\cdot i)}$.

$\mathrm{y} = \frac{\mathrm{Cx+D}}{\mathrm{Ax+B}}$ and $\mathrm{y} = \frac{\mathrm{C}}{\mathrm{A}} + \frac{\mathrm{D}-\frac{\mathrm{B\cdot C}}{\mathrm{A}}}{\mathrm{Ax+B}}$,

where $\mathrm{A} = \Omega^* \cdot i$, $\mathrm{B} = -\Omega^*(1-\alpha+i)$, $\mathrm{C} = -i(1-\alpha+\Omega^*)$, $\mathrm{D} = \Omega^* \cdot i$,

$\frac{\mathrm{C}}{\mathrm{A}} = \frac{-i(1-\alpha+\Omega^*)}{\Omega^*\cdot i}$, and $\frac{\mathrm{F}}{\mathrm{A}} = \frac{-\Omega^*\cdot i-\frac{\Omega^*(1-\alpha+i)i(1-\alpha+\Omega^*)}{\Omega^*\cdot i}}{\Omega^*\cdot i}$, $\quad Width_{n(\beta^*)} = \sqrt{\left|\frac{\mathrm{F}}{\mathrm{A}}\right|}$.

$VA_{n(\beta^*)} = \frac{\Omega^*(1-\alpha+i)}{\Omega^*\cdot i} = \frac{-\mathrm{B}}{\mathrm{A}}$. $\quad HA_{n(\beta^*)} = \frac{-i(1-\alpha+\Omega^*)}{\Omega^*\cdot i} = \frac{\mathrm{C}}{\mathrm{A}}$.

$Width_{n(\beta^*)} = \sqrt{|(\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1 + \Omega^* + i))|}$.

$Shape_{n(\beta^*)} = \sqrt{2|(\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1 + \Omega^* + i))|}$.

$Curvature_{n(\beta^*)} = 1/\sqrt{2|(\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1 + \Omega^* + i))|}$.

$\mathrm{A} = 0.18679323$. $\mathrm{B} = -1.82504322$. $\mathrm{C} = -0.2732736$. $\mathrm{D} = 0.18679323$.
C/A=−1.46297379.

−B/A=9.77039275. $\mathrm{F} = -2.483197165 = 0.18679323 - \frac{0.49873613}{0.18679323} = \mathrm{D} - \frac{\mathrm{BC}}{\mathrm{A}}$.

$\left(\mathrm{y} - \frac{\mathrm{C}}{\mathrm{A}}\right)\left(\mathrm{x} + \frac{\mathrm{B}}{\mathrm{A}}\right) = \frac{\mathrm{F}}{\mathrm{A}}$. For $n(\beta^*)$: $(y + 1.46297379)(x - 9.77039275) = -13.2938285$.

5-3 $\widetilde{\boldsymbol{\beta^*}}(\boldsymbol{n})$:

$\widetilde{\beta^*}(n) = \frac{(\Omega^*\cdot i-\Omega^*(i-(1-\alpha)))\cdot n + i(1-\alpha)}{\Omega^*\cdot i\cdot n+i(1-\alpha+\Omega^*)}$ from $\widetilde{\beta^*}(n) = 1 - \frac{\Omega^*\cdot n(i+(1-\alpha))+\Omega^*\cdot i}{\Omega^*\cdot i\cdot n+i(1-\alpha+\Omega^*)}$,

setting $\widetilde{\beta^*} = 1 - \beta^*$ and starting with $\beta^*(n) = \frac{\Omega^*\cdot n(i+(1-\alpha))+\Omega^*\cdot i}{\Omega^*\cdot i\cdot n+i(1-\alpha+\Omega^*)}$.

$f = i(1-\alpha) - \frac{i(1-\alpha+\Omega^*)(\Omega^* i-\Omega^*(i-(1-\alpha))}{\Omega^* i}$.

And, $f = i(1-\alpha) - \frac{i(1-\alpha+\Omega^*)(\Omega^*(-(1-\alpha))}{\Omega^* i} = (1-\alpha)(i + (1-\alpha+\Omega^*)$.

$\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax+b}}$. $\quad \mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d}-\frac{\mathrm{b\cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$.

$\mathrm{a} = \Omega^* \cdot i$, $\mathrm{b} = i(1-\alpha+\Omega^*)$, $\mathrm{c} = \Omega^* \cdot i - \Omega^*(i + (1-\alpha))$, $\mathrm{d} = i(1-\alpha)$, $\mathrm{e} = \frac{\Omega^*\cdot i-\Omega^*(i+(1-\alpha))}{\Omega^*\cdot i}$, and $\mathrm{f} = i(1-\alpha) - \frac{i(1-\alpha+\Omega^*)\{\Omega^*\cdot i-\Omega^*(i-(1-\alpha))\}}{\Omega^*\cdot i} = \mathrm{d} - \frac{\mathrm{b\cdot c}}{\mathrm{a}}$,

$\mathrm{f} = 2.4832 = 0.8709 \times (0.0993 + 0.8709 + 1.8811)$.

And, using $\mathrm{f} = (1-\alpha)\{i + (1-\alpha+\Omega^*)\}$, $\frac{\mathrm{f}}{\mathrm{a}} = \frac{(1-\alpha)\{i+(1-\alpha+\Omega^*)\}}{\Omega^*\cdot i}$, where $Width_{\widetilde{\beta^*}(n)} = \sqrt{\left|\frac{\mathrm{f}}{\mathrm{a}}\right|}$.

$VA_{\widetilde{\beta^*}(n)} = \frac{-i(1-\alpha+\Omega^*)}{\Omega^* \cdot i} = \frac{-\mathrm{b}}{\mathrm{a}}$. $HA_{\widetilde{\beta^*}(n)} = \frac{\Omega^* \cdot i - \Omega^*(i+(1-\alpha))}{\Omega^* \cdot i} = \frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{\widetilde{\beta^*}(n)} = \sqrt{\left|\frac{(1-\alpha)\{i+(1-\alpha+\Omega^*)\}}{\Omega^* \cdot i}\right|}$. $Shape_{\widetilde{\beta^*}(n)} = \sqrt{2\left|\frac{(1-\alpha)\{i+(1-\alpha+\Omega^*)\}}{\Omega^* \cdot i}\right|}$.

$Curvature_{\widetilde{\beta^*}(n)} = 1/\sqrt{2\left|\frac{(1-\alpha)\{i+(1-\alpha+\Omega^*)\}}{\Omega^* \cdot i}\right|}$.

E.g., a=0.18679323=1.8811×0.0993, b=0.2732736=0.0993×2.752,
c=−1.63825=0.18679323−1.8811×(0.0993+0.8709), d=0.08648=0.0993×0.8709, where
x=*n*=0.00972, then,
c·*n*+d=0.070505=−1.63825×0.00972+0.08648 and
a·*n*+b=0.27508923=0.18679323×0.00972+0.2732736, and thus,
$\widetilde{\beta^*}$=0.2565=0.070556÷0.27508923.

$\mathrm{f} = \left(\mathrm{d} - \frac{\mathrm{bc}}{\mathrm{a}}\right) = 2.48319681 = 0.08648 - \frac{0.2732736 \times -1.63825}{0.18679323}$. $\frac{\mathrm{f}}{\mathrm{a}} = 13.2938266 = \frac{2.48319681}{0.18679323}$

And, using $\mathrm{f} = (1-\alpha)\{i + (1-\alpha+\Omega^*)\}$, $\frac{\mathrm{f}}{\mathrm{a}} = 13.26585428 = \frac{2.47797177}{0.18679323}$ (no error in the Excel).

$\widetilde{\beta^*} = 0.2565 = \frac{-1.63825}{0.18679323} + \frac{2.48319681}{0.27508923} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d} - \frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$.

$VA_{\widetilde{\beta^*}(n)} = -1.46297379 = \frac{-0.2732736}{0.18679323} = \frac{-\mathrm{b}}{\mathrm{a}}$. $HA_{\widetilde{\beta^*}(n)} = -8.77039 = \frac{-1.63825}{0.18679323} = \frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{\widetilde{\beta^*}(n)} = 3.64607 = \sqrt{13.2938266}$.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $\widetilde{\beta^*}(n)$: $(y + 8.77039)(x + 1.46297379) = 13.2938266$.

5-4 $\boldsymbol{n(\widetilde{\beta^*})}$:

$n(\widetilde{\beta^*}) = \frac{\{i(1-\alpha+\Omega^*)\}(1-\beta^*) - i(1-\alpha)}{\Omega^* \cdot i(1-\beta^*) + \Omega^*(1-\alpha)}$.

Here starting with $\beta^*(n) = \frac{\Omega^* \cdot n(i+(1-\alpha)) + \Omega^* \cdot i}{\Omega^* \cdot i \cdot n + i(1-\alpha+\Omega^*)}$ and using $n(\beta^*) = \frac{i(1-\alpha+\Omega^*)\beta^* - \Omega^* \cdot i}{-\Omega^* \cdot i \cdot \beta^* + \Omega^*(1-\alpha+i)}$, where

$n(\widetilde{\beta^*}) = \frac{\Omega^* \cdot i(1-\beta^*) + \Omega^*(1-\alpha) - i(1-\alpha+\Omega^*)\beta^*}{-\Omega^* \cdot i \cdot \beta^* + \Omega^*(1-\alpha+i)} = 1 - \frac{i(1-\alpha+\Omega^*)\beta^* - \Omega^* \cdot i}{-\Omega^* \cdot i \cdot \beta^* + \Omega^*(1-\alpha+i)}$.

$\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d} - \frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$.

Appendices

$a = \Omega^* \cdot i$. $b = \Omega^*(1-\alpha)$. $c = i(1-\alpha+\Omega^*)$. $d = -i(1-\alpha)$.

a=0.18679323. b=1.63825. c=0.2732736=0.0993×2.752. d=−0.08648.

c/a=1.46297379. −b/a=−8.7703928. $f = -2.48319681 = -0.08648 - \frac{0.447690475}{0.18679323} = d - \frac{bc}{a}$. f/a=−13.2938266.

$\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$. For $n(\widetilde{\beta^*})$: $(y - 1.46297379)(x + 8.77039) = -13.2938266$.

Special cases: $\boldsymbol{\beta^*}$ versus $\widetilde{\boldsymbol{\beta^*}} = \mathbf{1} - \boldsymbol{\beta^*}$, and $\boldsymbol{\alpha}$ versus $\widetilde{\boldsymbol{\alpha}} = \mathbf{1} - \boldsymbol{\alpha}$

6-1 $\boldsymbol{\beta^*(i)}$:

$\beta^*(i) = \frac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{\{(1-\alpha)+\Omega^*(1+n)\}i}$.

Here using $\beta^* = \frac{\Omega^*(i + i\cdot n + n(1-\alpha))}{i((1-\alpha)+\Omega^*(1+n))}$.

$y = \frac{cx+d}{ax}$ and $y = \frac{c}{a} + \frac{d}{ax}$.

$a = (1-\alpha) + \Omega^*(1+n)$. b=0. $c = (1+n)\Omega^*$. $d = (1-\alpha)\Omega^* \cdot n$. $\frac{f}{a} = \frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha)+\Omega^*(1+n)}$.

$VA_{\beta^*(i)} = 0 = \frac{-b}{a}$. $HA_{\beta^*(i)} = \frac{(1+n)\Omega^*}{(1-\alpha)+\Omega^*(1+n)} = \frac{c}{a}$.

a=2.7702843=0.8709+1.8811×1.00972, b=0 c=1.899384292,

f=d=0.015923789=0.8709×1.8811×0.00972. f/a=0.00574807=0.015923789÷2.7702843.

$VA_{\beta^*(i)} = 0 = \frac{-b}{a}$. $HA_{\beta^*(i)} = 0.68562793 = \frac{c}{a}$. $Width_{\beta^*(i)} = \sqrt{\frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha)+\Omega^*(1+n)}}$.

$Shape_{\beta^*(i)} = \sqrt{2\frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha)+\Omega^*(1+n)}}$. $Curvature_{\beta^*(i)} = 1/\sqrt{2\frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha)+\Omega^*(1+n)}}$.

$\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$. For $\beta^*(i)$: $(y - 0.68562793)(x + 0) = 0.00574807$.

6-2 $\widetilde{\beta^*}(i)$:

$\widetilde{\beta^*}(i) = \frac{(1-\alpha)i-(1-\alpha)\Omega^*\cdot n}{((1-\alpha)+\Omega^*(1+n))i}$, where $\widetilde{\beta^*} = 1-\beta^*$.

Here starting with $\beta^*(i) = \frac{(1+n)\Omega^*\cdot i+(1-\alpha)\Omega^*\cdot n}{\{(1-\alpha)+\Omega^*(1+n)\}i}$ and, using

$\widetilde{\beta^*}(i) = \frac{((1-\alpha)+\Omega^*(1+n))i-(1+n)\Omega^*\cdot i-(1-\alpha)\Omega^*\cdot n}{((1-\alpha)+\Omega^*(1+n))i} = 1 - \frac{(1+n)\Omega^*\cdot i+(1-\alpha)\Omega^*\cdot n}{((1-\alpha)+\Omega^*(1+n))i}$.

$\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d}}{\mathrm{ax}}$, where f=d.

$VA_{\widetilde{\beta^*}(i)} = 0 = \frac{-\mathrm{b}}{\mathrm{a}}$. $HA_{\widetilde{\beta^*}(i)} = \frac{1-\alpha}{(1-\alpha)+\Omega^*(1+n)} = \frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{\widetilde{\beta^*}(i)} = \sqrt{\left|\frac{-(1-\alpha)\Omega^*\cdot n}{(1-\alpha)+\Omega^*(1+n)}\right|}$. $Shape_{\widetilde{\beta^*}(i)} = \sqrt{2\left|\frac{-(1-\alpha)\Omega^*\cdot n}{(1-\alpha)+\Omega^*(1+n)}\right|}$.

$Curvature_{\widetilde{\beta^*}(i)} = 1/\sqrt{2\left|\frac{-(1-\alpha)\Omega^*\cdot n}{(1-\alpha)+\Omega^*(1+n)}\right|}$.

a=2.7702843=0.8709+1.8811×1.00972, b=0 c=0.8709,
f=d=−0.015923789=−0.8709×1.8811×0.00972. c/a=0.31432938.
f/a=−0.00574807=0.015923789÷2.7702843.

$Width_{\widetilde{\beta^*}(n)} = 0.0758160 = \sqrt{\left|\frac{\mathrm{f}}{\mathrm{a}}\right|}$.

$\left(\mathrm{y}-\frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x}+\frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $\widetilde{\beta^*}(i)$: $(y-0.31432938)(x+0) = -0.00574807$.

6-3 $\boldsymbol{\alpha}(\boldsymbol{i})$:

$\alpha(i) = \frac{\{(1+n)\Omega^*-\beta^*-\beta^*\Omega^*(1+n)\}i+\Omega^*\cdot n}{-\beta^*\cdot i+\Omega^*\cdot n}$, starting with $\beta^* = \frac{(1+n)\Omega^*\cdot i+(1-\alpha)\Omega^*\cdot n}{i((1-\alpha)+\Omega^*(1+n))}$.

$\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d}-\frac{\mathrm{b\cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$.

$\mathrm{a} = \beta^*$, $\mathrm{b} = -\Omega^*\cdot n$, $\mathrm{c} = -\{(1+n)(1-\beta^*)\Omega^*-\beta^*\}$, $\mathrm{d} = -\Omega^*\cdot n$.

$\mathrm{f} = -\Omega^*\cdot n - \frac{\Omega^*\cdot n\{(1+n)(1-\beta^*)\Omega^*-\beta^*\}}{\beta^*} = -\left(\mathrm{d}+\frac{\mathrm{b\cdot c}}{\mathrm{a}}\right)$. $\frac{\mathrm{f}}{\mathrm{a}} = \frac{-\Omega^*\cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{\beta^{*2}}$.

$VA_{\alpha(i)} = \frac{\Omega^*\cdot n}{\beta^*} = \frac{-\mathrm{b}}{\mathrm{a}}$. $HA_{\alpha(i)} = \frac{-\{(1+n)(1-\beta^*)\Omega^*-\beta^*\}}{\beta^*} = \frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{\alpha(i)} = \sqrt{\left|\frac{-\Omega^*\cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{\beta^{*2}}\right|}$. $Shape_{\alpha(i)} = \sqrt{2\left|\frac{-\Omega^*\cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{\beta^{*2}}\right|}$.

$Curvature_{\alpha(i)} = 1/\sqrt{2\left|\frac{-\Omega^*\cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{\beta^{*2}}\right|}$.

$\alpha(i) = \frac{-0.25630793i-0.018284292}{0.7435i-0.018284292}$.

$\mathrm{a} = 0.7435 = \beta^*$. $\mathrm{b} = -0.018284292$. $\mathrm{c} = -0.25630793$. $\mathrm{d} = -0.018284292$.

Appendices

$\mathrm{f} = -0.011981118 = -0.01828429 - \frac{-0.004686409}{0.7435}.$

$\frac{\mathrm{f}}{\mathrm{a}} = -0.0161145 = \frac{-0.011981118}{0.7435}.$

$VA_{\alpha(i)} = 0.024592 = \frac{-\mathrm{b}}{\mathrm{a}}$ $HA_{\alpha(i)} = 0.34473158 = \frac{-0.25630793}{-0.7435} = \frac{\mathrm{c}}{\mathrm{a}}.$

$Width_{\alpha(i)} = 0.126942928 = \sqrt{|-0.0161145|}$. $Shape_{\alpha(i)} = 0.17952437 =$

$\sqrt{2|-0.0161145|}.$

$Curvature_{\alpha(i)} = 5.570274386 = 1/\sqrt{2|-0.0161145|}.$

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}.$ For $\alpha(i)$: $(y - 0.34473158)(x - 0.024592) = -0.016114484.$

6-3-2 $\alpha(n)$, newly added, to cope with stop-macro inequality:

$\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d} - \frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$; the same as $\alpha(n)$.

