Chapter 10

Qualitative Democracy Levels-Measure in Hyperbola Functions, towards Moderation = the Origin in the Plane

Foreword to Chapter 10

In the endogenous-equilibrium, hyperbola functions are derived geometrically using endogenous equations in the two-dimensional plane and united with human philosophy based on towards moderation. The origin of the plane shows an ultimate point of moderation but immeasurable. All points close to this ultimate point are measured accurately. The distance between horizontal and vertical asymptotes constitutes an optimal range of measure. Human, society, and economic democracy and respective levels are historically and most wholly expressed by hyperbola functions composed of basic ratio functions: the capital-labor ratio, the capital-output ratio, the relative share of capital, the rate of return, the growth rate of output, the rate of technological progress, Total Factor Productivity (*TFP*), the rate of change in population, inverse of speed years for convergence, the valuation ratio, and the qualitative coefficient $beta^*$. This chapter formulates twelve hyperbola functions, among which eight are reduced to linear. Empirically by country, this chapter discovers that eight linear functions are exactly surrogate functions for measuring the qualitative levels of democracy, leading to a few new-facts on linear functions, closer to the literature, asymmetric, at a point of convergence.

Signposts to Chapter 10: democracy; qualitative level; measure; data; database; external; endogenous; statistics; historical; geometrical; the two-dimensional plane; hyperbola; origin; moderation; philosophy; human; social; economic; optimum range; basic ratio functions; the market principles; endogenous equilibrium

Chapter 10, *HEU*

1. Introduction

Democracy differs from natural science and belongs to social science connected with human consciousness and politics and political economy. Democracy may be one which should not be counted or measured. Democracy may belong to behavioral science. However, the author's scientific approach avoids behavior and stays within a purely endogenous system or the *EES*, i.e., "*Earth Endogenous System* (the 1st Ed. in 2013; the 2nd Ed. in 2014)." The *EES* is essentially nature-oriented; money M2-neutral to the NDI=Y (national disposable net income=output, under three equality of incomes) and returns, consumption-neutral to technology, the relative share of capital-neutral to stop macro-inequality, deficit-neutral, politics-neutral, and spirituality-neutral.

This chapter is one of two chapters for applying the curvature of hyperbola function to qualitative measure of the real assets endogenously. This chapter intensively and scientifically measures qualitative democracy levels numerated by using the curvature in the two dimension plane. The base comes from the other chapter, "Optimum functions-measure numerated by using the curvature in the two-dimensional plane." The hyperbola does not include a maximum or minimum equation as in parabola. Instead, the hyperbola gives us much room for select an optimum range of objective measure. The author advocates that the hyperbola and the curvature constitute a methodology most flexibly selecting social and economic policies.

The other chapter is much more general in measuring the curvature of hyperbola function. 'The hyperbola function' here is distinguished with 'the hyperbolic function' in mathematics, which is geometrically connected with six trigonometric functions. This chapter receives the essence of the curvature measure from the other chapter so that the author abbreviates repeating explanations in this chapter. Introduction here serves a brief summing up of the curvature-measure.

Four elements, *a*, *b*, *c*, and *d* for hyperbola standard type, the case of no zero. Additionally, $e = \frac{c}{d}$. $f = d - \frac{b \cdot c}{a}$. $\frac{f}{a} = \frac{1 - b \cdot c}{a^2}$.

$y = h(x)$ by setting $h(x) = \frac{cx+d}{ax+b}$. $y = \frac{c}{a} + \frac{d - \frac{b \cdot c}{a}}{ax+b} = \frac{c}{a} + \frac{f}{ax+b}$.

Five attributes of the hyperbola: (1) $\text{VA} = \frac{-b}{a}$; (2)$\text{HA} = \frac{c}{a}$; (3)$\text{Width} = \sqrt{\left\lfloor \frac{f}{a} \right\rfloor}$; (4) $\text{Sharpe} = \sqrt{2\left\lfloor \frac{f}{a} \right\rfloor}$; (5) $\text{Curvature} = \frac{1}{\text{Sharpe}} = 1/\sqrt{2\left\lfloor \frac{f}{a} \right\rfloor}$; Or $\text{Curvature} = 1/\sqrt{2\left\lfloor \frac{1 - b \cdot c}{a^2} \right\rfloor}$.

$x \cdot y = 1.0000$ or $y = 1.0000/x$. $1 = cx + d$. If c=0, d=1.0. If a=1.0 and b=0, $ax + b = x$. As a result, $y = 1.0000/x$ holds under a=1.0, b=0, c=0, and d=1.0.

$\text{Curvature} = 1/\sqrt{2\left\lfloor\frac{1-b\cdot c}{a^2}\right\rfloor} = 1/\sqrt{2\left\lfloor\frac{1-1}{1}\right\rfloor} = 1/\sqrt{2\left\lfloor\frac{0}{1}\right\rfloor}$. It implies Curvature =0 or Curvature overlaps the origin of the plane.

Conclusively, it is impossible for us to obtain the values of curvature by the value of x on the two-dimensional plane, the y axis and x axis. The author needs to apply the standard type of hyperbola to the test of the curvature.

When an independent variable is fixed, the value of the curvature is constant naturally. This idea leads us to measure hyperbola functions by making parameters fixed and designing the qualitative levels of democracy. The author establishes new fact-findings for transformations between endogenous equations and hyperbola functions. These new processes are condensable and scientifically illustrated by BOX 1, 2, and 3. These results are empirically shown using Table A1 and Figures A1, A2, and A3, by country, at the end. These essence and substances are interpreted step by step before Conclusions.

2. Review of the Literature: To Step Broader and Deeper

Historically, the author glances at the trends and stream of democracy by country. In a word, human history is one of democracy. It implies that human life is involved in democracy and essentially broader and deeper. Suppose democracy is another word of learning-by-doing. Human's learning-by-doing is our life itself. Human's learning-by-doing, however, strongly influenced by external or internal conflicts. Human among other mammal animals is destined to live with each own decision-making, apart from God's will or Nature rules and by individual. It means that eating, sleeping and family-love differs from others so that one may be against the other. The truth may be that one is one among the whole or the Nature and the Universe, commonly and regardless of religion, philosophy, and thought. Human must earn money to live on. Greedy starts here so that social and economic circumstances are all connected with each other. In short, democracy and human greedy last forever. It implies that social and economic behaviors enlarge by nature.

For review of the literature, the author is fortunate to be able to find the best paper that proves the essence of democracy using actual fresh data for

voting and political business cycle, before and after election. The best paper is Nordhaus, William, D. (pp.169-190, 1975), such that solving problems by the use of differentials and integral calculus, with 25 equations and eight geometrical graphs, Figure 1 to Figure 8. In particular, the author here cites his Figure 8, as below using **Fig. 1**. His Figure 8 shows 'the political business cycle' discovered by Nordhaus' legwork data in nine countries, Australia, Canada, France, Germany, Japan, NZ, Sweden, UK, and US, 1947-1972. His Figure 8 clarifies cyclical symmetry drawn by two ratios, Inflation Rate and Unemployment Rate.

Nordhaus (ibid.) empirically proves the political business cycle. The author indicates here the first appearance literature: geographically for imperfect competition, J. Robinson (1933); mathematically, Hicks, J. R. (1932, 1935) for the elasticity of substitutions, σ, and now; Nordhaus (ibid.) for integrating geographical and mathematical, based on actual statistics data. Uniquely, Nordhaus (ibid.) matches the essence of purely endogenous under perfect competition measured accurately, with inflation and unemployment rate measured endogenously, using the KEWT database, 1960/90- 2011. Typically, the author points out symmetry versus asymmetry here in that σ =1.0000 guarantees symmetry and σ =1.0000 asymmetry with no exception, as the KEWT database proved this fact-finding.

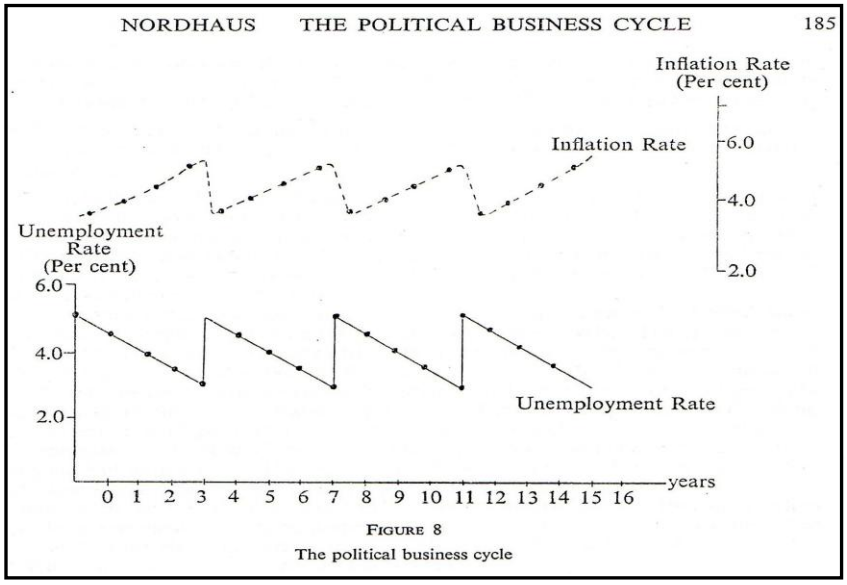


FIGURE 8
The political business cycle

Data source: explained by Note 2 of Nordhaus' (p.185, ibid.): The sources of the data are four papers, [13] , [14], [15] for OECD data, [22] for UK, Dept. of Employment Gazette, various issues, and [23] for United Nations, Statistical Yearbook, various issues.

Fig. 1 Political business cycle of Nordhaus, W. D. (Figure 8, p.185, 1975)

Gradually, researches on democracy spread wider and deeper connected with human life and economies. A core of economies for people is the relation between profits/returns and growth of output, or the first appearance of Phelps, Edmund, S. (638-643, 1961); Phelps, Edmund, S. (793-814, 1965), though its seeds research earlier started by Samuelson, Paul, A. (1938 to 1942). The rate of return, the growth rate of national disposable net income, poverty and wealth, all the economic policies must be based on the real assets, financial/market assets, and externals; all of these are connected each other. Particularly, the author pays attentions to: Sraffa, Piero (535-550, 1926); Robinson, Joan (1933); Samuelson, Paul, A. (486-496, 1965); Treisman, Daniel (399-457, 2000); and Carothers, T. (5-21, 2002). Further to: Ross, M. L. (325-, April 2001), Diamond, L. (21-35, 2002), Hegre, H; Ellingsen, T.; Gates, S., Gleditsch, N. P. (33-48, 2001), Levisky, S., and, Way, L. A. (51-65, 2002), mostly in *Journal of Democracy*.

Daniel, Treisman's (433-435, 438, ibid.) cross-national study is impressive in empirics. Among other hypotheses: Hypothesis 4 (H4) is reinforced by his Fig. 1 **'**Residual Unexplained corruption, Transparency International 1998 rating,' in his Conclusion. It implies that dynamic and static are compatible and everything changes beyond space and time. His figure matches the qualitative levels of democracy measured by the curvature, by individual, family, local area, town and city, country, and the world, as proved in the KEWT database, by year and over years.

Historically, democracy and the literature are cooperative and progress gradually, back and forth, towards a universal level. Cyclically, democracy and the literature march together, back and forth, towards a universal level. Each country has its own culture, as seemingly and radically shown in Thailand, Philippines, Myanmar, and Sri-Lank. Global and individuals and people practice learning by doing without stop. Dynamic and stable simultaneously mark the passage of time.

The above fact-findings vividly influence on the development of social and economic science. A problem may be: Social and economic science cannot be so flexible and dynamic as long as it is scientific or staying at the two dimension plane. Question: Is it really so? Answer is No. Scientific is enough staying at the two dimension plane. Why? The qualitative levels of democracy are measured by the curvature by hyperbola function, as earlier stated in the *EES*. The curvature by hyperbola function is based on endogenous equations with no assumption and under perfect competition or full-employment with no inflation/deflation.

3. How to Understand the Essence of Democracy by Country

Democracy is one of most debatable and serious objects not only in sociology, economics, behavioral, politics, mathematics, topology, and natural and social sciences but also in legislature, government, court of justice, political parties, leaders, decision-makers, lawyers, bureaucrats, people, and individuals. The clue of democracy spreads widely, with philosophical and historical backgrounds and land geographical by country, area, continents, and by culture and civilization.

Democracy changes beyond space and time, along with human life and society. The market principle apparently remains unchanged along with money used for exchanges in goods and services. The essence of democracy may remain unchanged but, why does the quality or level of democracy ever changing historically? Someone advocates 'radical' democracy while others deny even democracy similarly to against-capitalism. Do democracy and capitalism march together or not? What are wrong with democracy and capitalism? The author absorbs harmoniously and wholly all of these extremes and theoretically partial conclusions into the endogenous-equilibrium that reinforces the market principles.