$\mathrm{a} = -\Omega^*,\ \mathrm{b} = -\beta^* \cdot i,\ \mathrm{c} = -\Omega^*(1 - i(1-\beta^*)),\ \mathrm{d} = i \cdot \Omega^* - \beta^* \cdot i(1+\Omega^*).$

6-4 $\tilde{\alpha}(i)$:

$\tilde{\boldsymbol{\alpha}}(i) = \frac{(1+n)(1-\beta^*)\Omega^* \cdot i}{\beta^* \cdot i - \Omega^* \cdot n},$

setting $\tilde{\alpha} = 1 - \alpha$, and starting with $\beta^* = \frac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{i\left((1-\alpha) + \Omega^*(1+n)\right)}.$

$\mathrm{y} = \frac{\mathrm{cx}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{-\frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}.$

$\mathrm{a} = \beta^*,\ \mathrm{b} = -\Omega^* \cdot n,\ \mathrm{c} = (1+n)(1-\beta^*)\Omega^*,\ \frac{\mathrm{f}}{\mathrm{a}} = \frac{\Omega^* \cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{(\beta^*)^2}.$

$VA_{\tilde{\alpha}(i)} = \frac{\Omega^* \cdot n}{\beta^*} = \frac{-\mathrm{b}}{\mathrm{a}}.$ $HA_{\tilde{\boldsymbol{\alpha}}(i)} = \frac{(1+n)(1-\beta^*)\Omega^*}{\beta^*} = \frac{\mathrm{c}}{\mathrm{a}}.$

$Width_{\tilde{\boldsymbol{\alpha}}(i)} = \sqrt{\left|\frac{\Omega^* \cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{(\beta^*)^2}\right|}.$ $Shape_{\tilde{\boldsymbol{\alpha}}(i)} = \sqrt{2\left|\frac{\Omega^* \cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{(\beta^*)^2}\right|}.$

$Curvature_{\tilde{\boldsymbol{\alpha}}(i)} = 1/\sqrt{2\left|\frac{\Omega^* \cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{(\beta^*)^2}\right|}.$

$\tilde{\alpha}(i) = \frac{0.48719207i}{0.7435i - 0.018284292}.$

a=0.7435. $\mathrm{b} = -0.018284292 = -\Omega^* \cdot n.$ $\mathrm{c} = 0.48719207 = (1+n)(1-\beta^*)\Omega^*.$

$\mathrm{f} = 0.011981116 = \frac{0.00890796}{0.7435} = \frac{\Omega^* \cdot n\{(1+n)(1-\beta^*)\Omega^*\}}{\beta^*}.$

$\frac{\mathrm{f}}{\mathrm{a}} = 0.016114484 = \frac{0.00890796}{0.7435^2} = \frac{-\mathrm{bc}}{\mathrm{a}^2}$.

$VA_{\tilde{\alpha}(i)} = 0.024592 = \frac{-\mathrm{b}}{\mathrm{a}}$. $HA_{\tilde{\alpha}(i)} = 0.655268338 = \frac{0.48719201}{0.7435} = \frac{\mathrm{c}}{\mathrm{a}}$.

$Width_{\tilde{\alpha}(i)} = 0.12694284 = \sqrt{0.016114484}$. $Shape_{\tilde{\alpha}(i)} = 0.179524282 = \sqrt{2 \times 0.016114484}$.

$Curvature_{\tilde{\alpha}(i)} = 5.570277117 = 1/\sqrt{2 \times 0.016114484}$.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $\tilde{\alpha}(i)$: $(y - 0.655268338)(x - 0.024592) = 0.016114484$.

Philosophy Hidden in 'Circle + Hyperbola' and Implication of Ellipse: from the Viewpoint of KEWT Database 6.12 & 7.13

Let us (with readers) geometrically consider the essence of the KEWT database as a purely endogenous system. The author presents the philosophy of hyperbola properly but related to universal nature. The philosophy springs out from the KEWT database (for the comparison of Kamiryo Endogenous World Table (KEWT) with the current representative several databases in the literature, see Chapter 6).

The KEWT database stays at two dimensions or Plane. Circle is most fitted for Plane while ellipse is drawn at any dimensional above two. The literature prefers linear vs. non-linear rather than circle vs. ellipse. This is because no economist uses the circle possibly related to hyperbola and non-linear. Mathematically, circle is related to *sin* and *cosin* and also exponent, e^x. Also e^x is lucky to be familiar with imaginary numbers that prevail everywhere academically today. Nevertheless hyperbola, most clearly compared with the case of ellipse, expresses philosophy of Positive and Negative Principle discovered thousands of years ago in old China, with a zero point.

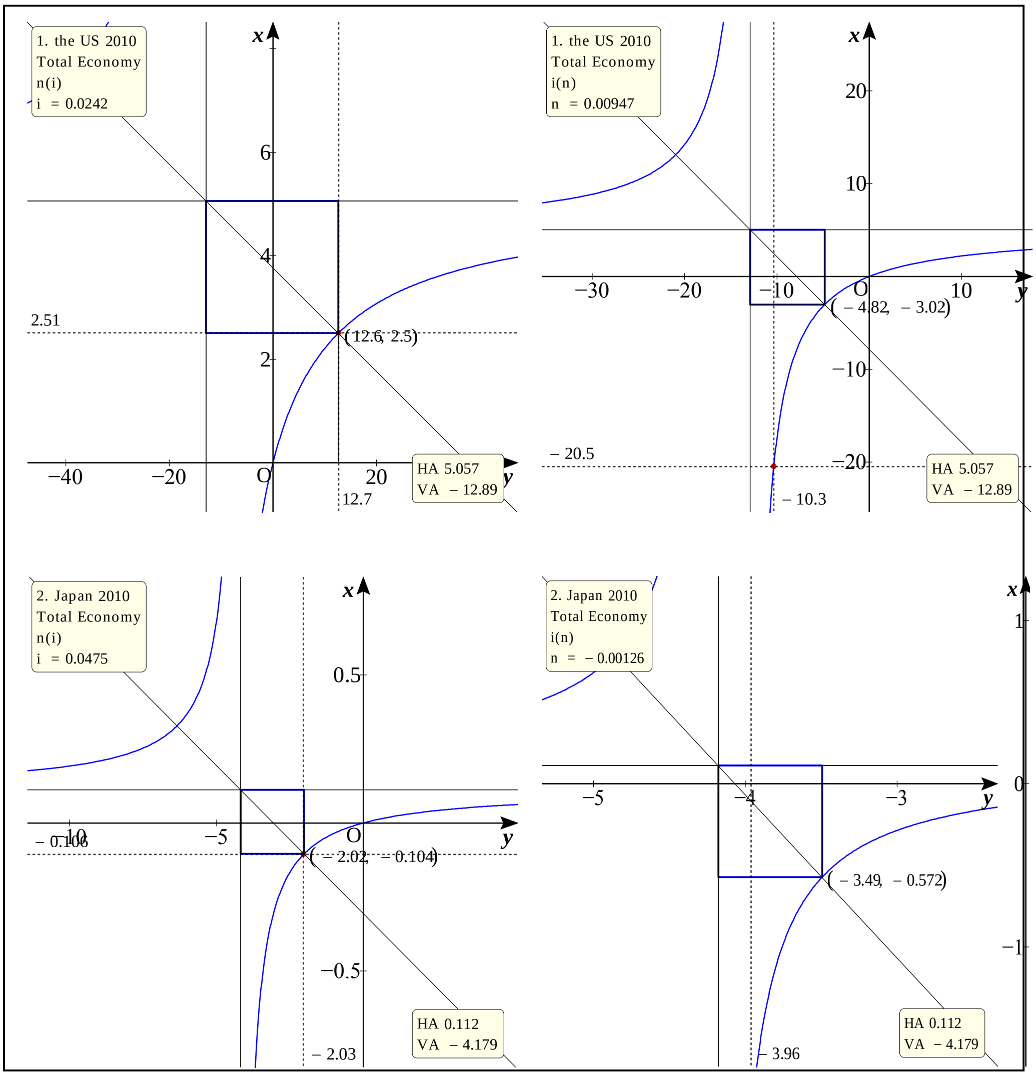
Why does KEWT database stick to two-dimensional Plane? The KEWT database realizes causes and effects simultaneously by discrete-year and under no assumption. All the parameters and variables are measured accurately with seven primary endogenous parameters which determine the speed years by country, sector, year, and over years; beyond space and time. Also, endogenous equations are each reduced to hyperbola in the KEWT database. Even the rate of inflation or deflation is measured using the rate of return hyperbola function to the ratio of net investment to disposable endogenous income. This endogenous fact shows that Plane implicitly includes space and time as one dimension. Iyonoishi (2012) proves theoretically and empirically (using familiar goods such as *Japanese Sudare/bamboo blind* and *banana's rind*) the existence of 'elusive Higgs bosom' (see Chapter 10). Her discovery implies that the real world expresses six dimensions regardless of number of dimensions, i.e., even in two-dimensional Plane.

Appendices

Appendix C. Hyperbola graphs

Appendix C. shows hyperbola graphs each by each by country, using KEWT 6.12, 1990-2010. Each graph shows endogenous results of 2010. Yet, each is suggestive in the current situation, broadly covering from the past to the future, similarly to Graphic Dynamics (GD) by sector simulated in Chapter 8.

BOX C-1 No difference between i(n) and n(i): examples 2010



Appendix C. shows 13 hyperbola cases of 12 selected countries, each case on two pages, after **Table C1** as follows:

C1: Speed (i), speed (n), and Omega(beta); Totally six pages

C2: n(i), beta*(i), r*(i), Omega(i), and alpha(i); Totally ten pages

C3: i(n), beta*(n), r*(n), Omega(n), and alpha(n); Totally ten pages

Table C1 Data needed for elements *a b c d* of hyperbolas

TOTAL	i=I/Y	n	α	Ω	β^*	δ_0	gradient=c/b; intercept=d/b		
1. US	0.0242	0.00947	0.2081	1.9974	0.9386	0.7462	r*(n)	9.18669	0.01721
2. Japan	0.0475	(0.00126)	0.0962	3.6885	0.7837	(0.0138)	reduces to	2.61417	0.02936
3. Australia	0.1428	0.01033	0.0925	1.9613	0.7305	0.3244	linear.	0.92455	0.03761
4. France	0.0642	0.00481	0.1255	1.6336	0.6950	0.4040	Others	2.87523	0.06296
5. Germany	0.0333	(0.00134)	0.0924	1.6231	0.6177	(0.0097)	make all	4.54981	0.06303
6. the UK	0.0340	0.00536	0.1729	1.4073	0.7128	0.6242	hyperbolas.	7.21029	0.08419
7. China	0.5341	0.00617	0.5428	3.1712	0.8793	0.4187		1.31879	0.16303
8. Inidia	0.2163	0.01374	0.1953	1.6014	0.7023	0.4513		1.38819	0.10285
9. Brazil	0.2033	0.00872	0.1461	1.6668	0.6873	0.3511		1.12319	0.07783
10. Mexico	0.2334	0.00949	0.1942	1.9201	0.7293	0.3417		1.23056	0.08949
11. Russia	0.1364	(0.00355)	0.2545	0.8114	0.5101	6.1484		3.98600	0.32785
12. S.Afric	0.1518	0.00758	0.0971	1.3871	0.6347	0.4078		1.06971	0.06190
G	$i_G=I_G/Y_G$	n_G	α_G	$\Omega_G=K_G/Y_G$	β^*_G	δ_{0G}	gradient=c/b; intercept=d/b		
1. US	0.5966	0.0095	0.1734	2.7319	0.7794	0.2037	r*(n)	0.4323	0.05938
2. Japan	0.3202	(0.0013)	(0.2739)	7.2225	0.8456	(0.1625)	reduces to	(1.0506)	(0.03924)
3. Australia	0.1656	0.0103	0.0224	0.9693	0.5307	1.2535	linear.	0.2749	0.02024
4. France	0.1571	0.0048	(0.1315)	1.1962	0.5329	(0.3611)	Others	(1.6728)	(0.10190)
5. Germany	0.0874	(0.0013)	(0.1172)	1.1430	0.4967	11.1867	make all	(2.8064)	(0.10626)
6. the UK	0.0458	0.0054	(0.5415)	2.3285	0.7112	0.0623	hyperbolas.	(16.777)	(0.14263)
7. China	0.3328	0.0062	0.2364	1.8028	0.7136	0.3546		1.1194	0.12421
8. Inidia	0.4692	0.0137	0.2079	3.2909	0.8266	0.2373		0.5912	0.05506
9. Brazil	0.1784	0.0087	0.1614	2.0024	0.7354	0.3206		1.2998	0.06926
10. Mexico	0.4488	0.0095	0.3037	3.3115	0.8397	0.2769		0.8894	0.08329
11. Russia	0.2932	(0.0035)	0.0836	0.9738	0.5086	1.7727		0.6485	0.08812
12. S. Afric	0.0791	0.0076	(0.1614)	1.1370	0.5515	0.3788		(3.8140)	(0.11300)
PRI	$i_{PRI}=I_{PRI}/Y_{PR}$	n_{PRI}	α_{PRI}	$\Omega_{PRI}=K_{PRI}/Y_I$	β^*_{PRI}	δ_{0PRI}	gradient=c/b; intercept=d/b		
1. US	(0.1517)	0.00947	0.2188	1.7718	0.6624	0.1512	r*(n)	(2.0356)	0.14278
2. Japan	(0.0132)	(0.00126)	0.1785	2.9022	0.8402	0.3580	reduces to	(16.035)	0.04133
3. Australia	0.1360	0.01033	0.1135	2.2575	0.7681	0.3202	linear.	1.1249	0.03863
4. France	0.0332	0.00481	0.2111	1.7793	0.7728	0.5293	Others	8.3046	0.07869
5. Germany	0.0198	(0.00134)	0.1448	1.7432	0.6318	(0.0288)	make all	11.6462	0.09867
6. the UK	0.0317	0.00536	0.3140	1.2253	0.7164	0.7807	hyperbolas.	14.0049	0.18122
7. China	0.5768	0.00617	0.6078	3.4615	0.9025	0.4421		1.3348	0.16735
8. Inidia	0.1627	0.01374	0.1926	1.2430	0.6505	0.6497		1.9481	0.12815
9. Brazil	0.2119	0.00872	0.1408	1.5520	0.6683	0.3725		1.0760	0.08135
10. Mexico	0.1877	0.00949	0.1710	1.6249	0.6919	0.3998		1.4085	0.09188
11. Russia	0.0895	(0.00355)	0.3056	0.7628	0.5082	9.2943		7.1422	0.42595
12. S. Afric	0.1700	0.00758	0.1617	1.4496	0.6589	0.4362		1.5436	0.09987

Notes:

1. Unit by axis do not change VA and HA.
2. beta and 1-beta are shown by using beta 1-beta is expressed in each graph; similarly, alpha and 1-alpha.
3. *r**(*n*) exceptionally reduces to linear due to a=0, where gradient=c/d and intercept=d/b.