It is natural for democracy by country to change and seldom have the same quality over years. Nevertheless, our society have no measure to numerate the quality level by year and over years. Dynamic and balanced equilibrium changes minute by minute, resulting in the qualitative level of democracy. Dynamic and balanced equilibrium changes year by year, resulting in the qualitative level of democracy. Dynamic and balanced equilibrium changes decade by decade, resulting in the qualitative level of democracy. In short, staticly and dynamically, the qualitative level of democracy changes, although the essence and direction of democracy never change. What are the essence and direction of democracy?

The essence of democracy is, of course, of the people, for the people, and by the people, although the order depends on the qualitative level of democracy. The direction is people and individuals by country, from government, interest-groups, selfish political parties, and wrong leaders and decision-makers who prefer money to human life and organic happiness that integrates body and mind. The direction is a concrete essence of democracy and accordingly, the perception and will that practice day and night towards moderation avoiding extremes and integrating dynamic and static. In other words, ultimate essence of democracy is human life itself beyond space and time. Human is destined to train for higher spirituality, believing the absolute existence, the Nature, and

God. Ultimate essence of democracy differs from various religions and, regardless of atheism or not, democracy is a way to moderation. The ultimate point of moderation is not measured since it is the origin of the two-dimension in the plane. However, numerous points of close-to-the ultimate origin are measurable. These points are essentially instable when these approach to the vertical asymptote. These points are essentially stable when these are apart from the vertical asymptote. These points are reversed and, instable when these cross over the vertical asymptote. Two extremes in the actual world correspond with the crossing narrow range in the endogenous-equilibrium. The author never blames the two extremes since the two extremes inevitably stand at such narrow ranges.

Similarly, capitalism or socialism: Both stand at partial theories and do not guarantee sustainable growth when consumption increases raising the wage rate. The goal of economic life is to increase the consumption per capita. Do statistics-oriented economists realize the consumption per capita? Answer is 'never realizable' since economists and decision-makers execute statistics-oriented methodologies under increasing deficits and debts. The market principles do not express causes at all since the principles show absolute prices by good and services and never aggregate the total amounts accurately. The endogenous-equilibrium inversely measures aggregated total amounts as national disposable net income ($NDI=C+S=W+\Pi$), simultaneously with accurate capital stock over years. A constant capital-output ratio is preserved over years and tentative stock at the beginning is replaced by no given capital stock.

The endogenous-equilibrium further reinforces statistics-oriented data proving a fact-finding that statistics data are always within a certain range of endogenous data of the KEWT database. The KEWT database take original data by country, each ten from the real assets and fifteen from the financial/market data, in *International Financial Statistical Yearbook*, IMF, 1960/90 to 2012/13. The levels of democracy are measured in the endogenous-equilibrium and accordingly, using the KEWT database by country and over years.

As a result, the endogenous-equilibrium and the KEWT database found eight nature-neutrals: Money M2-neutral of the financial/market assets to the real assets and accordingly the real rate of return is zero; consumption-neutral to technology and output; the relative share of capital-neutral to stop macro-inequality; the deficit-neutral to NDI if and only if deficit=zero; politics-neutral, and spirituality-neutral. These neutrals are proved in separate chapters

and do not refer to in this chapter for simplicity. This chapter indirectly suggests what processes realize politics-neutral and spirituality-neutral. Democracy levels are closely related to politics-neutral and spirituality-neutral.

In short, democracy, economies, and societies are tightly connected with each other. Someone indicates that behavioral economics and democracy may be a main area of deepening the discussions. However, this chapter avoids this idea. This chapter sticks to measure scientifically the democracy levels by country using the two-dimensional plane. Particularly, this chapter proposes new fact-findings for democracy-levels, not commonly based on geometrically connected with six trigonometric functions but solely based on the two-dimensional plane. Consumption per capita is attributed to national taste, preferences, culture, and history, independently of the rate of technological progress, which transudes in the level of democracy.

4. How to Increase in Consumption per Capita Socially and Economically?

The author understands that the qualitative levels of democracy are not independent of a stable economy, as the author glanced at in the previous section 2. Currently, there are several questions and debates for economic policies based on statistics data. These are some examples or experiments by item:

(1) They have three poles; 1) financial policy to increase M2 beyond traditional levels, 2) soundness of fiscal policy by raising consumption tax rate under floating exchange system, and 3) structural reform towards perfect competition in macro and micro economies. Here, the author replaces ‘strategies towards robust growth’ by policy, so as to match the endogenous-equilibrium; structural reform in the macro real assets. Is this argument pertinent?

(2) They advocate that by conquering deflation economies, macro and micro, recover growth and then, consumption per capita and the rate of unemployment will naturally increase. Is this logic true theoretically and empirically?

(3) They advocate that macro detachment, e.g., 15 %, between supply and demand should be solved first of all. Is this logic true theoretically and empirically?

(4) They advocate that the potential growth rate is 1.5 % supposedly and able to raise two times or more by increase new net investment. Is this logic true

theoretically and empirically?

(5) They advocate that they prefer full-employment to the increase in the real wage rate. Is it empirically possible for us to separate employment and the real wage rate under moderate inflation rate, say 2 %? Is the tradeoff between the rate of inflation and the rate of unemployment true? What rule sustainably holds between price movements, the rate of employment, and the real wage rate?

(6) Does new phase of monetary easing not result in bad influence on the world economies? Someone expresses the results 'chronic deflation and new type of bubbles.' How do they estimate or forecast the assets-bubbles?

(7) How to vividly maintain vitality and energy in mature society and under decreasing population country? They say worldwide researches under severe competition. How do they integrate theory and practice actually, apart from words of ideas and philosophies?

The author replies to the above arguments by uniting respective by item into whole theory and practice. The author's concrete answers support the qualitative levels of democracy so as to match the endogenous-equilibrium. The essence of the current economic analysis matches the essence of the qualitative levels of democracy. Both essences have taken root on our earth, universe, and human. Vertical/partial debates are based on the market principles and still friends for cooperative work on people's economy and society.

Three poles, monetary, fiscal, and financial/market policies are integrated into whole economic policies, whose base is fiscal policy as a core. The real assets polices are negative e while statistics financial/market policies are positive. Both sides constitute hyperbola measuring the negative and positive principle geometrically. Host is fiscal policy in the real assets and our society needs to obey fiscal policy, apart from an assumption that government cash flow-in and -out equals government saving less net investment. As a result, corporate tax reduction, subsidies or minus taxes, and government net investment are all misled.

The current economic analysis has many assumptions vertically or partially. Suppose perfect competition. Then, all of debates are actually realized and solved; full-employment, no inflation and no deflation, increases in consumption per capita and the real wage rate, no assets-bubbles, increase in net investment of enterprises and the private sector, stable and balanced BOP and deficit. Therefore, our society needs to connect statistics data with endogenous

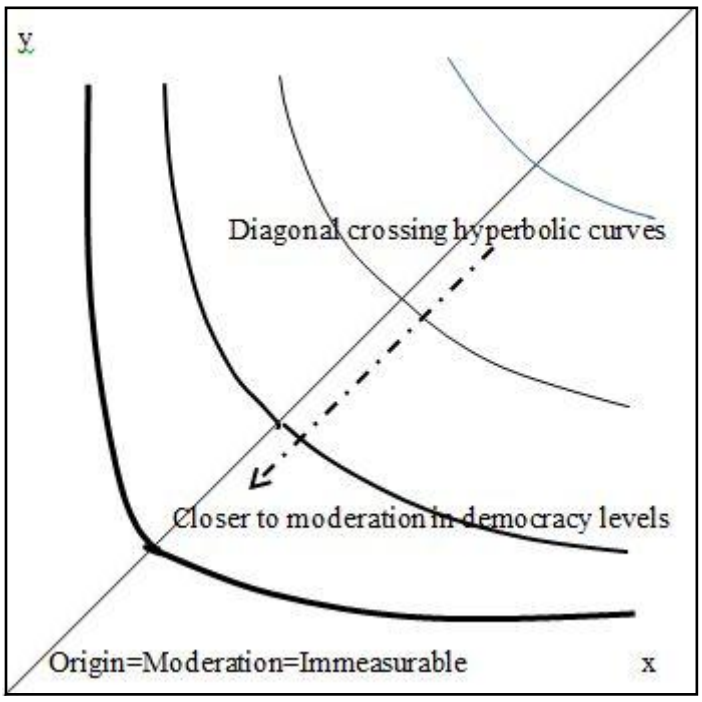
data.

The current economic analysis, instead, increases all kinds of government interventions. This process is against the truth and people feel uneasy and discomposed: government control destroys sound economic operations increasing no structural reforms and deficit by year. The qualitative levels of democracy inevitably reflect this process.

5. How to Measure the Qualitative Levels of Democracy: With Empirical Linear Results

This section wholly clarifies processes to measure and shows empirical results by country, limiting to eight linear functions reduced from endogenous equations.

The qualitative levels of democracy are measured by each using the curvature of hyperbola functions. **Fig. 2** shows its whole design. The measure for evaluation is modesties that express the qualitative levels of democracy. The origin shows an ultimate point of moderation that cannot be measured. Points A, B, C, and D each show various level of moderation. Point A is modest more than Point B since it is closer to the origin. Point C is modest more than Point D. **BOX 1** shows all the hyperbola functions with each type of characters by function. The type of characters is designed differently from hyperbolic functions mathematically connected with six trigonometric functions. For five characters of hyperbola functions, the KEWT database have weighted on the Width and Sharpe and their devices rather than the curvature but now, the author calculates the curvature directly in the KEWT database. Accordingly, our hyperbola functions could have unique measure for evaluation as above. BOX 1 does not include hyperbola functions for measuring the qualitative levels of democracy. The author here prepares for formulating the qualitative levels of democracy starting with BOX 2. **BOX 2** is a box that takes up plausible endogenous equations before reducing to hyperbola functions. The author is much obliged to a fable of ‘Solovia’ and ‘Oikos Nomos,’ explained by Phelps, E. S. (1961). Phelps, E. S. (638-643, 1961) has been a target of the design of measure for evaluating the qualitative levels of democracy.



Notes: Generally, the curvature in a hyperbola (standard type), $\mathrm{h(x)} = \frac{cx+d}{ax+b}$, is drawn by; $\text{Curvature} = \frac{1}{\text{Sharpe}}$. $\text{Curvature} = 1/\sqrt{2\left|\frac{f}{a}\right|}$. $\text{Curvature} = 1/\sqrt{2\left|\frac{1-b\cdot c}{a^2}\right|}$.

Implication of moderation: A reduced form of endogenous equation has the origin by supposing VA=0 and HA=0. This origin is a final target of human being but, not measurable, while the curvature closer to the origin the higher the qualitative level of democracy. Democracy is stable and optimum range is narrower, so that people are happier, body and mind, and politics-neutral prevails steadily.

Fig. 2 How to numerate the qualitative levels of democracy by country

How can the author directly formulate ratio relationship between the capital-labor ratio and the propensity to consume? The author thinks, this is worthy of further development in economics and econometrics. The author challenges for other ratio relationships related the capital-output ratio, using the propensity to consume, the rate of change in population, and the speed coefficient. **BOX 3** shows these preliminary arrangements. A constant capital-output ratio holds over years and with no initial value given in the KEWT database. This fact is our axiom discovered after numerous experiments by applying the capital-output ratio to this fact in the KEWT database. Therefore, the author designed eight ratio functions each as reduce form of endogenous equations: $c(k)$, $c(n)$, $\Omega(n)$, $c(\Omega)$, $\Omega(c)$, $k(\Omega)$, $\Omega(k)$, and $\lambda^*(\Omega)$. Each ratio function avoids given policy parameters and replaces these by measured parameters and variables since there is no external or exogenous in the endogenous-equilibrium. Endogenous equations are transformed to hyperbola functions as follows (avoiding *Italic*, here):

(1) $c(k) \rightarrow y = \frac{d}{ax}$: From to $(1-\alpha) = \frac{c}{(rho/r)}$ & $k = \frac{\alpha/(1-\alpha)}{(r/w)}$ to $y = \frac{d}{ax}$; b=0, c=0, $a = (r/w)$, $d = \alpha(rho/r)$.

(2) $\Omega(n) \rightarrow y = \frac{d}{ax+b}$: From $\Omega = \frac{-i(1-\alpha)\beta^*}{i(1+n)\beta^*-(i(1+n)+n(1-\alpha))}$ to $y = \frac{d}{ax+b}$; c=0, $a = i(1-\beta^*)+(1-\alpha)$, $b = i(1-\beta^*)$, $d = i \cdot \beta^*(1-\alpha)$.