Appendices

C1: Speed (i), speed (n), and Omega(beta)

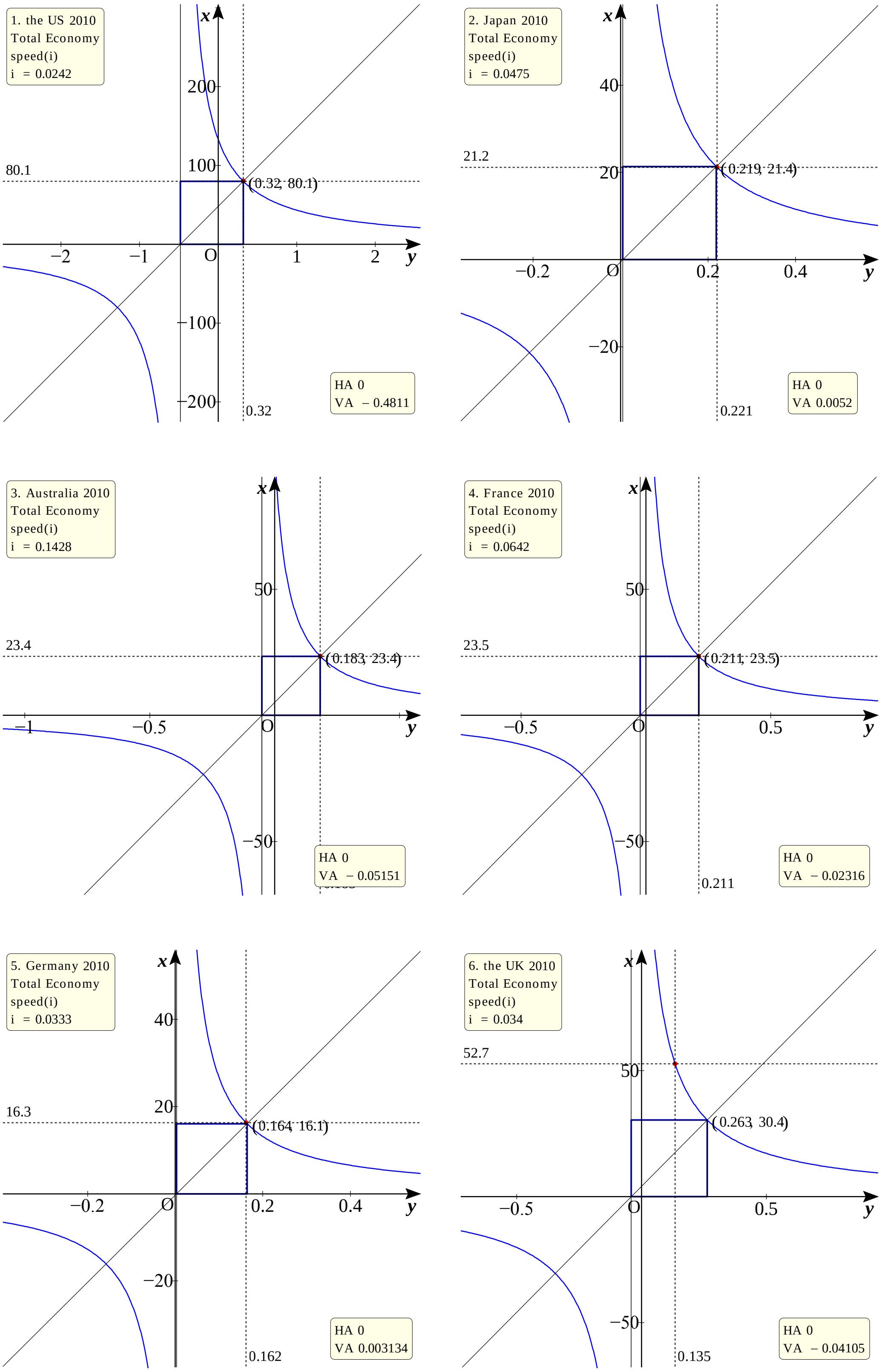


Figure 1-1 speed(*i*) by country, 2010

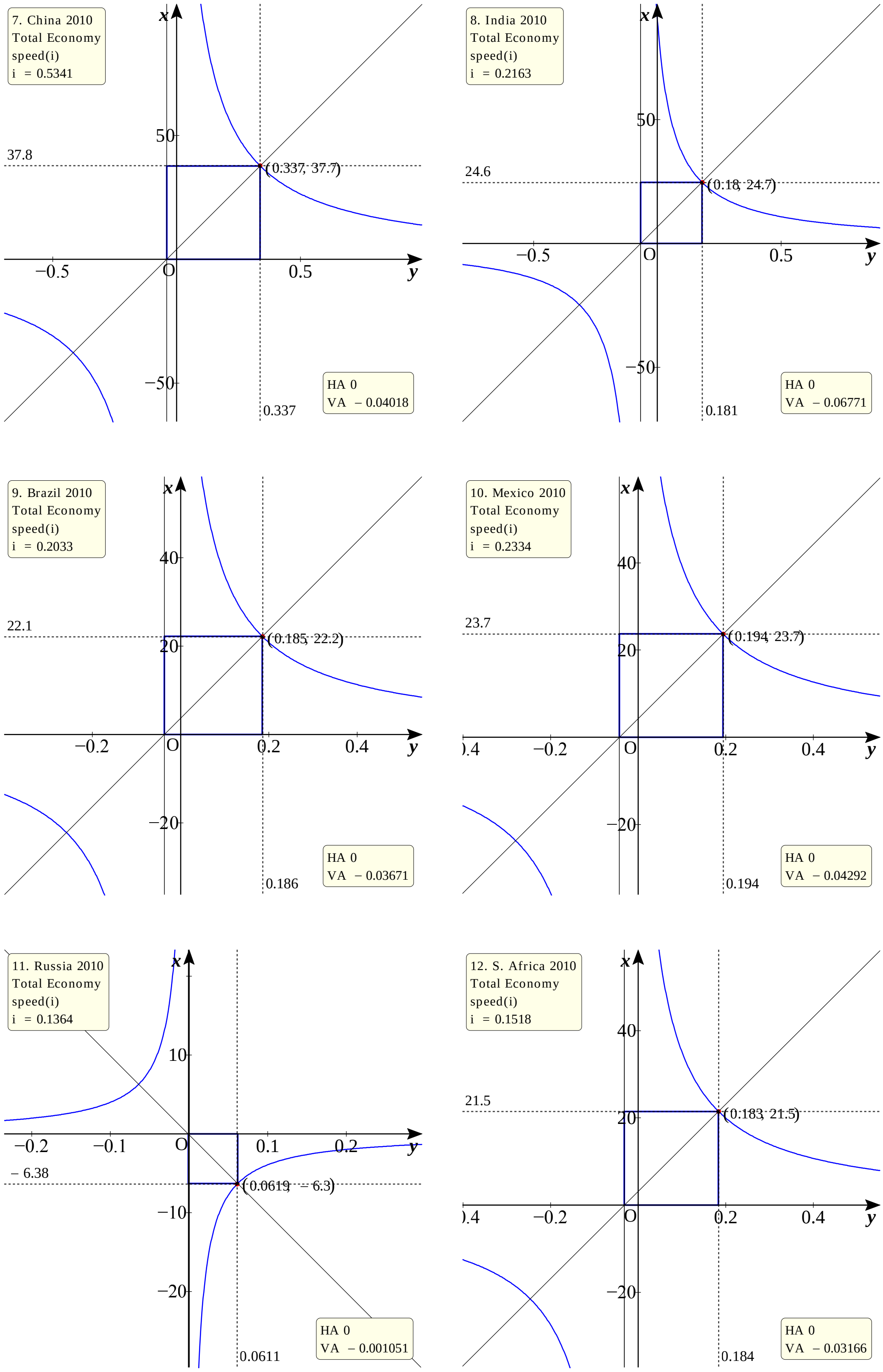


Figure 1-2 speed(i) by country, 2010

Appendices

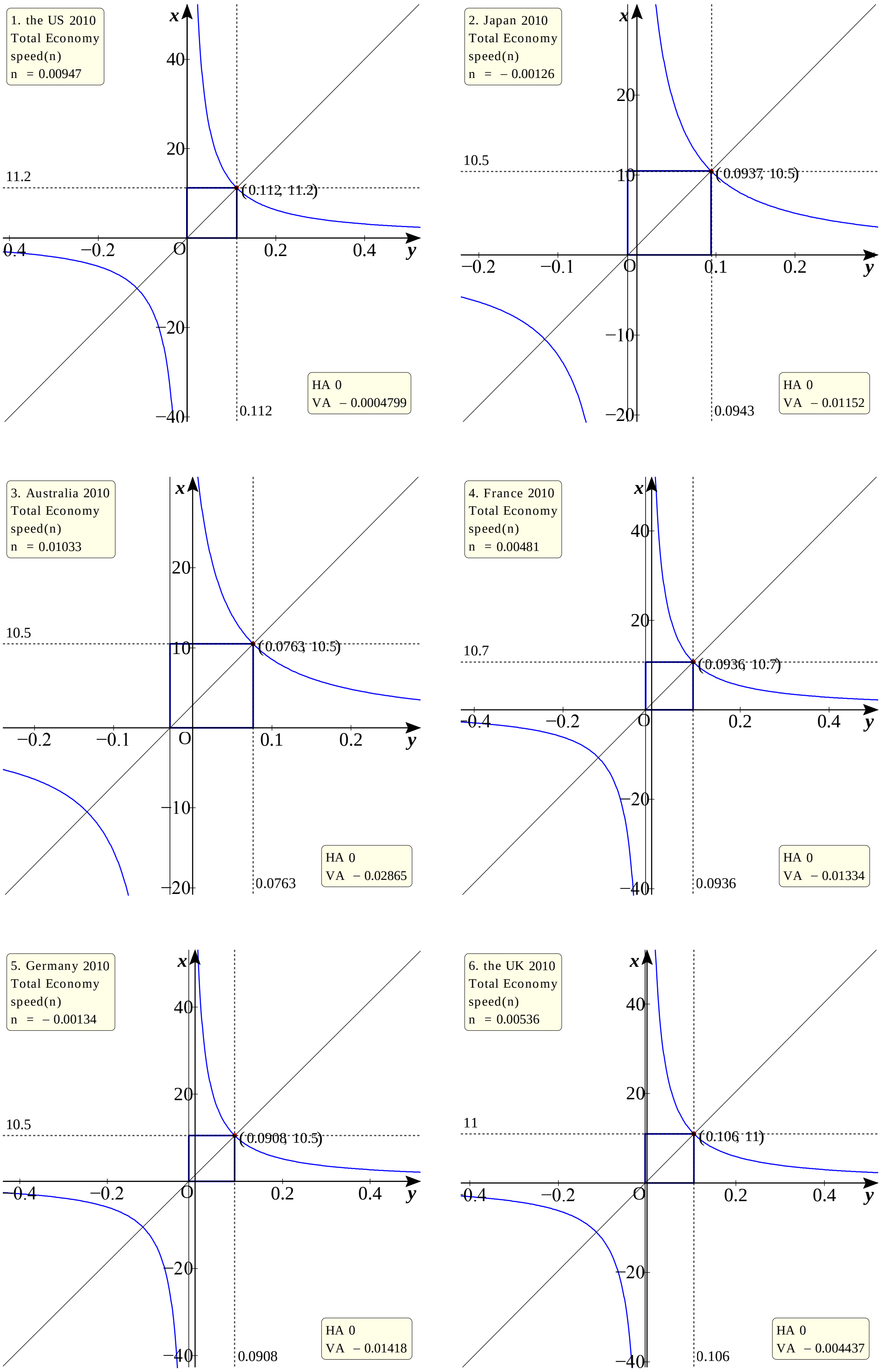


Figure 2-1 speed(n) by country, 2010

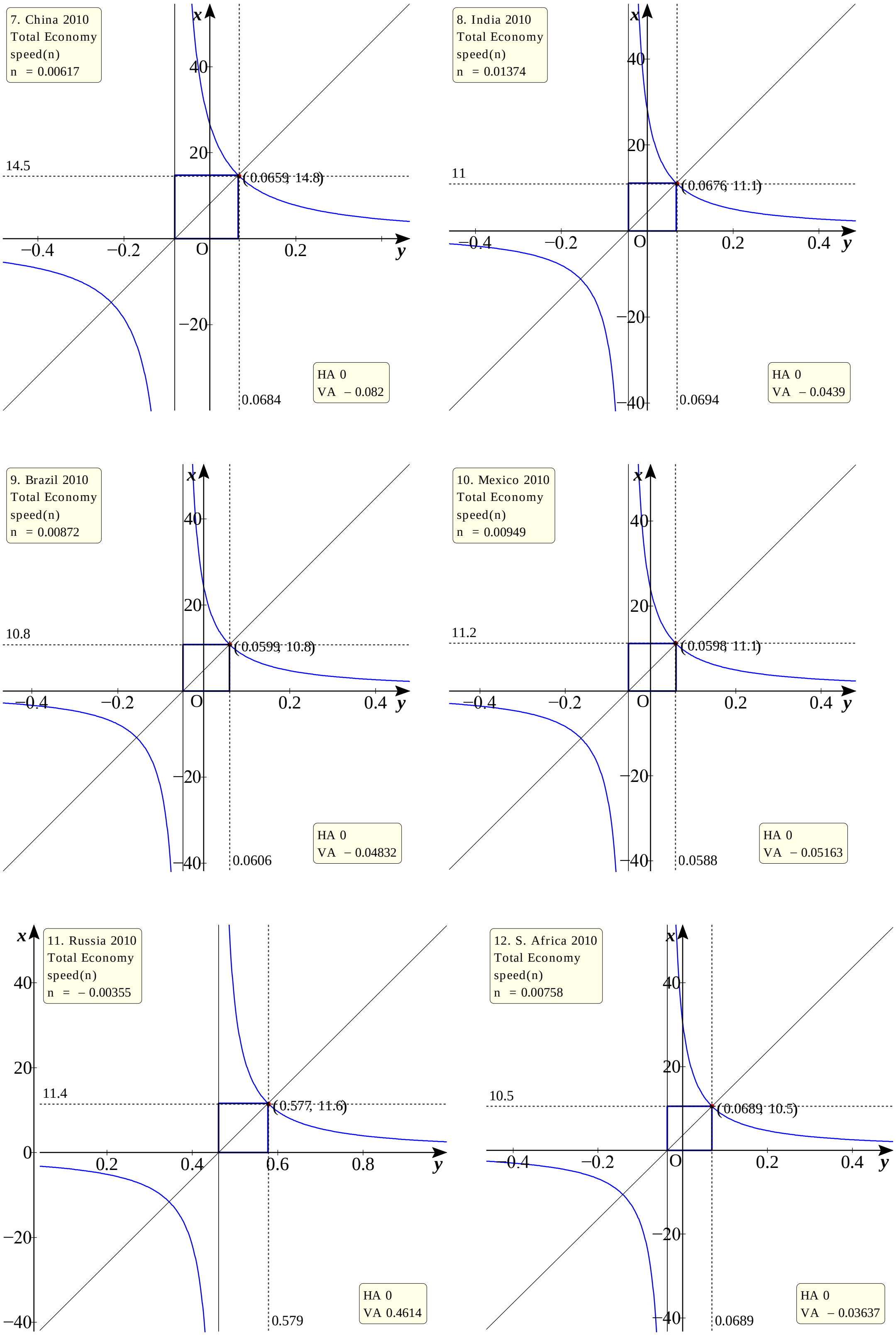


Figure 2-2 speed(n) by country, 2010

Appendices

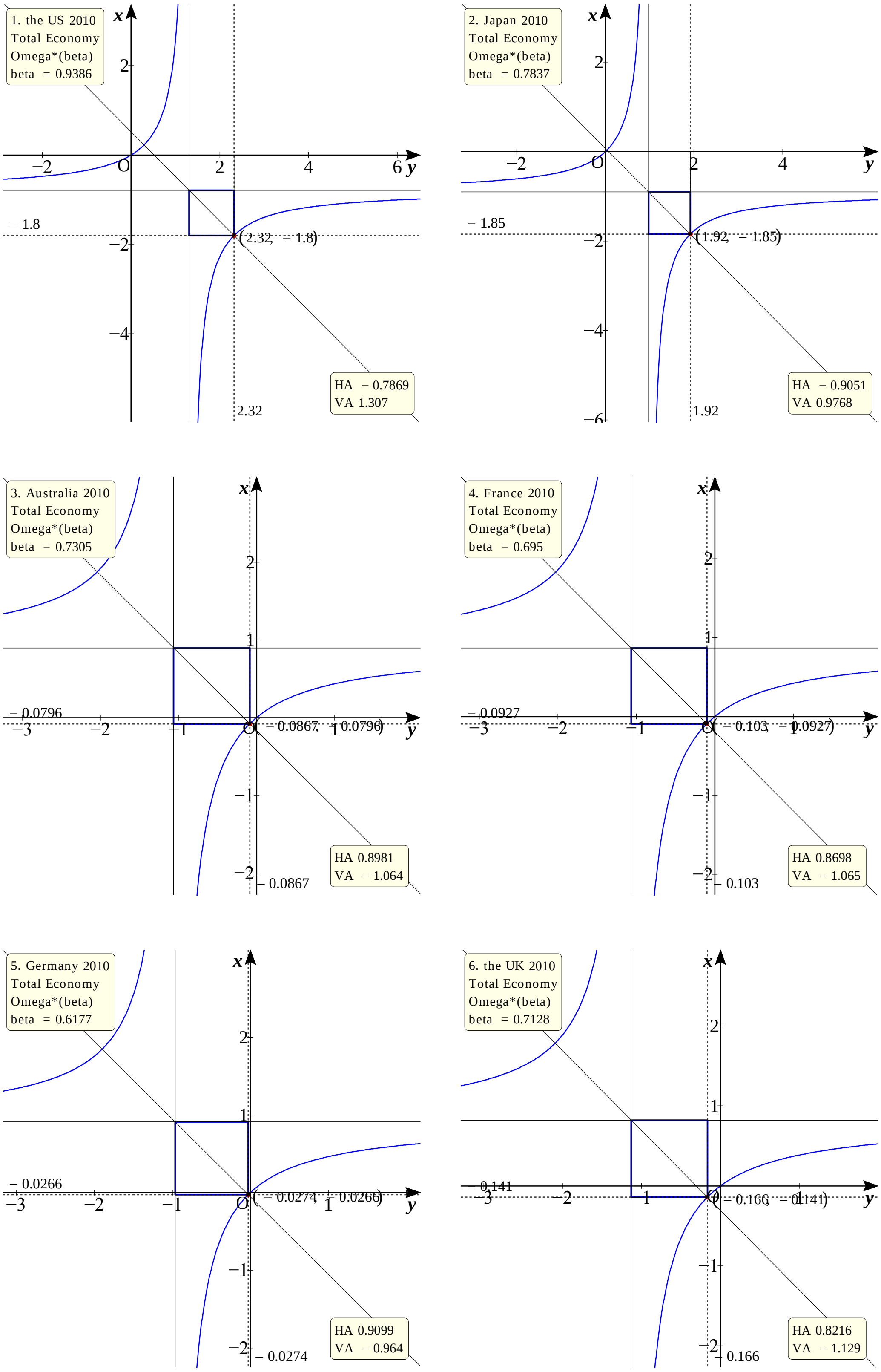


Figure 3-1 Omega(beta*) by country, 2010

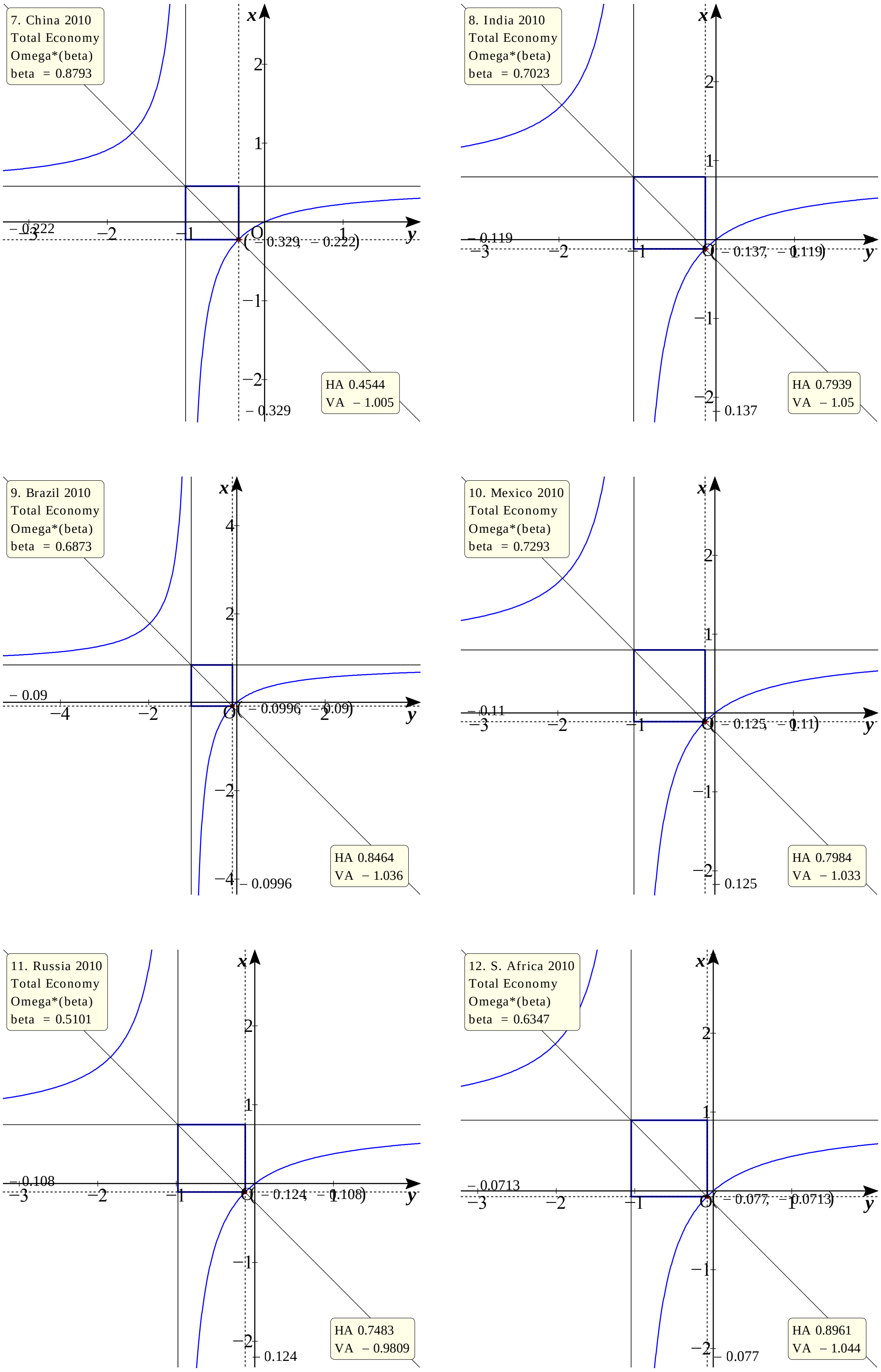


Figure 3-2 Omega(beta*) by country, 2010

Appendices

C2: n(i), beta*(i), r*(i), Omega(i), and alpha(i)

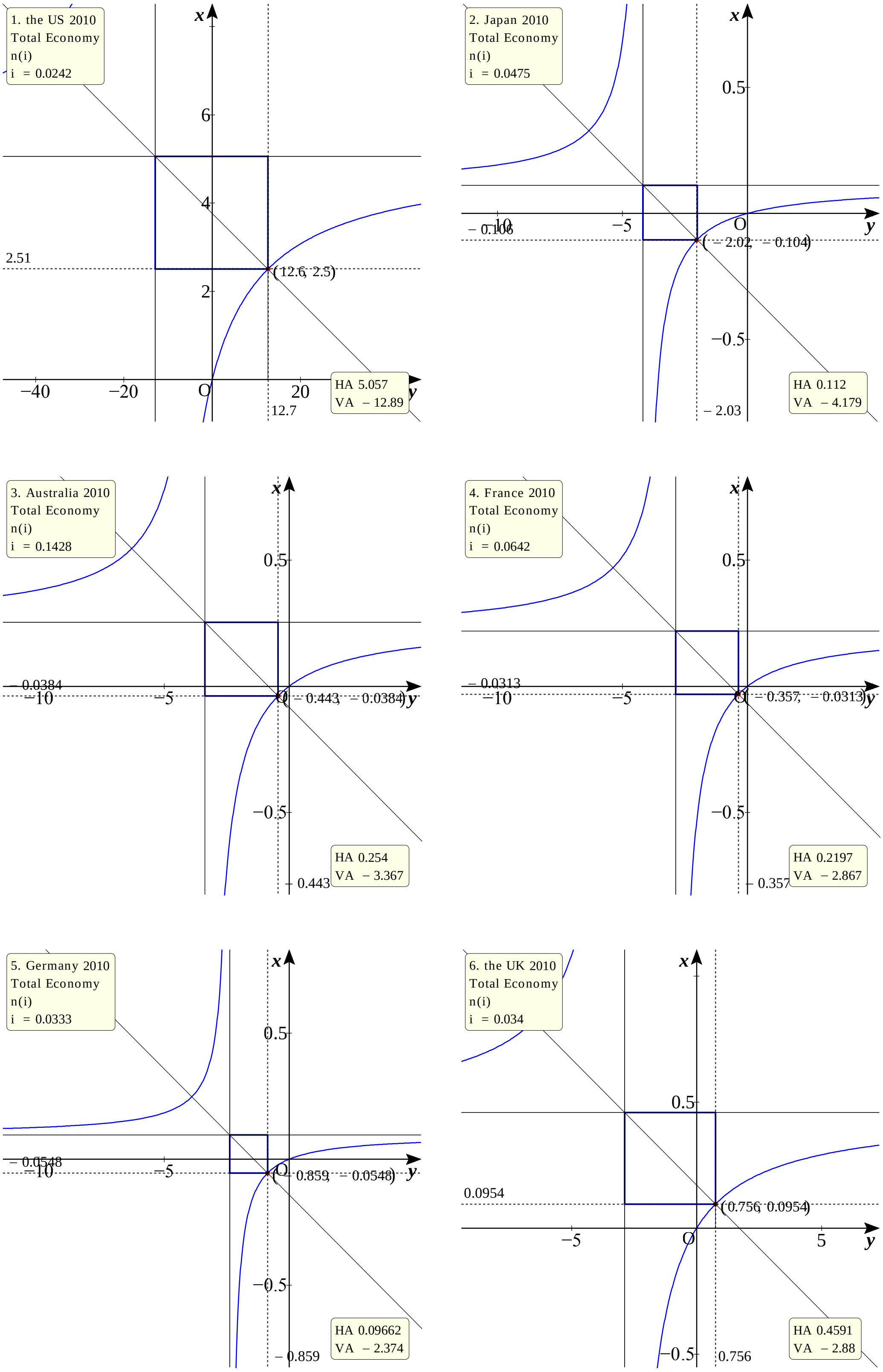


Figure 4-1 *n*(*i*) by country, 2010

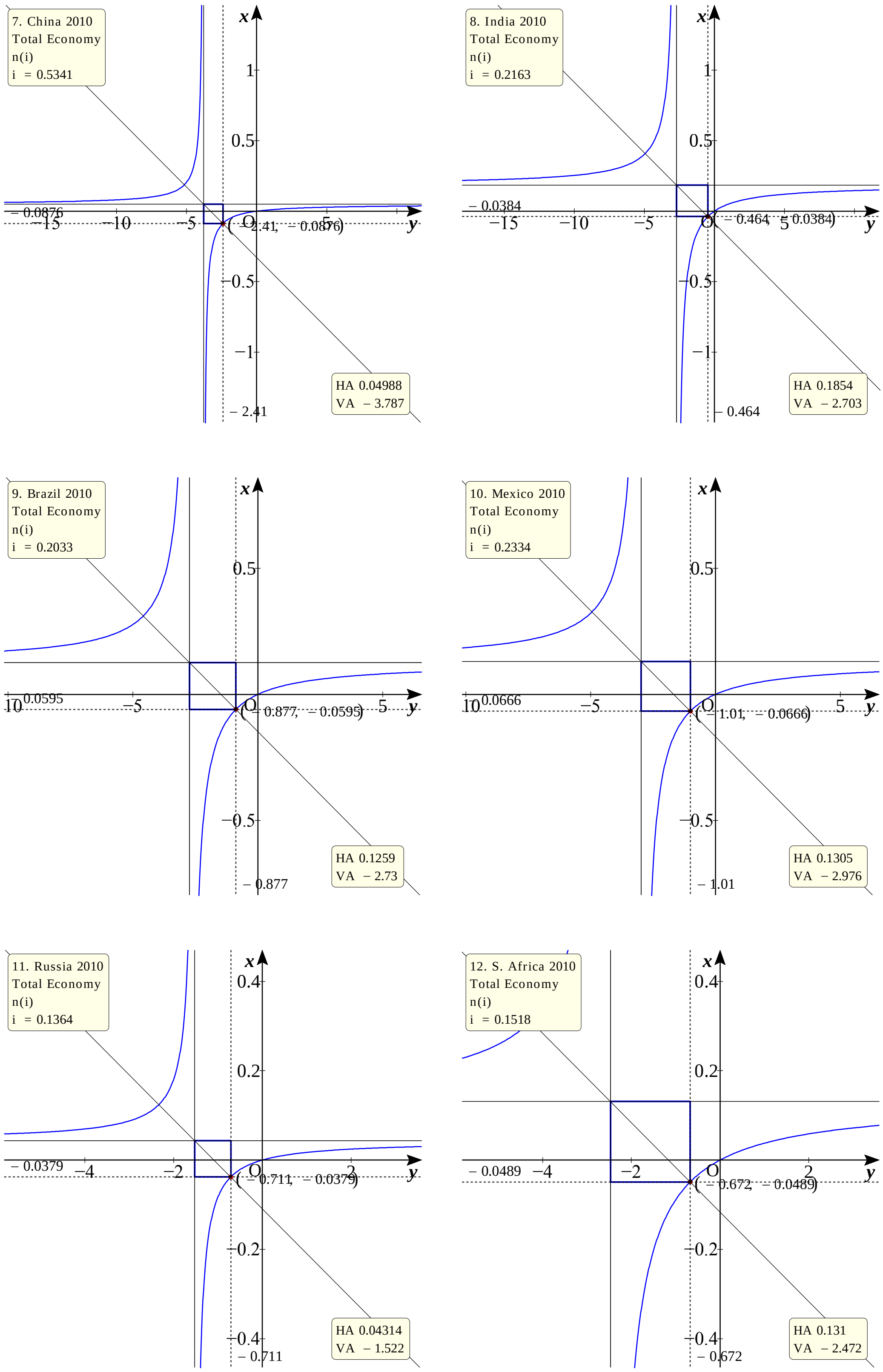


Figure 4-2 $n(i)$ by country, 2010

Appendices

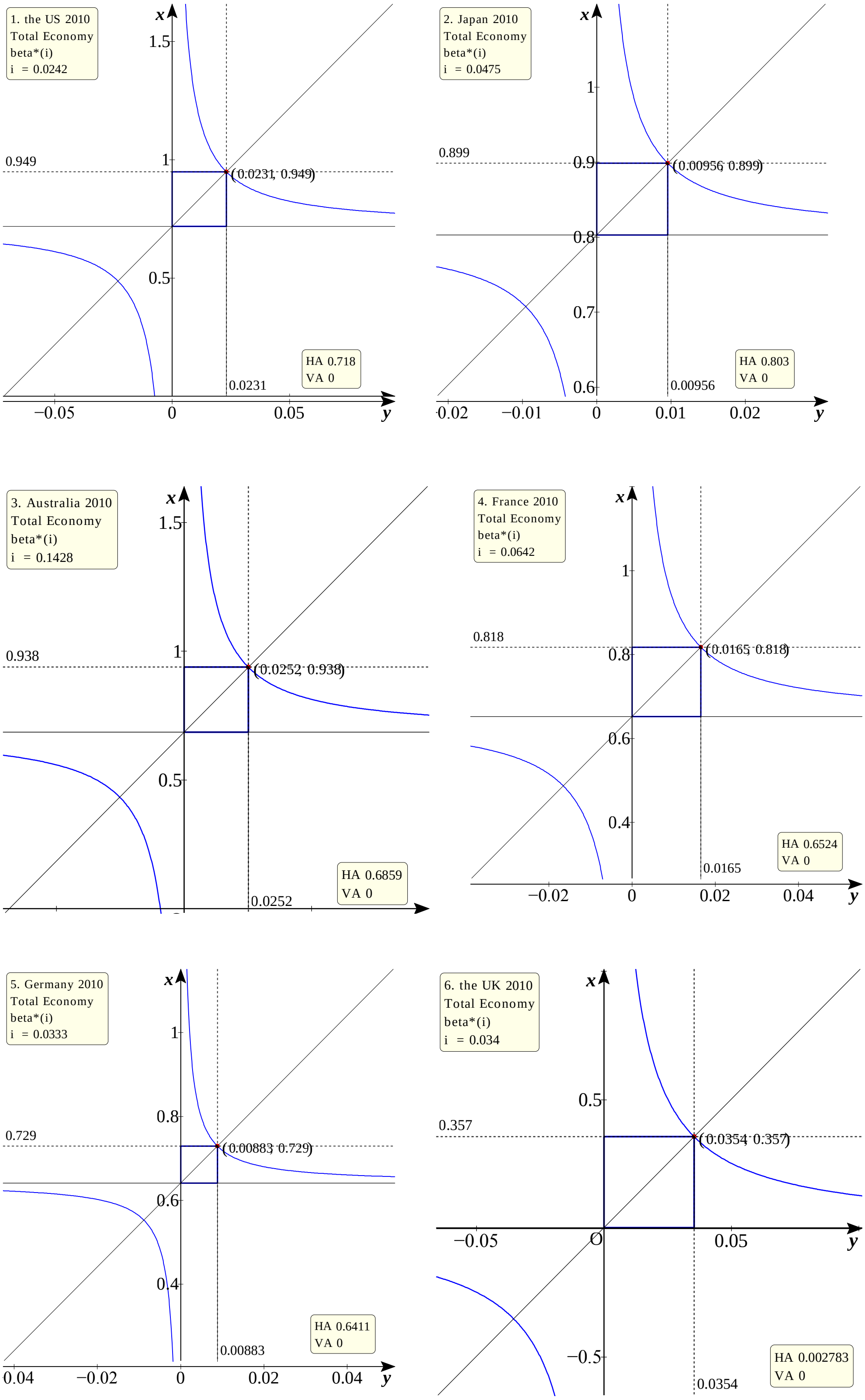


Figure 5-1 beta*(i) by country, 2010

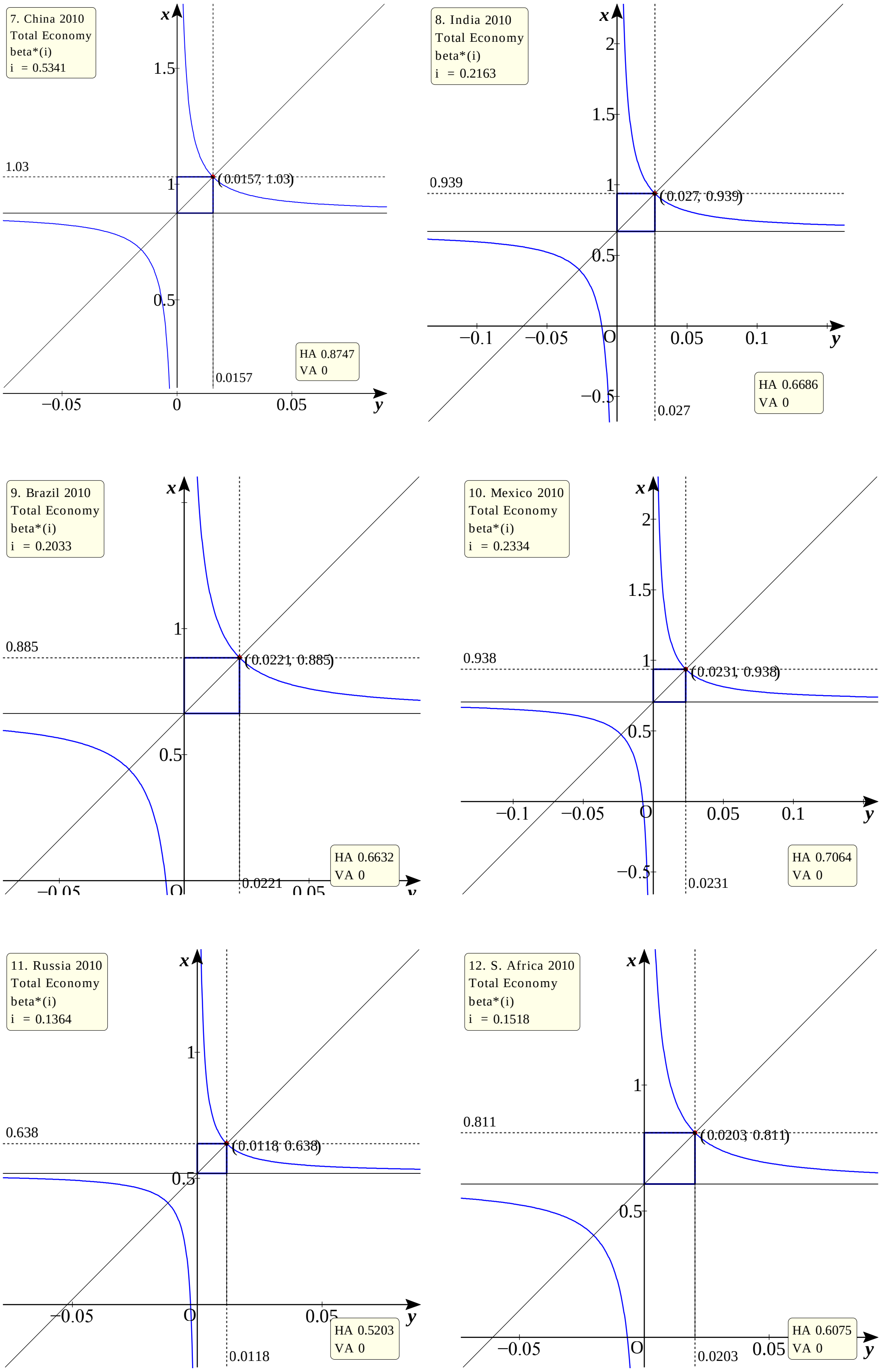


Figure 5-2 beta*(i) by country, 2010

Appendices

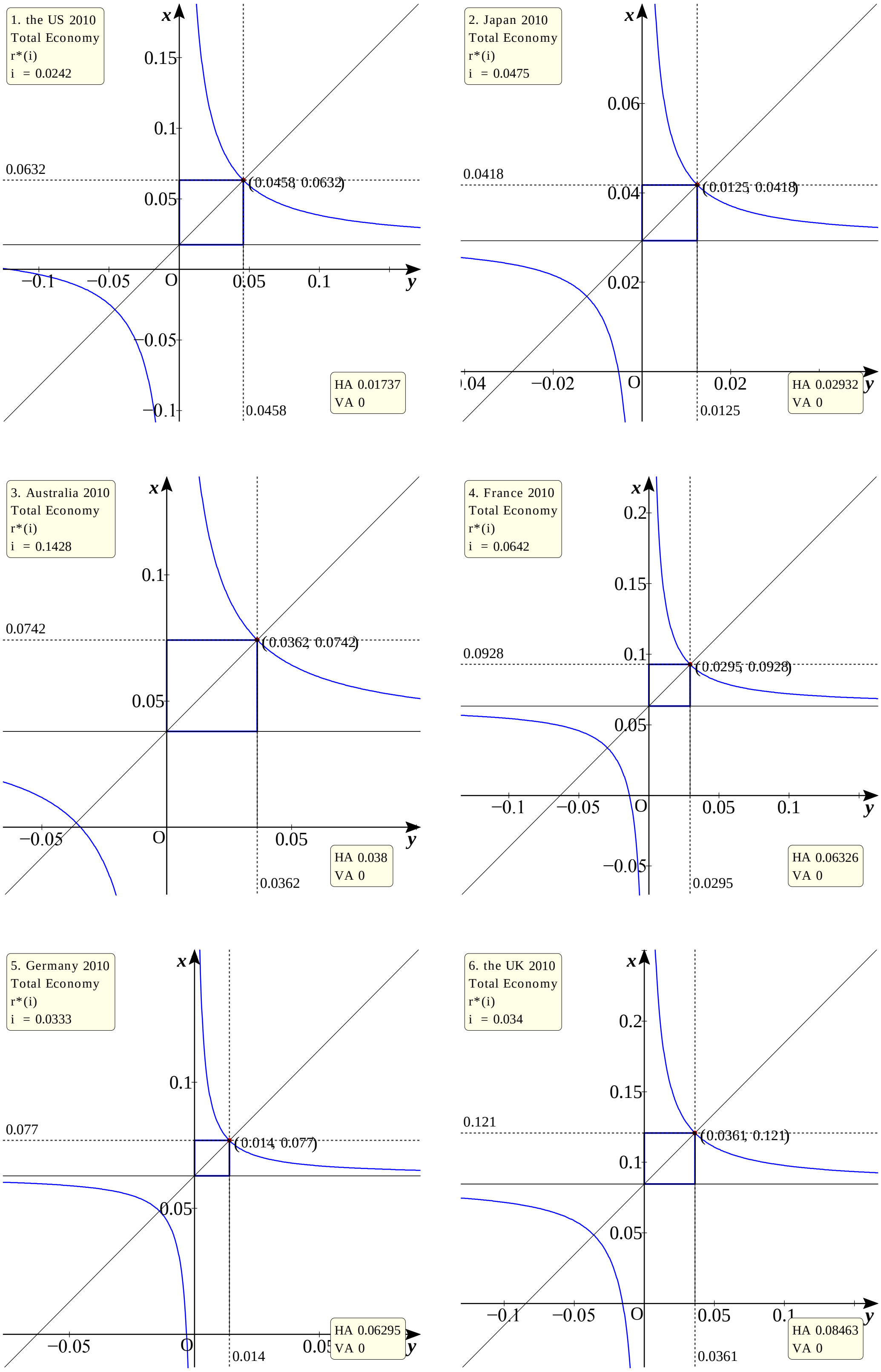