(3) $c(\Omega) \rightarrow y = ax + b$: From $(1-\alpha) = \frac{c}{(rho/r)}$ & $\alpha = \Omega \cdot r$ to $y = ax + b$; $a = -rho$, $b = \frac{rho}{r}$.

(4) $c(n) \rightarrow y = \frac{d}{ax+b} + \left(\frac{rho}{r}\right)$: From $\Omega = \frac{-i(1-\alpha)\beta^*}{i(1+n)\beta^*-(i(1+n)+n(1-\alpha))}$ & (2) and (3) to $y = \frac{d}{ax+b} + \left(\frac{rho}{r}\right)$; c=0, $a = i(1-\beta^*)+(1-\alpha)$, $b = i(1-\beta^*)$, $d = -rho \cdot i \cdot \beta^*(1-\alpha)$. Exceptionally $c(n)$ have to be formulated by combining two formulations $\Omega(n)$ and $c(\Omega)$.

(5) $\Omega(c) \rightarrow y = ax + b$: From $(1-\alpha) = \frac{c}{(rho/r)}$ & $\alpha = \Omega \cdot r$ to $y = ax + b$; $a = \frac{-1}{rho}$, $b = \frac{1}{r}$.

(6) $\Omega(k) \rightarrow Y = A + BX$: [1] From $TFP = A = \frac{k^{1-\alpha}}{\Omega}$ or $\Omega = \frac{k^{1-\alpha}}{A}$ to $Y = A + BX \leftarrow y = ax^b$; $a = A = (1-\alpha)$, $b = B = LN(TFP)$, where TFP is endogenously measured (as a measured variable, instead of a given policy parameter in the literature).

(7) $k(\Omega) \rightarrow Y = A + BX$:[2] From $TFP = A = \frac{k^{1-\alpha}}{\Omega}$ or $\Omega = \frac{k^{1-\alpha}}{A}$ to $Y = A + BX \leftarrow y = ax^b$; $a = \frac{1}{1-\alpha}$, $b = B = \left(\frac{1}{1-\alpha}\right) LN(TFP)$.

(8) $\lambda^*(\Omega) \rightarrow y = \frac{d}{ax+b}$: From $\lambda^* = (1-\alpha)(n - g_A^*)$ to $y = \frac{d}{ax+b}$; a=1, c=0, $b = i(1-\beta^*)\big((1-\alpha)+(1+n)\big)$.

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[1] Suppose the case of $\Omega(k) = \frac{k}{r \cdot k + w}$. This case expresses a hyperbola function. The author smells out a linear function and, prefers a linear function to a hyperbola function.

[2] An exceptional reason for $k(\Omega) \neq LN(k)(LN(\Omega))$ under $\Omega = \Omega^* = \Omega_0$:

The author uses either λ^* or β^*, $\beta^* = \frac{\Omega^*\big(n(1-\alpha)+i(1+n)\big)}{i(1-\alpha)+\Omega^* \cdot i(1+n)}$.

(1) $\lambda^* = (1-\alpha)n + (1-\delta_0)g_A^*$. $\Rightarrow$ $\lambda^* = (1-\alpha)(n + i(1-\beta^*))$ under $\alpha = \delta_0$.

(2) $\Omega^* = \frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)(1+n)+n(1-\alpha)}$. $\Rightarrow$ $\Omega(1-\alpha) = \frac{\beta^* i(1-\alpha)}{i(1-\beta^*)(1+n)+n(1-\alpha)}$.

Using $k^{1-\alpha}(\Omega)$ and $\Omega = \frac{C \cdot \Omega}{-A \cdot \Omega + B}$ as a reduced form, can one solve $k^{1-\alpha} = \frac{C \cdot \Omega}{-A \cdot \Omega + B}$? Of course, it is impossible to solve it. If $k^{1-\alpha} = \frac{C \cdot \Omega}{-A \cdot \Omega \cdot B}$ were derived, it is possible to solve, as proved in the case of $\lambda^*(\Omega)$, where $\lambda^*(\Omega) = LN(\lambda^*)(LN(\Omega))$ holds.

BOX 1 Twenty-five hyperbola equations and specific types of hyperbolas by element

1-1 speed(i). 1.2 i(speed). 1.3 speed(n). 1.4 n(speed).

2-1 r^*(i). 2-2 i(r^*). 2-3 r^*(n). 2-4 n(r^*). 3-1 Ω^*(i). 3-2 i(Ω^*).

3-3 Ω^*(n). 3-4 n(Ω^*). 4-1 i(n). 4-2 n(i). 4-3 $\Omega^*(\beta^*)$. 4-4 $\beta^*(\Omega^*)$.

5-1 β^*(n). 5-2 n(β^*). 5-3 $\widetilde{\beta^*}$(n). 5-4 n$\left(\widetilde{\beta^*}\right)$. 6-1 β^*(i). 6-2 $\widetilde{\beta^*}$(i).

6-3 α(i). 6-3-2 $\alpha(n)$, to cope with stop-macro inequality. 6-4 $\widetilde{\alpha}$(i).

Special cases: β^* versus $\widetilde{\beta^*} = 1 - \beta^*$, and α versus $\widetilde{\alpha} = 1 - \alpha$.

Specific types of hyperbolas by element, a, b, c, d, e, and f:

element=0 and two elements=0 or $y = \frac{c}{b}x + \frac{d}{b}$.

A fable explained by Phelps, E. S. (pp.638-643, 1961) is relationship between the capital-labor ratio on the *x* axis and the propensity to consume on the *y* axis. BOX 1 does not include this relationship directly. Related endogenous equations are based on seven endogenous parameters and each shown in Notations in the *EES* (xxxvi-vii, ibid.):

i) a=0, the linear type: $y = \frac{cx+d}{b}$ or $y = \frac{c}{b}x + \frac{d}{b}$.

ii) b=0: $y = \frac{cx+d}{ax}$ and $y = \frac{c}{a} + \frac{d}{ax}$.

iii) c=0 and d=1: $y = \frac{1}{ax+b}$.

iv) d=0: $y = \frac{cx}{ax+b}$ and $y = \frac{c}{a} + \frac{-\frac{b \cdot c}{a}}{ax+b}$.

v) c=0: $y = \frac{d}{ax+b}$.

vi) No zero, the standard type: $y = \frac{cx+d}{ax+b}$ and $y = \frac{c}{a} + \frac{d-\frac{b \cdot c}{a}}{ax+b}$.

vii) $y = \frac{cx+1}{ax}$, where $b = 0$ and $d = 1$ (correctly adding):

BOX 2 Endogenous equations numerating a fable of 'Solovia' and 'Oikos Nomos', explained by Phelps, E. S. (1961)

Data source: Cited from Appendix of "*Earth Endogenous System*" (476-523, 2013) and adding two equations of one type, $y = (cx + 1)/ax$; Similarly to those hyperbola equations in Chapter 9, 'Optimum functions-measure, numerated by using curvature in the two-dimensional plane.'