Figure 6-1 *r**(*i*) by country, 2010

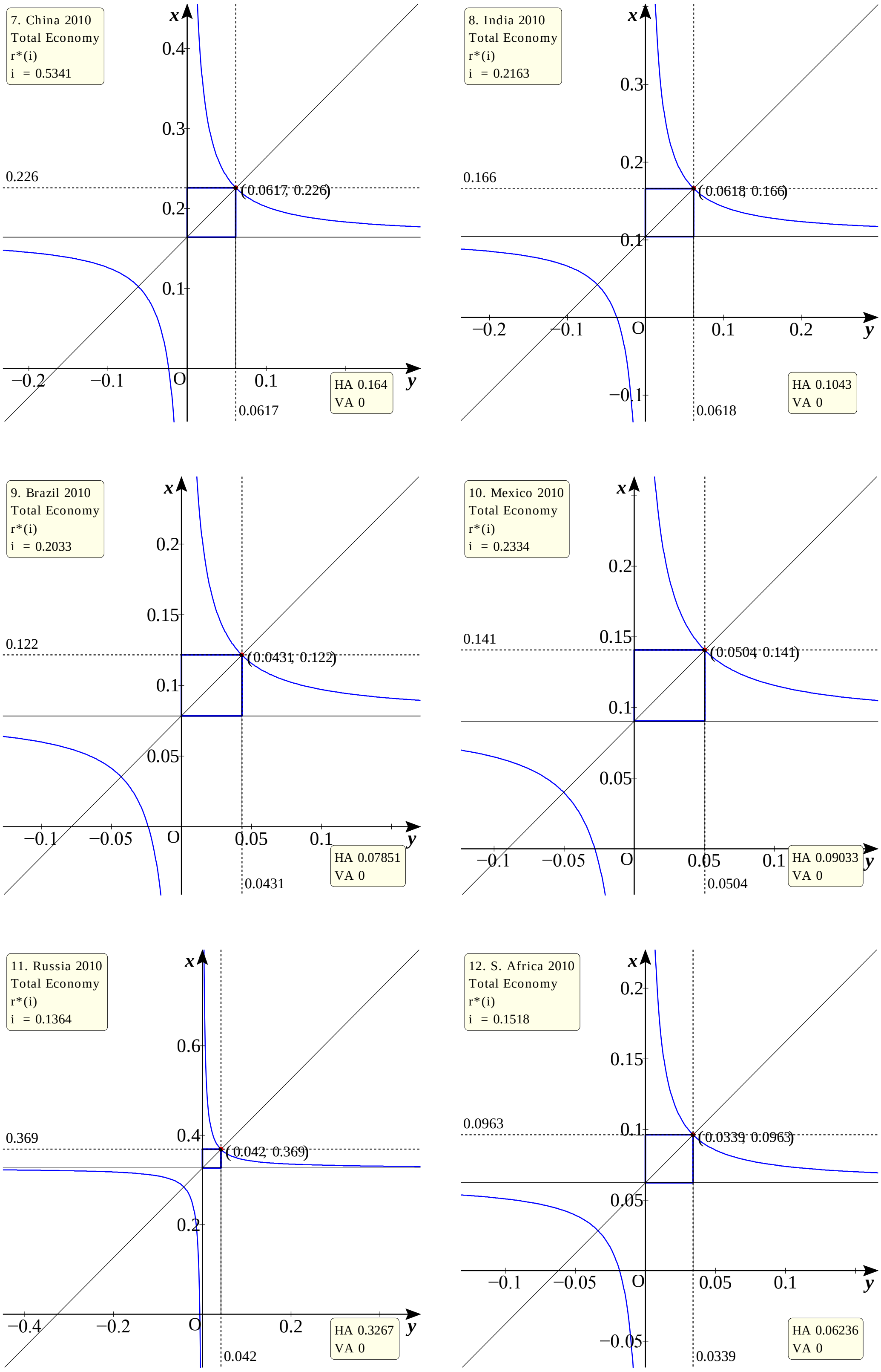


Figure 6-2 beta*(*i*) by country, 2010

Appendices

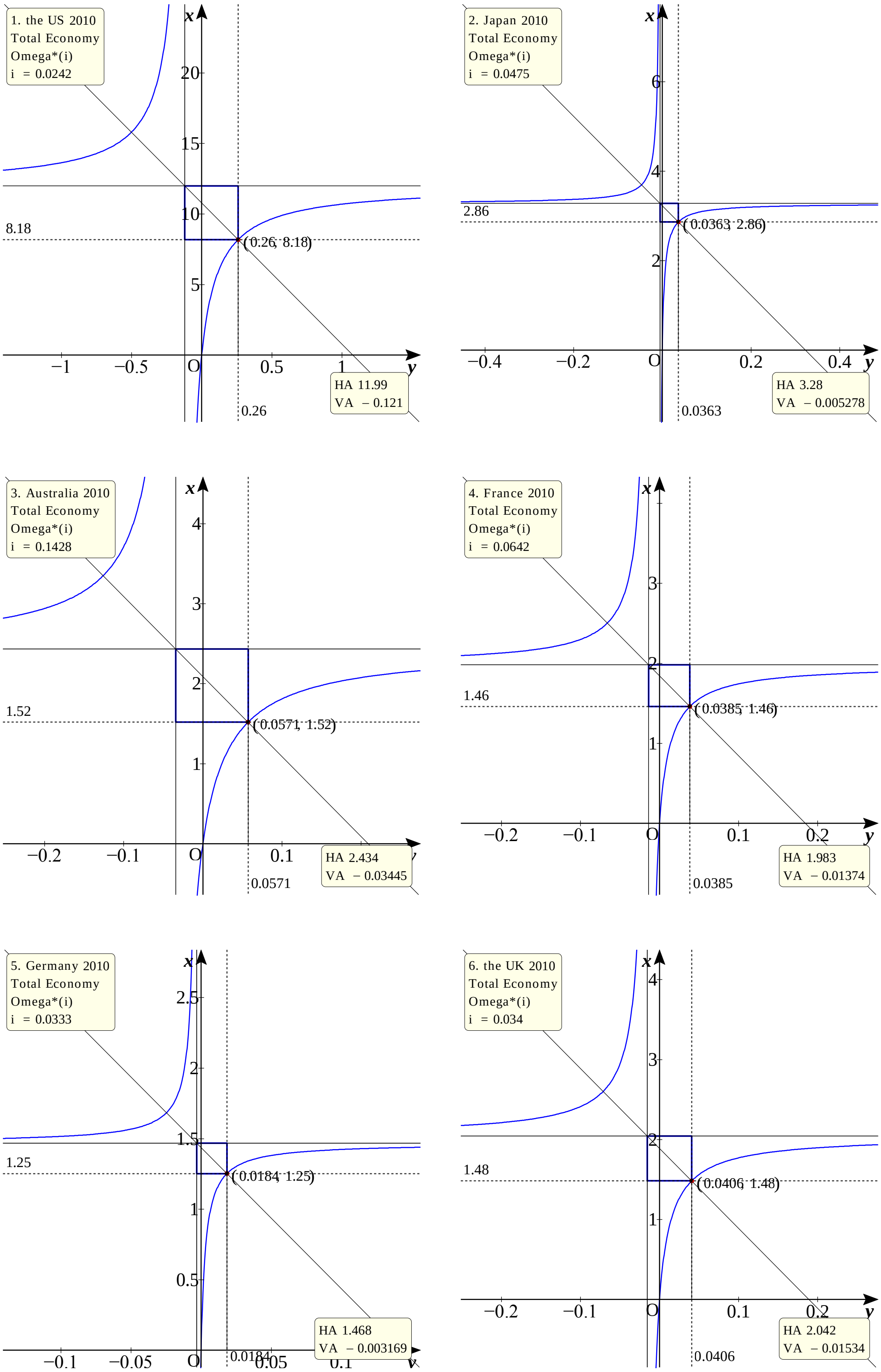


Figure 7-1 Omega(i) by country, 2010

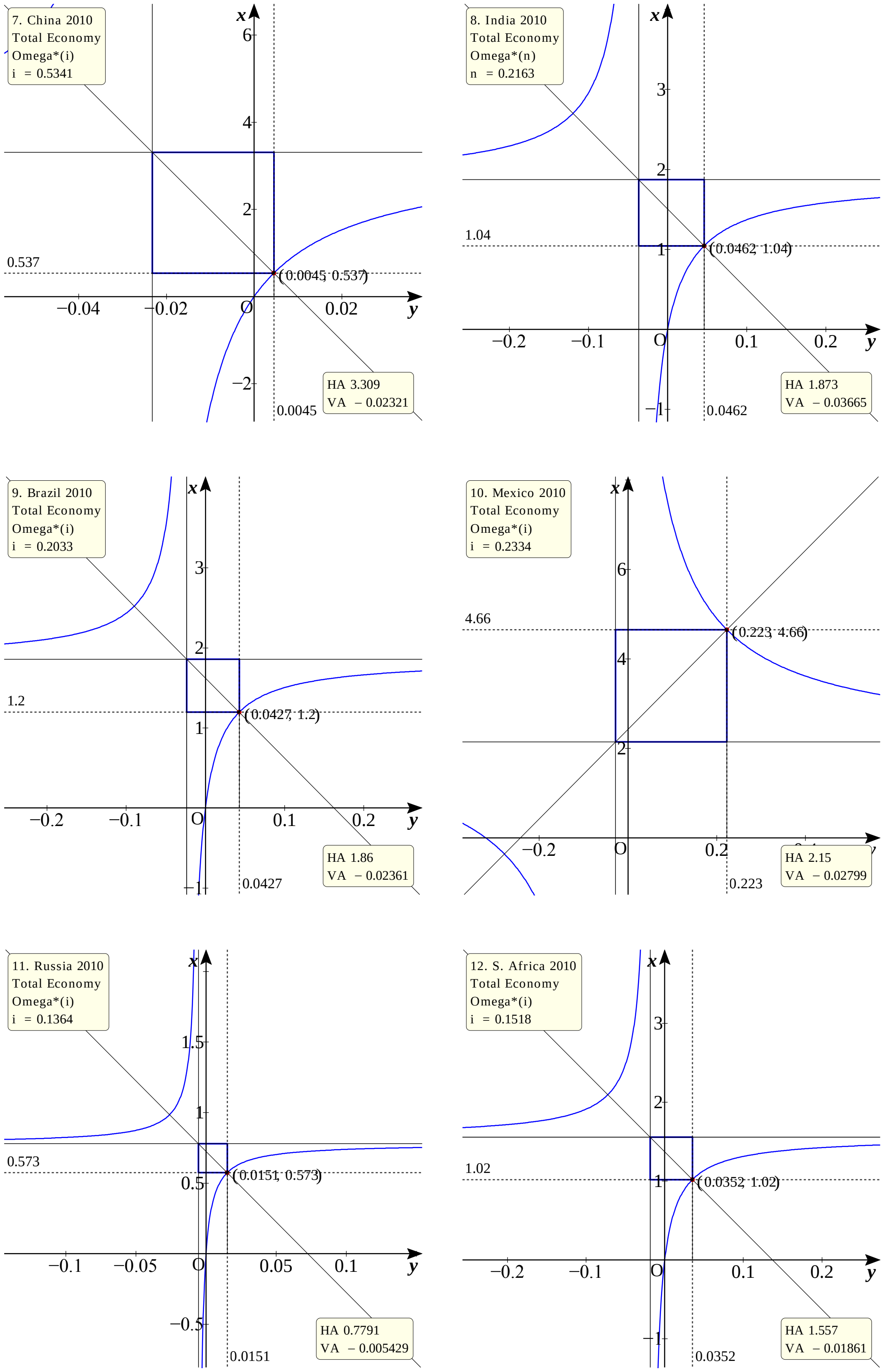


Figure 7-2 Omega(*i*) by country, 2010

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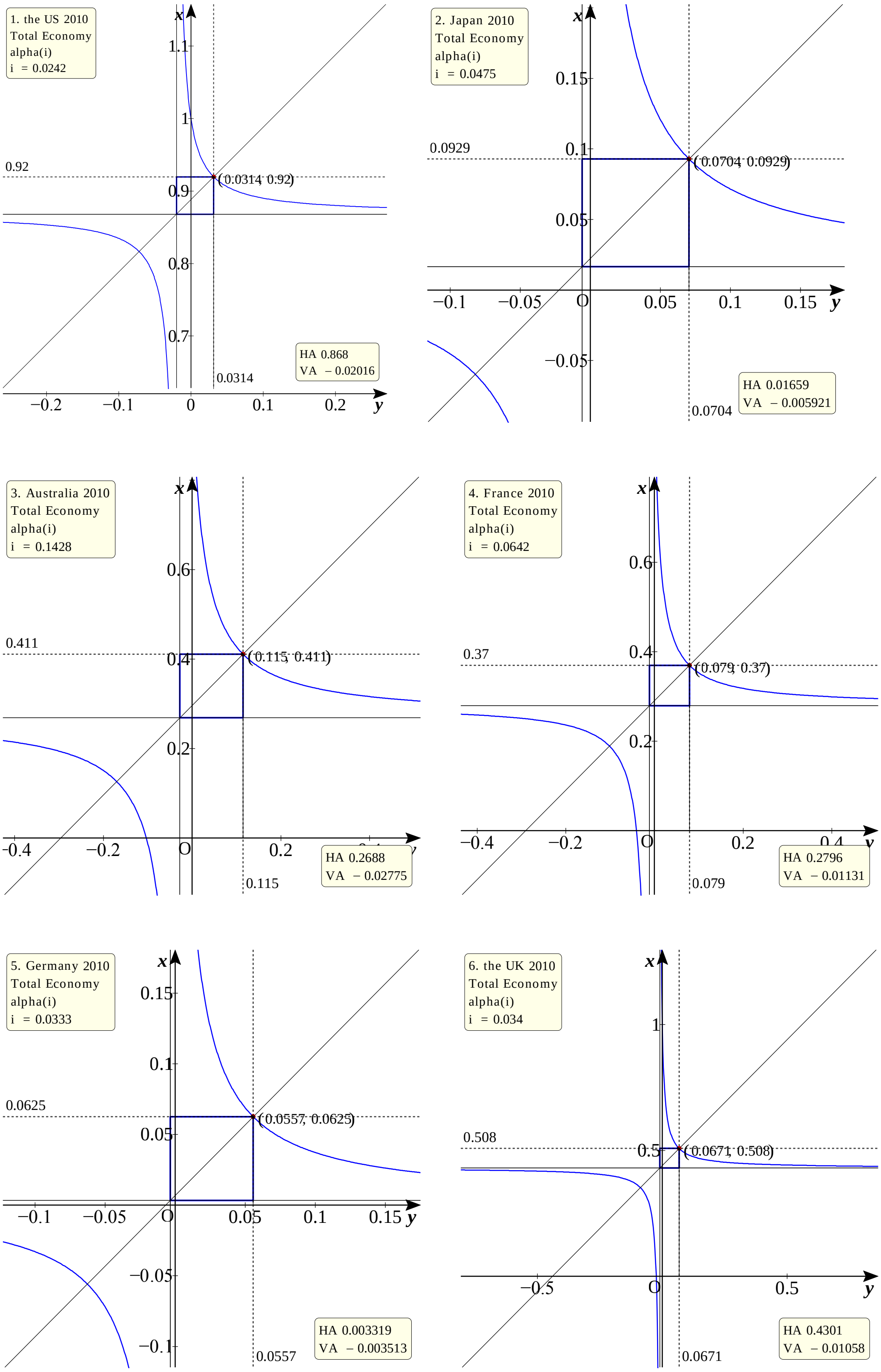

Figure 8-1 alpha(*i*) by country, 2010