Chapter 10, *HEU*

1. The relative share of capital, $\alpha = \Pi/Y$: $(1-\alpha) = \frac{c}{(rho/r)}$ and, the capital-labor ratio, $\frac{K}{L} = \frac{(\alpha/(1-\alpha)}{(r/w)}$ or $k = \frac{w \cdot \Omega}{1-r \cdot \Omega}$. $c = C/Y$ is the propensity to consume; (rho/r) is the relative discount rate for goods and services, by country and at the macro-level; $w = W/L$ is the wage rate; $\Omega = K/Y$ is the capital-output ratio and, $\Omega = \Omega^* = \Omega_0$ holds.
2. The rate of return, $r = \Pi/K$ and, $r = r^* = r_0$: $r = \alpha/\Omega$. If deficit is zero, this endogenous rate of return corresponds with the rate of profits in enterprises.
3. The elasticity of substitutions, $\sigma = \frac{-\Delta k/k}{\left(\Delta\left(\frac{r}{w}\right)\right)/\frac{r}{w}}$. MRS $= \Delta r/\Delta w$ is the marginal rate of substitution. In the KEWT database, the *sigma* sharply fluctuates by country, year and over years. In the transitional path by year, however, the *sigma* is exactly equal to 1.000000 (for recursive programming, see Chapter 16 in the *EES*).
4. The rate of technological progress and the growth rate of per capita output, each as FLOW: $g_A^* = i(1-\beta^*)$ and $g_y^* = g_A^*/(1-\alpha)$.
5. The growth rate of output, $g_Y^* = \frac{g_A^*(1+n)}{(1-\alpha)} + n$. The speed years, $1/\lambda^*$, are adjusted for moderate equilibrium, under $n_E = n$ or under full-employment.
6. The endogenous Phelps coefficient, $x = r^*/g_Y^*$ and, $x = \alpha/(i \cdot \beta^*)$: $r^* = \left(\frac{\alpha}{i \cdot \beta^*}\right) g_Y^*$. This endogenous coefficient corresponds with the exogenous Phelps coefficient. The endogenous cost of capital, $CC = C\ of\ C$: $CC = r^* - g_Y^*$.
7. $A(t) = \frac{k(t)^{1-\alpha}}{\Omega(t)}$. (See Note 11 on page 25, PhD thesis, Nov 2003). As a result,

   $g_{A(FLOW)}(t^*) = g_{TFP(STOCK)}(t^*)$, where A=total factor productivity (*TFP*) as STOCK.

   $g_{A(FLOW)}(t^*) = i(t^*) \cdot (1-\beta(t^*)$ and, $A_{TFP(STOCK)}(t^*) = A_0(1 +$ *gA(FLOW)t∗)1λ∗.*
8. The speed coefficient, $\lambda^* = (1-\alpha)(n - g_A^*)$, where $\delta_0 = \alpha$ and $g_A^* = i(1-\beta^*)$.

Already the author has had two linear functions, $r(n)$ and $n(r)$, exceptionally (see p.482, the *EES*, 2013). The above functions are divided into two groups, hyperbola and linear.

Hyperbola functions, four

$$c(k) \to y = \frac{d}{ax};\ \Omega(n) \to y = \frac{d}{ax+b};\ c(n) \to y = \frac{d}{ax+b} + \left(\frac{rho}{r}\right);\lambda^*(\Omega) \to y = \frac{d}{ax+b}\ .$$

Linear functions, eight

$$c(\Omega) \to y = ax + b;\quad \Omega(c) \to y = ax + b;\quad \Omega(k) \to Y = A + BX \text{ (using LN)};$$
$$k(\Omega) \to Y = A + BX \text{ (using LN)}.$$

Eight cases are: $c(k)$, $c(n)$, $\Omega(n)$, $c(\Omega)$, $\Omega(c)$, $k(\Omega)$, $\Omega(k)$, and $\lambda^*(\Omega)$. How can the author connect one dependent ratio with the other independent ratio? One and the other ratios for the above eight cases are all unknown in endogenous equations hitherto. The author discovers three new facts concretely for the above eight hyperbola functions. Also the author finds the real behind these concrete fact-findings:

New fact 1, for $\lambda^*(\Omega)$: Inverse of speed years, $1/\lambda^* = 1/\left((1-\alpha)n + (1-\delta_0)g_A^*\right)$, reduced to $1/\lambda^*_{\alpha=\delta_0} = (1-\alpha)(n + i(1-\beta^*))$. There is no difference between this equation and a corresponding equation, $(1-\alpha)(n + \text{given growth rate})$, formed by Sala-i-Martin, X. (PhD 1990, 1992) and Barro and Sala-i-Martin (1995), as clarified in a separate chapter. Starting from $\Omega(\alpha)$, $\lambda^*(\Omega)$ was finally proved theoretically and empirically (see **Table A1** and **Fig. 2**). The real is the same even though purely endogenous and exogenous in the literature appear differently. The real is deeply expressed by a constant capital-output ratio over years, as one axiom the author scarcely finds.

New fact 2, for $\Omega(c)$: Independent relation between the capital-output ratio and the propensity to consume is solved by using $\Omega(n)$, where the relative share of capital and the rate of change in population, $n_E = n$, is interrelated under full-employment. $\Omega(c)$ presents new fact-finding that the gradient value equals the intercept value by country and year and over years and that $\Omega(c)$ is symmetric always (see **Table A1** and **Fig. 2**). Symmetry spread endogenously and non-symmetry spread vertically based on prices and quantities separable in the literature.

New fact 3, for $k(\Omega)$, $\Omega(k)$**:** An identity (Kamiryo, PhD 2003) of $k = (TFP \cdot \Omega)^{\frac{1}{1-\alpha}}$ or $k^{1-\alpha} = TFP \cdot \Omega$ led to prove $k(\Omega)$ and $\Omega(k)$, using growth accounting theoretically and empirically (see **Table A1** and **Fig. 2**).

Resultantly, $k(\Omega)$ and $\Omega(k)$ wholly prove technological progress, flow and stock:

$g_{A(FLOW)}(t^*) = g_{TFP(STOCK)}(t^*)$, where A=total factor productivity (TFP) as STOCK.

$$g_{A(FLOW)}(t^*) = i(t^*) \cdot (1 - \beta(t^*)$$

$$A_{TFP(STOCK)}(t^*) = A_0(1 + g_{A(FLOW)}(t^*))^{1/\lambda^*}$$

All the parameters and variables are united, consistently, coherently, and simultaneously in parallel. One is one, not counted between two or three but one within a whole. This is the real. The author proves this fact theoretically and empirically.

Lastly, the author interprets linear results for four countries using Table A1 and, Fig. 2 and 3. This is because linear results are transparent mirrors of the qualitative levels of democracy. First of all, each country, the US, the UK, Japan, and Sweden, has its own culture and history, in harmony with the rate of technological progress, under the market principles. Leaders and policy-makers have done whatever policies they think best, depending on the structure of the balance of payments, deficit, and the balance in the private sector (or, households and enterprises).

Japan has enough room for various sets of economic policies to recover economic robustness, much more than other countries. Japan, however, has overlooked resultant deflation for two decades. Currently, Japan stepped into inflation policy and realized weak exchange rate. Due to the money-neutral, this policy does not improve the real asset realities so that policy-makers take all the policies possible apart from endogenous equilibrium. It implies that people and small enterprises have to endure all the losses in realities, inevitably falling into twine deficits.

Regarding ‘losses,’ it seems to have Japanese big companies won in strategies internationally and competitively. Corresponding big companies in other countries may suffer from share-down strategically. It implies that people in other countries have to endure all the losses similarly. The qualitative levels of democracy are most stable and sustainable historically when all the policies are people-oriented. It implies that low level of democracy is spiritually exported to other countries from one selfish country. Under the name of competition, unfavorable democracy spreads in industry-oriented countries over the world. Short-sighted company executives in one country must perceive Peter Drucker’s spirit. The author advocates that

sweat human temptations does not last long. Human mind and body are united. Spirit is not independent of economic competition. Perfect competition is measured endogenously, as proved in this chapter. The author defines it as a circuit of low level of democracy-export and -import.

Low level of democracy is, to some extent, next door to 'controlled economy.' Controlled economy can do whatever experiments to hide and guard against unfavorable results, as clarified historically. Controlled economy is most far from perfect competition that guarantees sustainability and, is apt to avoid structural reform interest groups dislike most. Social and economic realities become far from politics-neutral and spirituality-neutral. Democracy needs to accept wide ranges of thoughts and policies generously.

On the other hand, the US, the UK, and Sweden, each abide stable policies much more than those in Japan. Policy-makers have learned a lot by learning by doing. Experiments are more conservative for people, particularly in small populated countries, not limiting to the three countries in Table A1 and, Fig. A1, Fig. A2, and Fig. A3, each at the end of this chapter. Differences of economic robustness between Japan and three countries may come from be those in culture and civilization historically. Why do policy-makers in Japan not encourage traditional race characteristics for the last two thousand years or more? This is a short cut to recover from weak economy in the global economies. Some may confuse and mix global and short-sighted prosperity by international company. This confusion is wrong endogenously and proved using 86 countries in 1980/90 to 2011 by sector. Technological progress and preferences are compatible and managers must awake. This is a way of qualifying democracy.

6. Conclusions

This chapter focuses on how to measure qualitative levels of democracy.

The author got several by-products for measures, where the author finds eight linear functions, $c(k)$, $c(n)$, $\Omega(n)$, $c(\Omega)$, $\Omega(c)$, $k(\Omega)$, $\Omega(k)$, and $\lambda^*(\Omega)$, each as a reduced form of hyperbola functions. Total number of linear functions is eight, including two linear functions, $r(n)$ and $n(r)$, already found in the *EES*. Blessed by the natural science, eight linear functions are each simply measured without using graphs and, most fitted for qualitative levels by function, parameter, and variable. The most proud case is the qualitative levels of democracy.

Chapter 10, *HEU*

This chapter highlights three concrete fact-findings for four linear functions among eight linear functions: **New fact 1,** for $\lambda^*(\Omega)$; **New fact 2,** for $\Omega(c)$; **New fact 3,** for $k(\Omega)$ and $\Omega(k)$. These four linear functions correspond with a unique axiom of a constant capital-output ratio, $\Omega = K/Y = \text{const.}$. This axiom reinforces purely endogenous and the endogenous-equilibrium wholly. These four linear functions fundamentally guarantee robustness and sustainability of all the economic policies, the real, the financial/market, and externals.

Eight linear functions are all related to the qualitative levels of democracy. Two factors, *K* and *L*, are alive in a discrete endogenous Cobb-Douglas production function. Democracy is deeply involved in human life and social and economic science, even under the market principles and perfect competition with no assumption.

Empirical results, as shown in Table A1 and Figs 2 and 3, were inspected repeatedly in the transitional path by year, using recursive programing. Human is selfish by nature yet, human target is unselfish, as proved historically. Selfish leads to a miserable circuit of low level of democracy-export and -import, as the author discussed in the previous section. Human is connected with the real, hidden in three new facts above.

No problem and inconsistency occur between Twenty-five hyperbola (see BOX 1) and eight linear functions related to technology here and Robert, M., Solow's (1956, 1957) residual technology in the literature. Statistics data are always within a certain range of endogenous data, as long as both are based on the Cobb-Douglas production function. By the same reason, Keynesians, neo and new, clarify some aspect in the endogenous-equilibrium consistently, as proved in a separate chapter.

The qualitative levels of democracy show human history consistently and concretely. The author expects that this chapter may present better direction of democracy over decades. This chapter proves that economic and social information and openness are measured by country. This will contribute to the depth-cultivation of democracy in the near future. This is because in the literature there has been no method to measure qualitative levels hidden in an amount.

Our society cooperates with the literature under the market principles: One for statistics data and the other endogenous data march harmoniously, towards world peace and, people's welfare and happiness.

BOX 3 Transformed processes from endogenous equations to hyperbola functions numerating Phelps' (1961) fable

1.→ $(1-\alpha)=\frac{c}{(rho/r)}$, for $c=C/Y$.

2.→$k=\frac{w\cdot\Omega}{1-r\cdot\Omega}$, for $k=K/L$ and $\Omega=K/Y$.→$\Omega=\frac{k}{r\cdot k+w}$.

This shows $y=\frac{x}{ax+b}$ type, where c=1 and d=0.