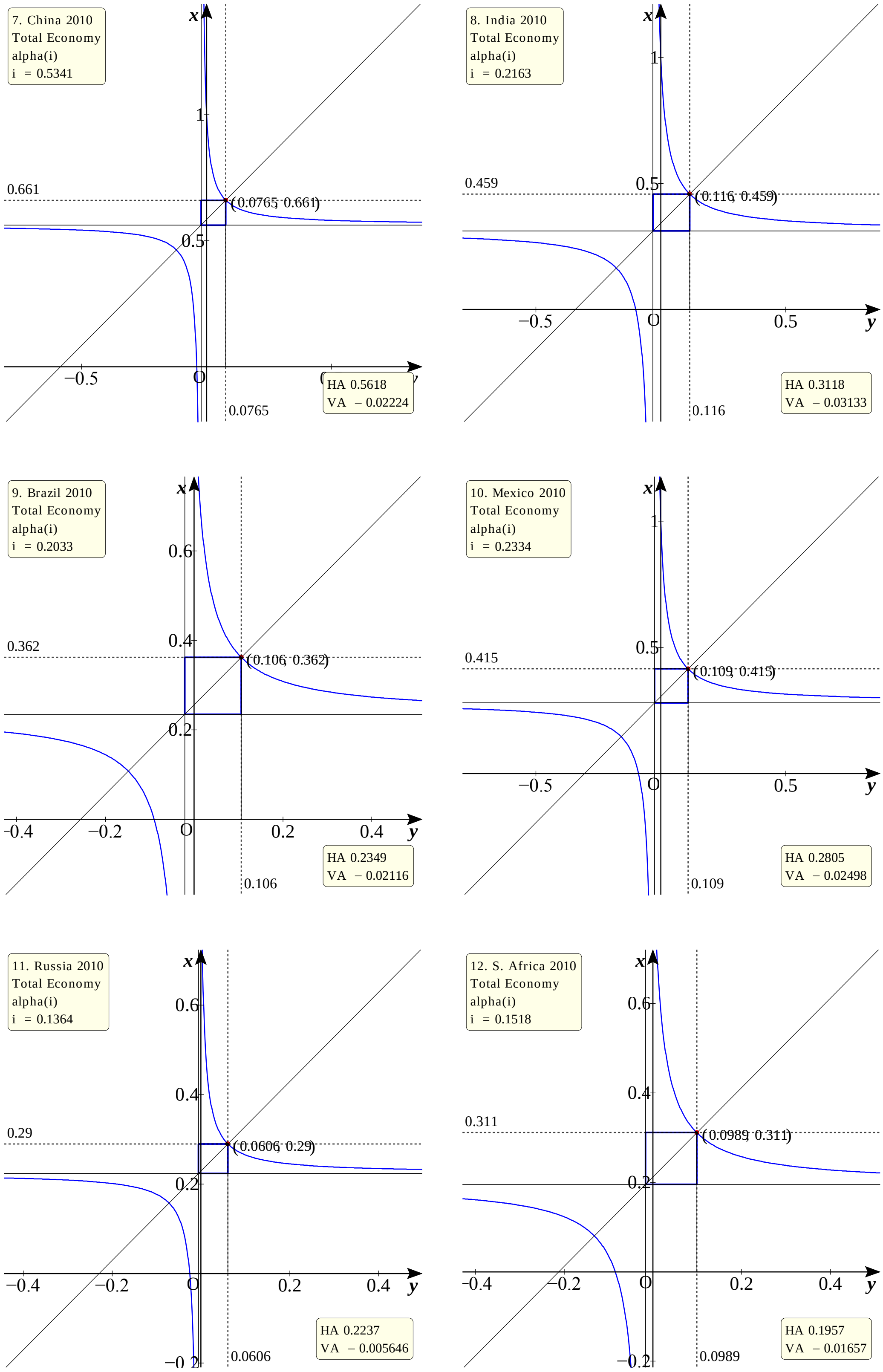

Figure 8-2 alpha(*i*) by country, 2010

Appendices

C3: i(n), beta*(n), r*(n), Omega(n), and alpha(n)

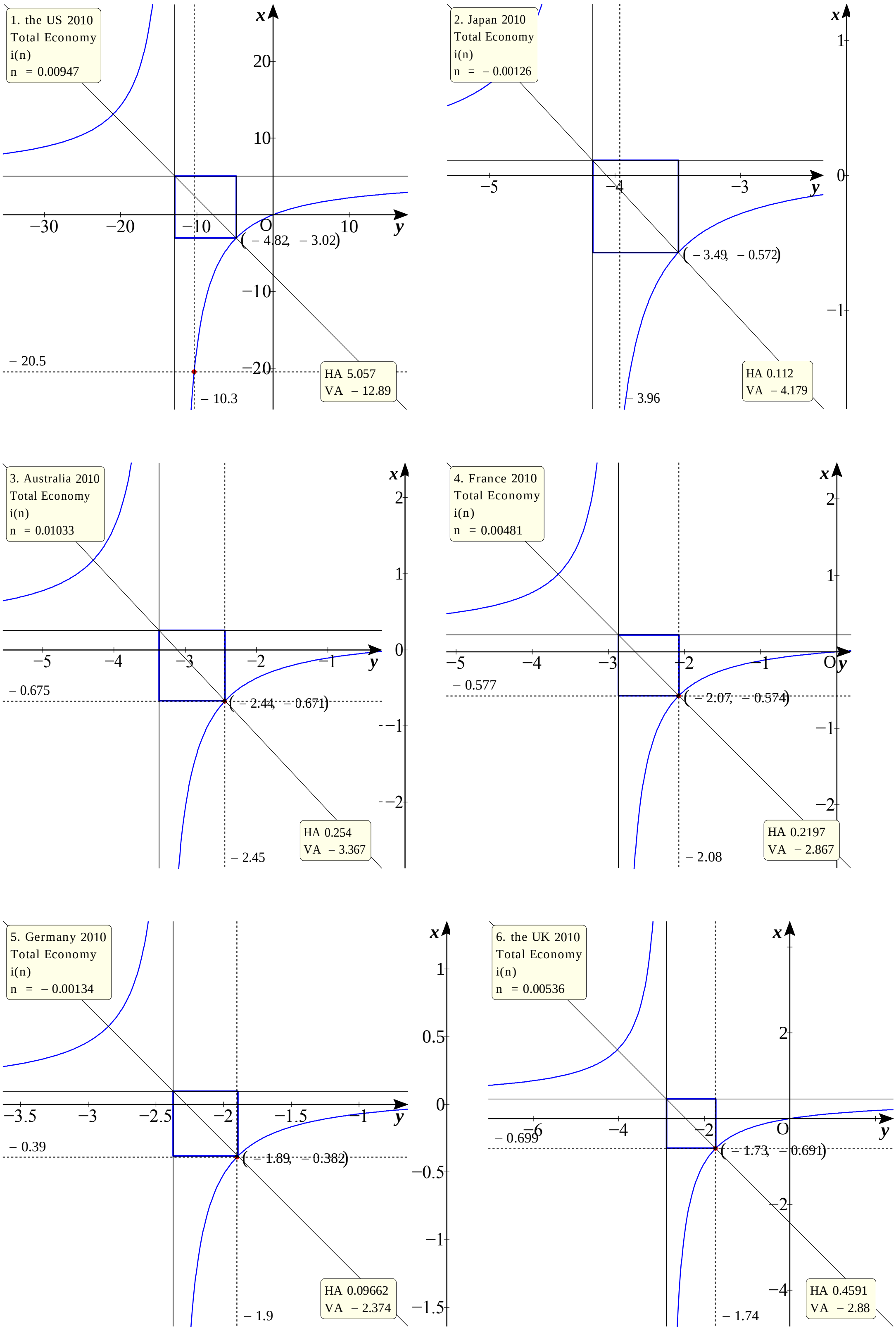

Figure 9-1 *i*(*n*) by country, 2010

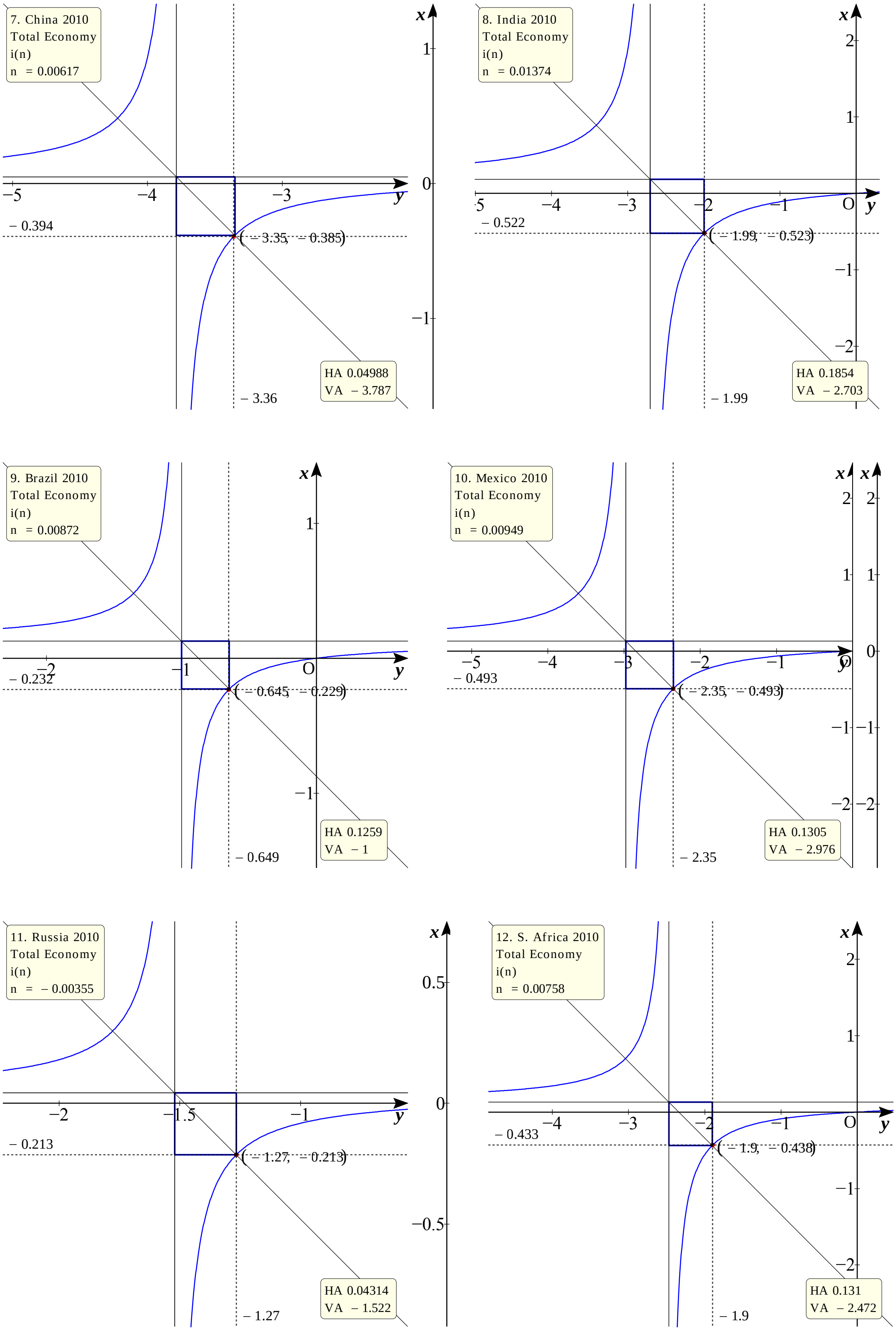

Figure 9-2 *i*(*n*) by country, 2010

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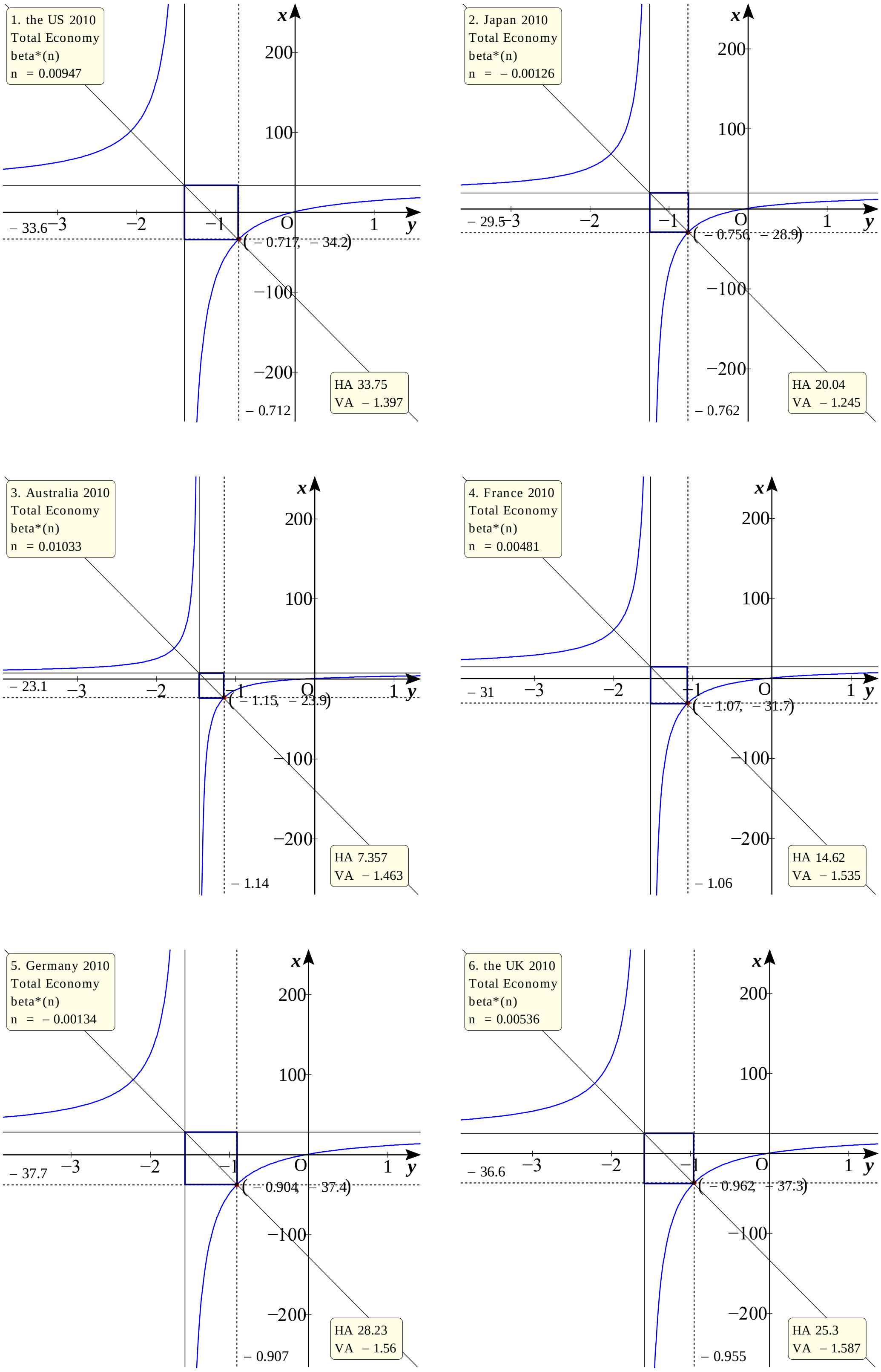

Figure 10-1 beta*(*n*) by country, 2010

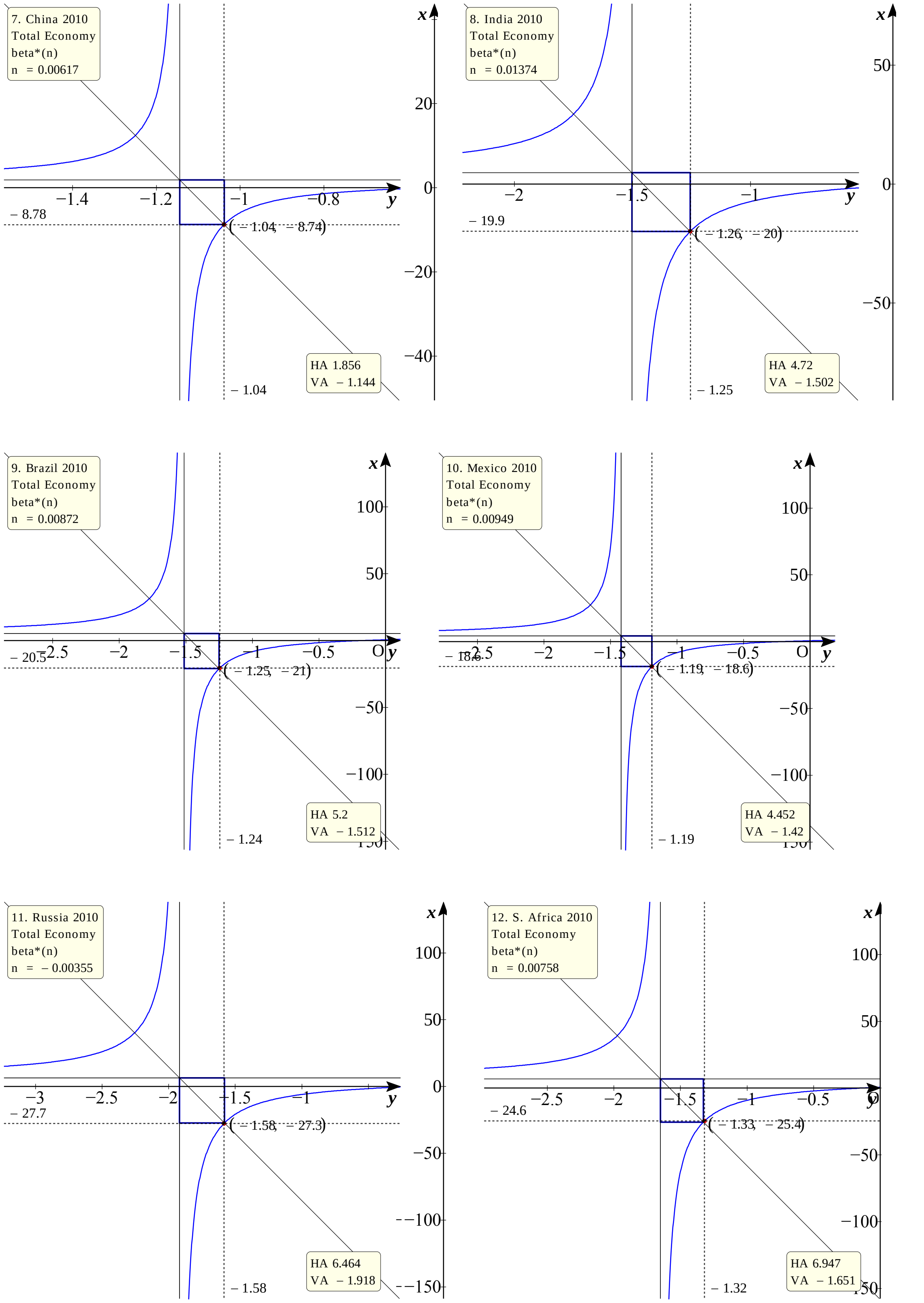

Figure 10-2 beta*(*n*) by country, 2010

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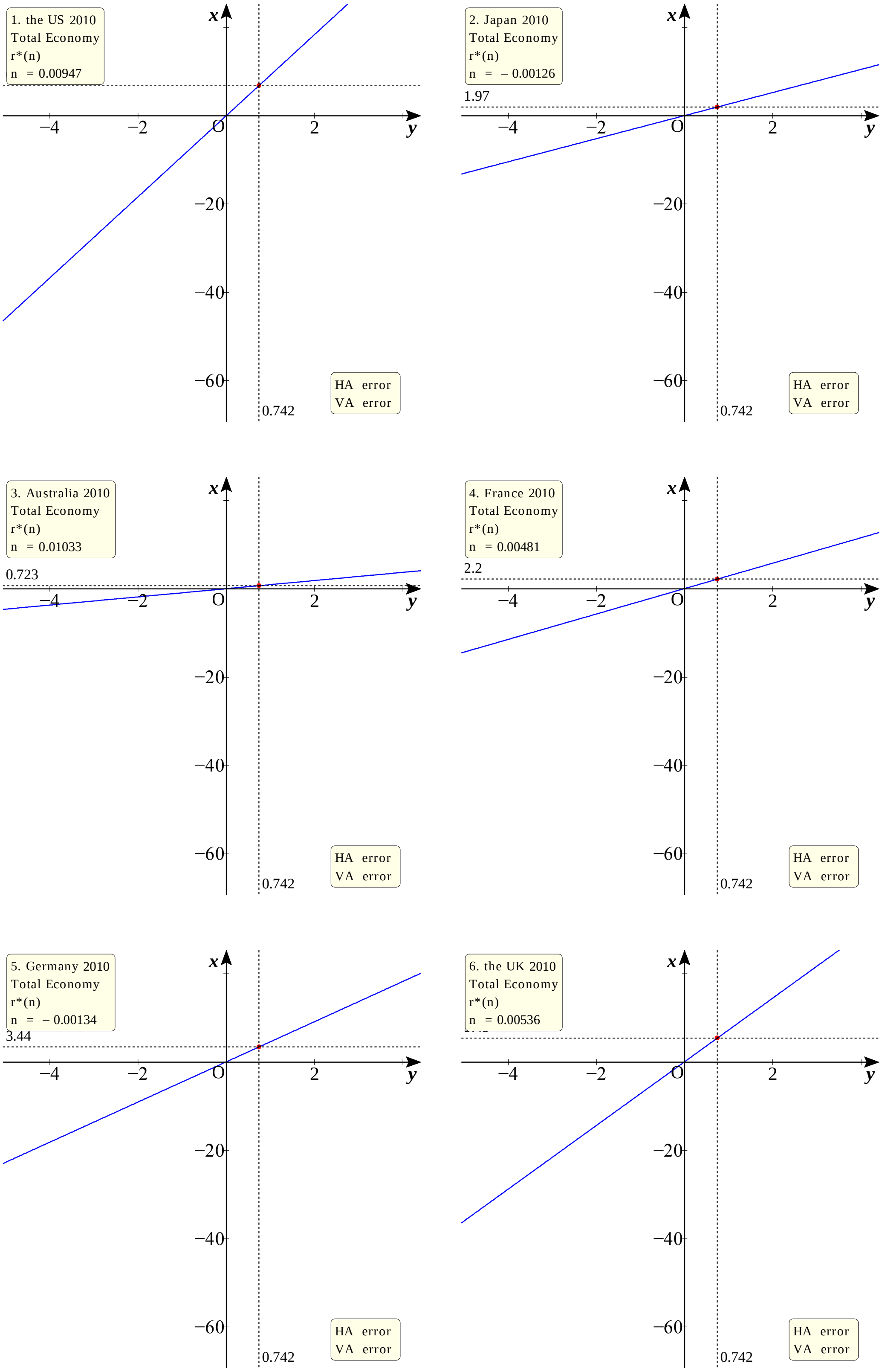

Figure 11-1 $r^*(n)$ by country, 2010

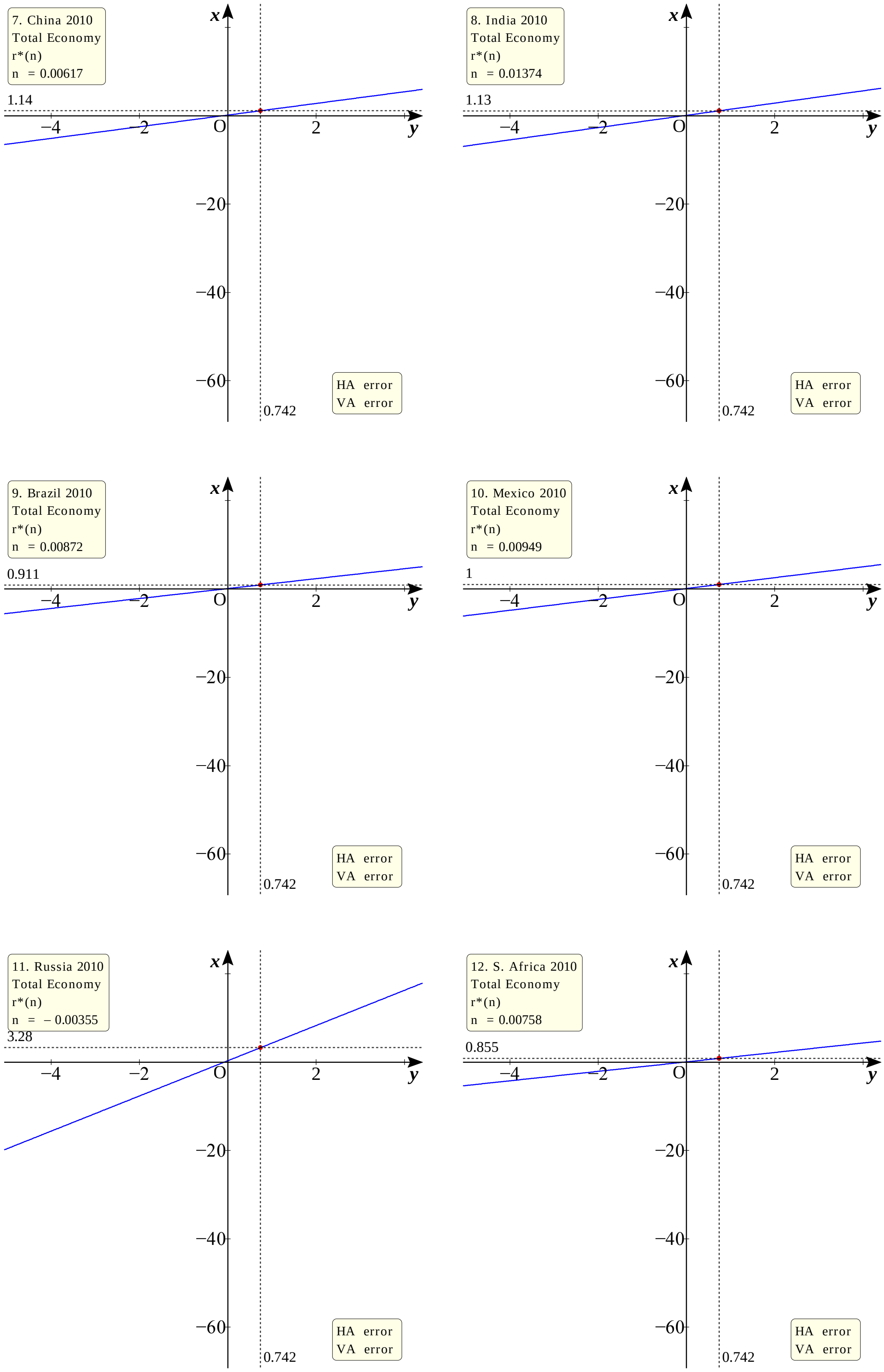

Figure 11-2 $r^*(n)$ by country, 2010

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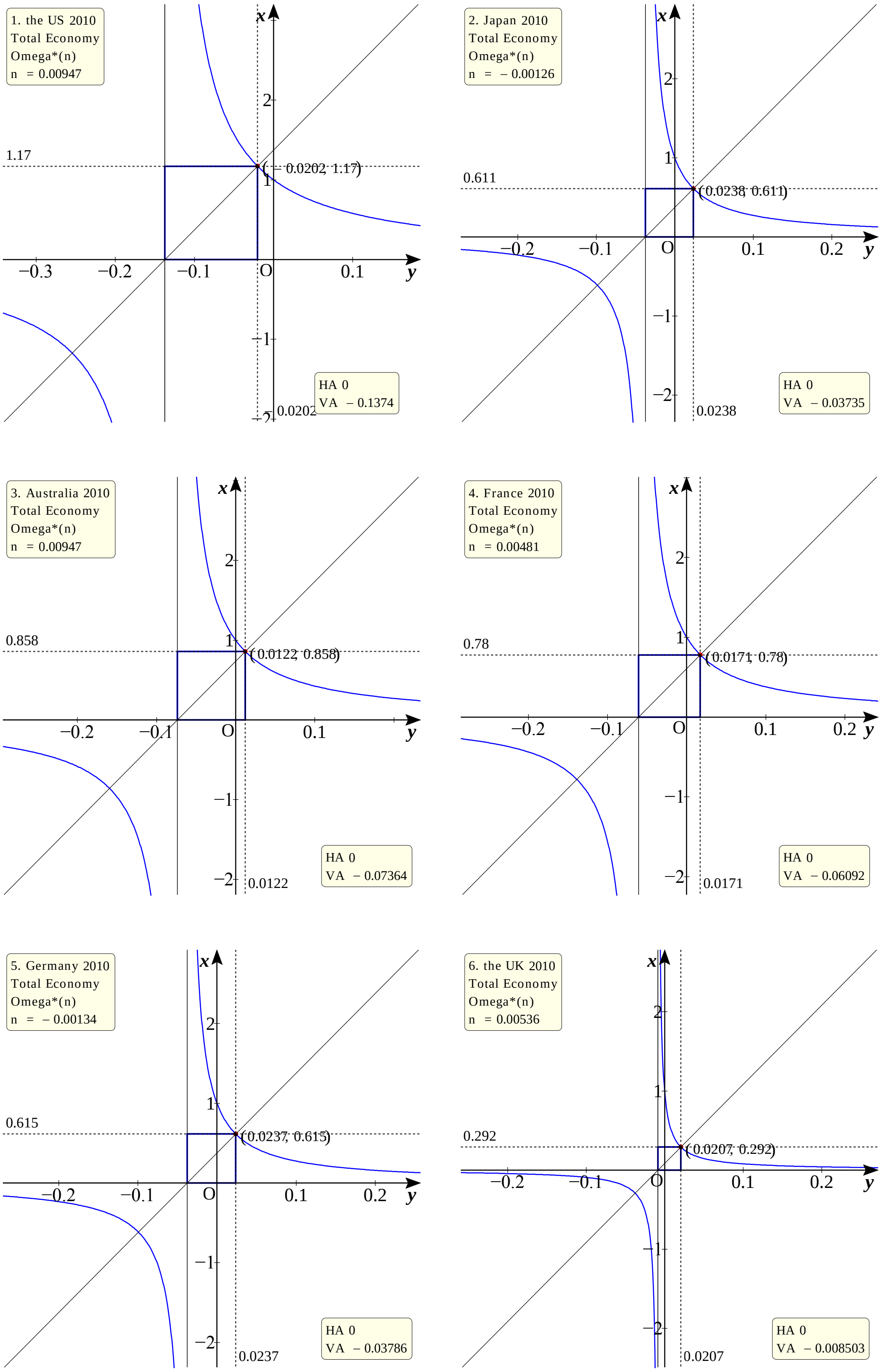


Figure 12-1 Omega(n) by country, 2010

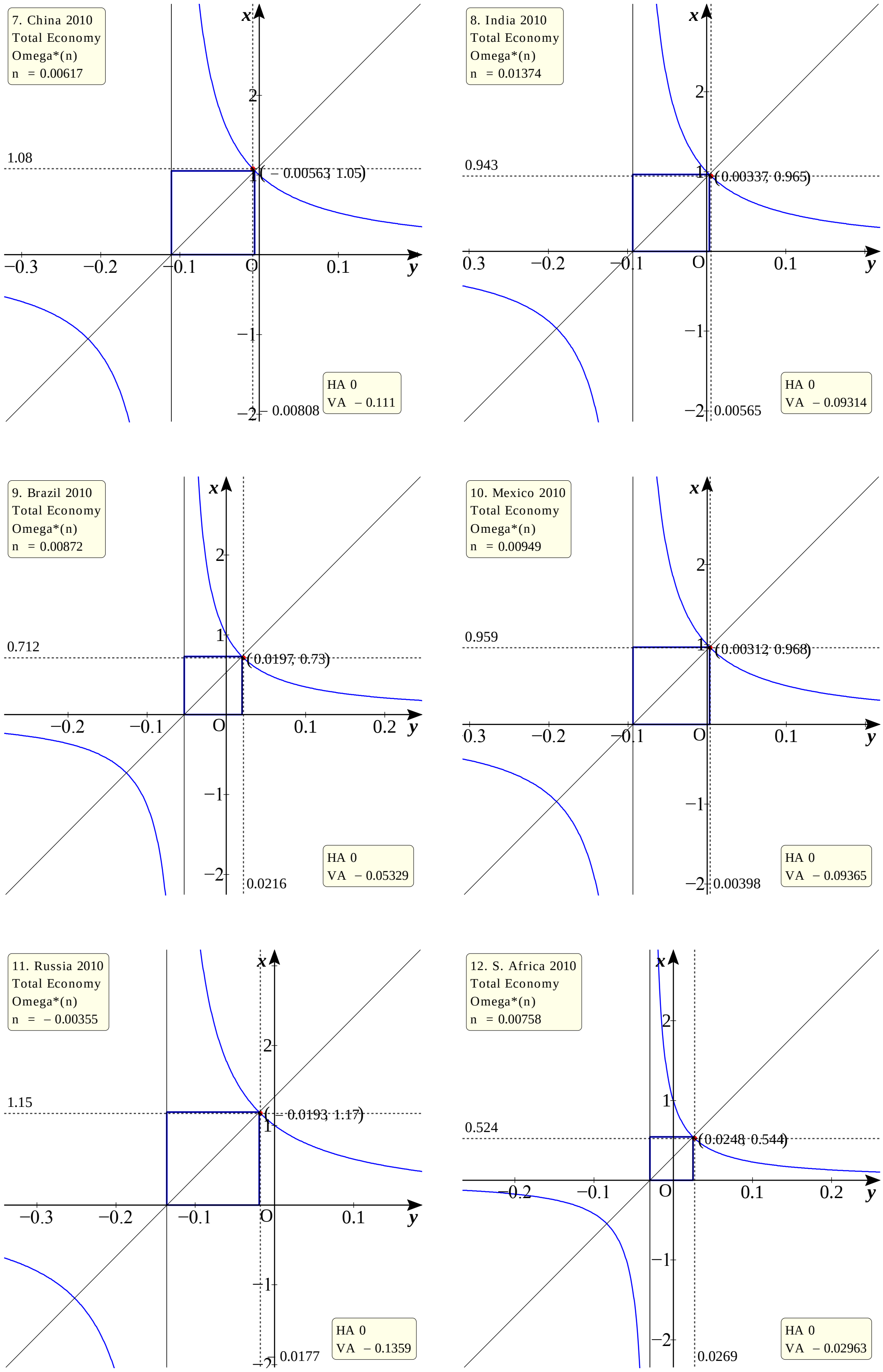


Figure 12-2 Omega(n) by country, 2010

Appendices

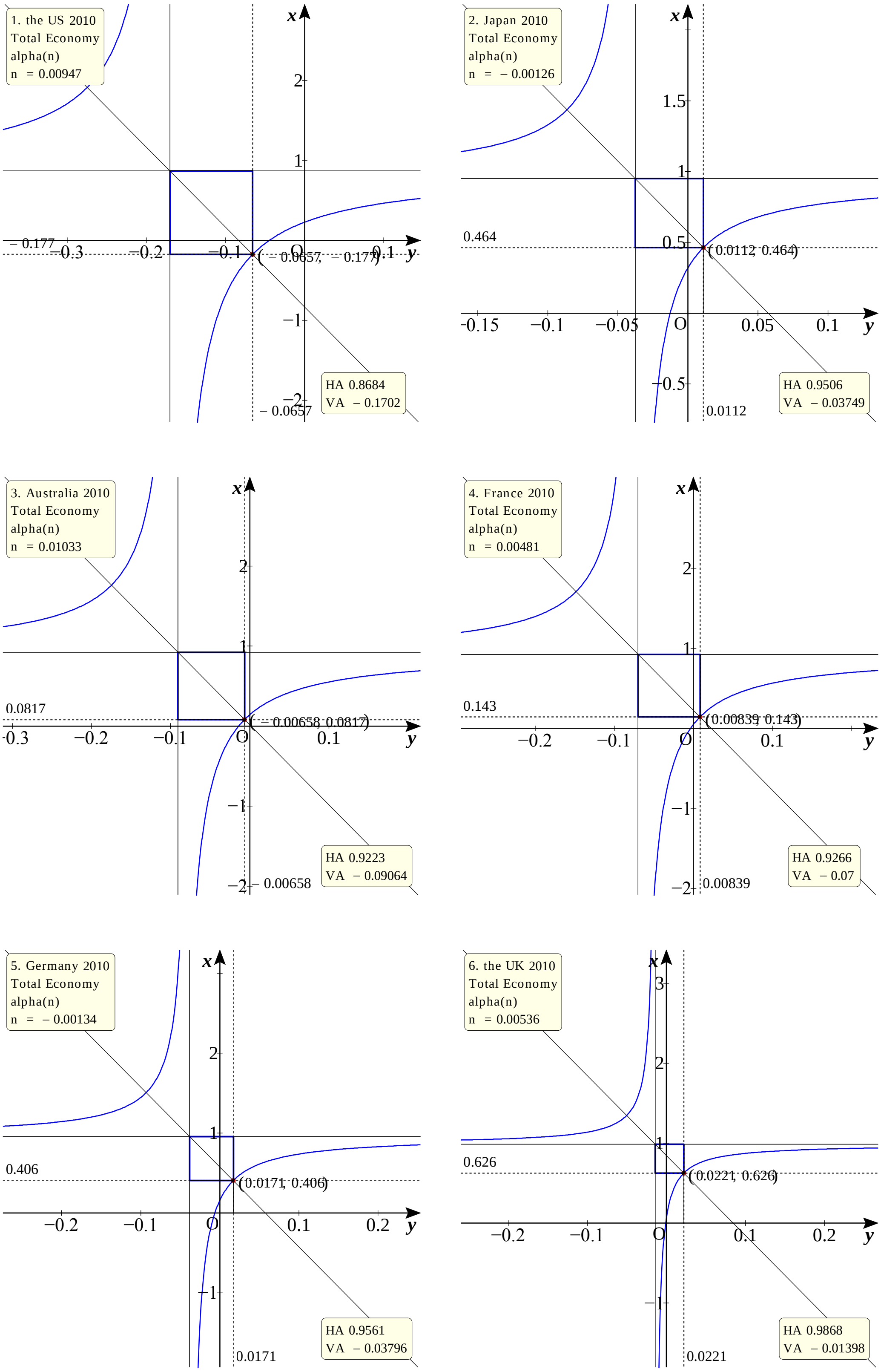


Figure 13-1 Alpha(n) by country, 2010

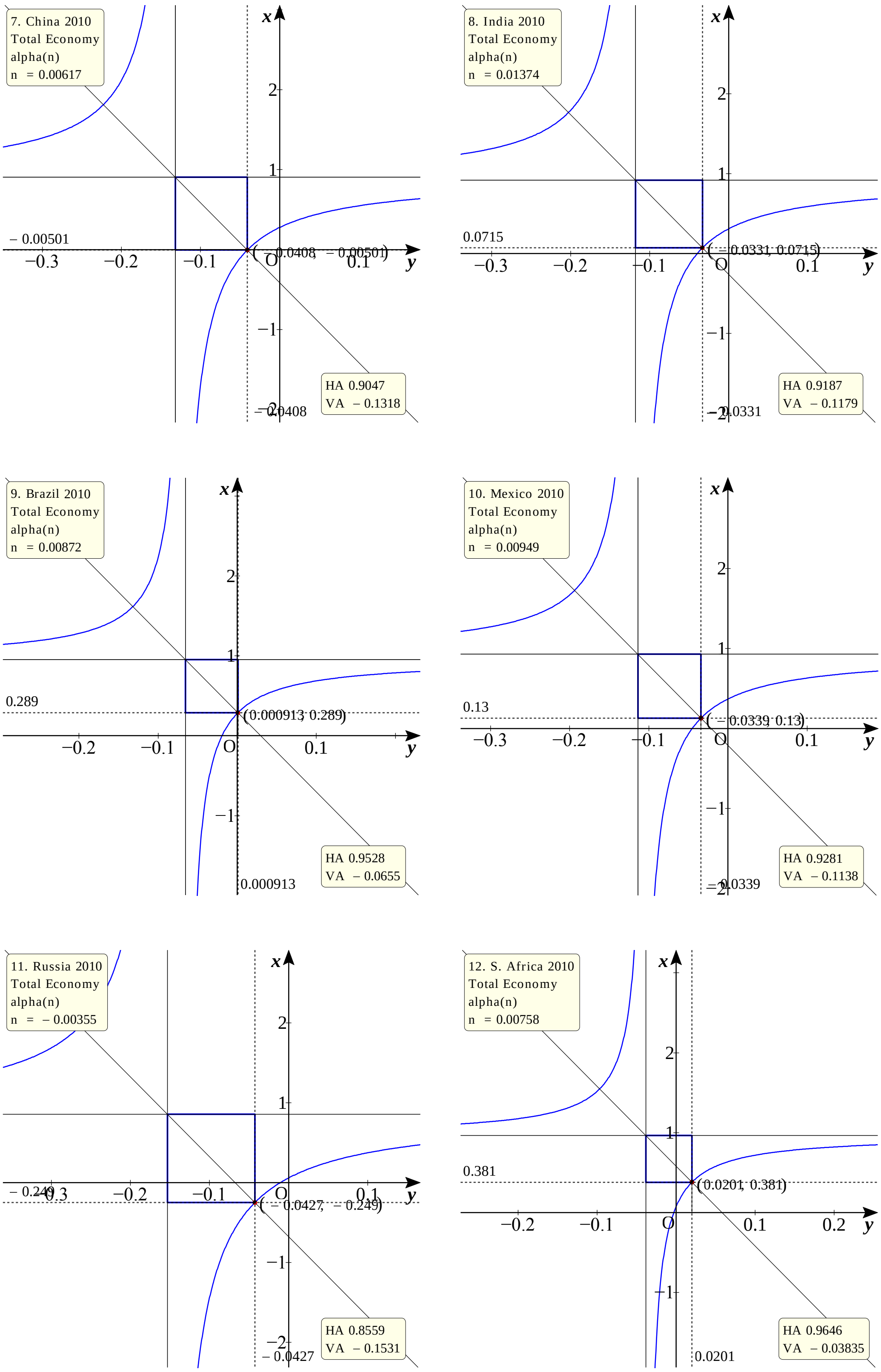


Figure 13-2 Alpha(n) by country, 2010

Appendices

Appendix D. *Endogenous equations and hyperbolas*

Eight basic equations[3] (Eq.1 to Eq.8) in the endogenous-equilibrium are shown as follows:

(1) The rate of technological progress, $g_A^* = i(1-\beta^*)$: The ratio of net investment to output, $i = I/Y$, and the quantitative net investment coefficient, β^*, or the qualitative net investment coefficient, $1-\beta^*$, where $\beta^* = \frac{\Omega^*(n(1-\alpha)+i(1+n))}{i(1-\alpha)+\Omega^* \cdot i(1+n)}$ and the relative share of capital $\alpha = \Pi/Y$.

(2) The growth rate of output, $g_Y^* = g_K^* = \frac{g_A^*(1+n)}{1-\alpha} + n$: The growth rate of population, n, and the rate of change in population in equilibrium, n_E. If $n = n_E$, it means full-employment.

(3) $r^* = g_Y^*\left(\frac{\alpha}{i \cdot \beta^*}\right)$: The endogenous golden rule coefficient is $\frac{\alpha}{i \cdot \beta^*}$, which solves the Petersburg paradox (see below). This is an extension of the (exogenous) Phelps (1961, 65) golden rule.

(4) The capital-output ratio, $\Omega^* = \frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)(1+n)+n(1-\alpha)}$.

(5) The rate of return, $r^* = \frac{\alpha}{\Omega^*} = \alpha\left(\frac{i(1-\beta^*)(1+n)+n(1-\alpha)}{\beta^* \cdot i(1-\alpha)}\right)$.

(6) The valuation value, $V = \frac{\Pi}{r^*-g_Y^*}$ and the valuation ratio, $v = r^*/(r^*-g_Y^*) = V/K$: The cost of capital is $r^* - g_Y^*$.

(7) The diminishing returns to capital (DRC) coefficient, $\delta_0 = 1 + \frac{LN(\Omega^*)}{LN(B^*)}$: the qualitative to quantitative coefficient, $B^* = \frac{(1-\beta^*)}{\beta^*}$.

(8) The speed years for convergence, $\frac{1}{\lambda^*} = \frac{1}{(1-\alpha)n+i(1-\delta_0)(1-\beta^*)}$.

The above equations are used for hyperbolic equations: For example, by using Eq. 8, $speed\ (i) = \frac{1}{(1-\beta^*)(1-\delta_0)i+(1-\alpha)n} = \frac{1}{ax+b}$ is derived (for other hyperbolas, see Appendices B and C above).

[3] For each proof of endogenous equations, see a synthesized separate paper (2010).

BOX D-1 Mechanics of main hyperbola equations using *rectangular equilateral triangle*

Fig.1.1 the Width for robustness

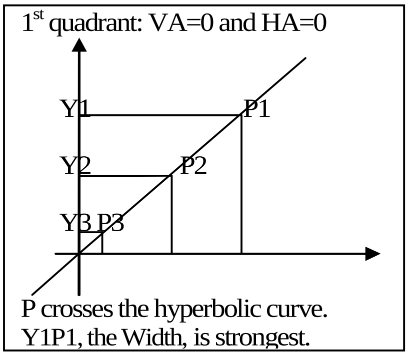


Fig.1.2 Plus and minus quadrants

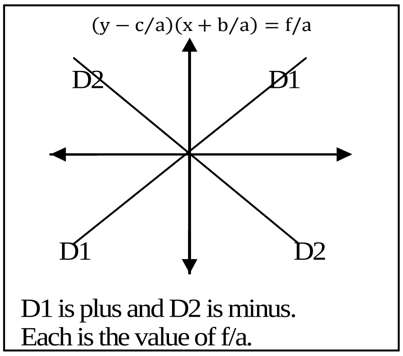


Fig.1.3 $r^*(i)$, where f/a>0

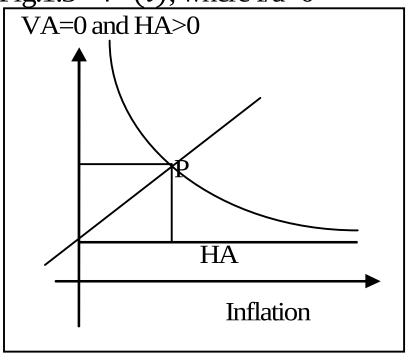


Fig.1.4 $r^*(i)$, where f/a<0

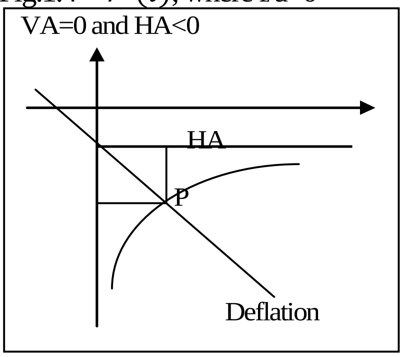


Fig.1-5 $speed\ (n)$

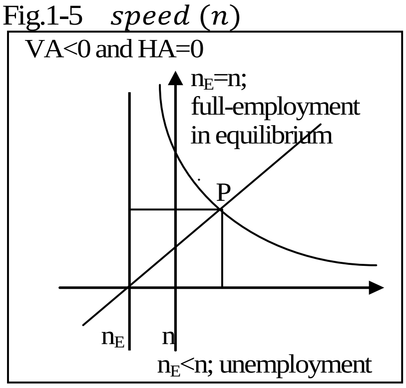
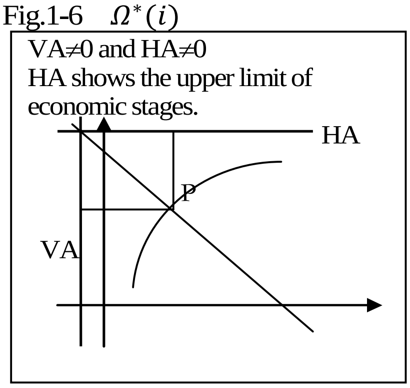


Fig.1-6 $\Omega^*(i)$



Note: Each sub-figure is useful for interpreting hyperbola graphs shown in Figures A1 to A6 below, as a good cut end. In particular, Fig.1-2 determines fundamental characteristic of each hyperbola graph, with respects of the rate of inflation/deflation and the upper limit of economic stages.

A new message on 31 Oct 2013:

The author is happy to be able to confirm that the author's hyperbola function in geometrical topology appears for the first time historically in the literature. During trip to the US in Oct 2013, the author could thankfully investigate this fact at several libraries.