3.→$\alpha=\Omega\cdot r$, for $\alpha=\Pi/Y$ and $\Omega=K/Y$.→$(1-\alpha)=1-\Omega\cdot r$.

$\rightarrow(1-\Omega\cdot r)=\frac{c}{(rho/r)}.\rightarrow c=(1-\Omega\cdot r)(rho/r).\rightarrow c=(rho/r)-rho\cdot\Omega$.

Adversely, $\Omega=-\frac{c}{rho}+\left(\frac{1}{r}\right)$.

This shows a linear function, $y=\frac{cx+d}{b}$ type, where c=rho and d=(rho/r).

The author finds a linear function type is limited to $r(n)$, $n(r)$ and here eight cases; $c(k)$, $c(n)$, $\Omega(n)$, $c(\Omega)$, $\Omega(c)$, $k(\Omega)$, $\Omega(k)$, and $\lambda^*(\Omega)$. The inverse of speed years for convergence here, λ^*, does not use the DRC coefficient, δ_0, but that satisfying $\alpha=\delta_0$ at convergence, where Total Factor Productivity and its growth rate perfectly match the rate of technological progress (see Notes in the *EES*).

4.→$TFP\cdot\Omega=k^{1-\alpha}$, $\Omega=K/Y$ and $k=K/L$.→$c(k)$ and $k(c)$ are each a linear function by using natural and/or common logarithm, $y=ax^B\Rightarrow Y=A+BX$.[3] The gradient value equals the intercept value, which implies asymmetric. This is a new fact-finding and constitutes a highlight in this chapter.

[3] Natural logarithm has a base of Napier, e=2.71828, where 'LN' is used for e^x. Common logarithm has '10' as a base, where 'log' is used for 10^x. Relation between logarithm and linear are commonly transformed. A and B are each given by fixing related parameters. It is not required to use $A(t)=\frac{k(t)^{1-\alpha}}{\Omega(t)}$ for the above transformation. It is possible for us to measure the transformation equations if capital stock is accurately measured consistently with all the variables and parameters.

Table A1 Linear functions, $c(\Omega), \Omega(c), \Omega(k), k(\Omega)$, each with gradient and intercept by country, 2011

	$c(\Omega) \rightarrow y = ax + b$:	$a = -rho$	$b = \frac{rho}{r}$
the US	c=-0.13816Ω+1.2166	-0.13816	1.2166
U K	c=-0.11886Ω+1.1145	-0.11886	1.1145
Japan	c=-0.03247Ω+1.0595	-0.03247	1.0595
SWEDEN	c=-0.0836Ω+0.9625	-0.08360	0.9625
	$\Omega(c) \rightarrow y = ax + b$:	$a = \frac{-1}{rho}$	$b = \frac{1}{r}$
the US	Ω=-7.23794c+7.2379	-7.23794	7.2379
U K	Ω=-8.41295c+8.4129	-8.41295	8.4129
Japan	Ω=-30.8011c+30.8011	-30.8011	30.8011
SWEDEN	Ω=-11.9624c+11.9624	-11.9624	11.9624
	Ω(k)	$a = A = (1 - \alpha)$	$b = B = LN(TFP)$
the US	Ω=0.8129k+2.9619	0.8129	2.9616
U K	Ω=0.8595k+2.6125	0.8595	2.6125
Japan	Ω=0.8841k+6.9820	0.8841	6.9820
SWEDEN	Ω=0.8665k+4.9654	0.8665	4.9654
	k(Ω)	$a = \frac{1}{1-\alpha}$	$b = B = \left(\frac{1}{1-\alpha}\right) LN(TFP)$
the US	k=1.2302Ω+3.6432	1.2302	3.6432
U K	k=1.1634Ω+3.0395	1.1634	3.0395
Japan	k=1.1311Ω+7.8973	1.1311	7.8973
SWEDEN	k=1.1541Ω+5.7306	1.1541	5.7306

Data source: The KEWT database, whose original data are from *IFSY*, IMF.

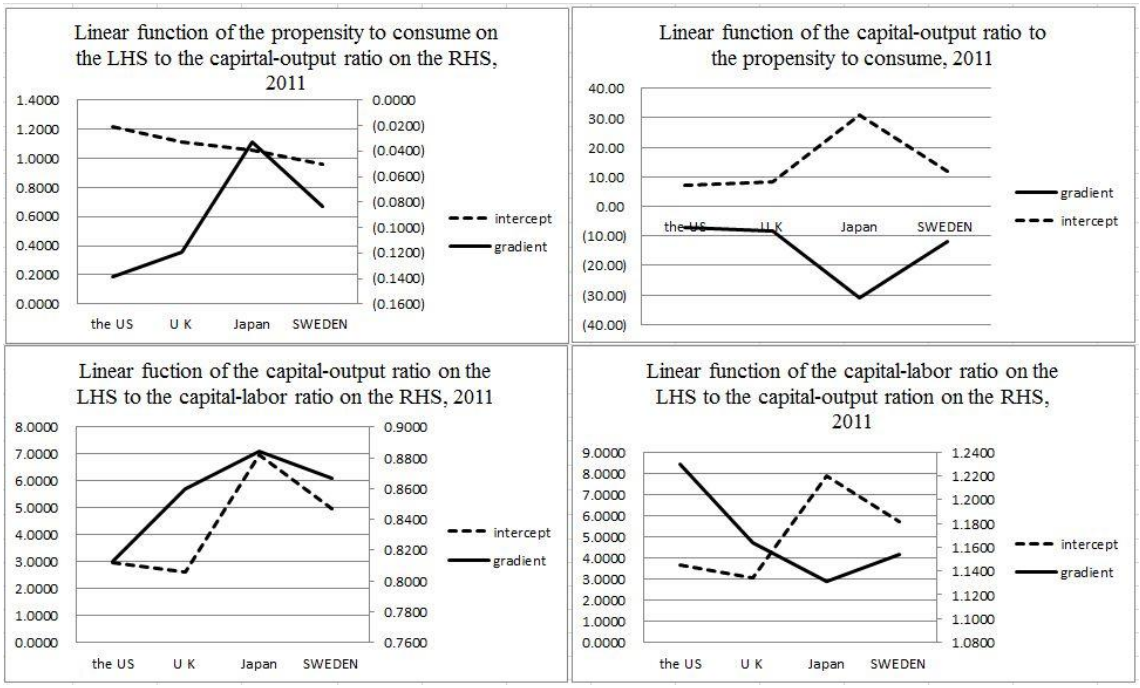
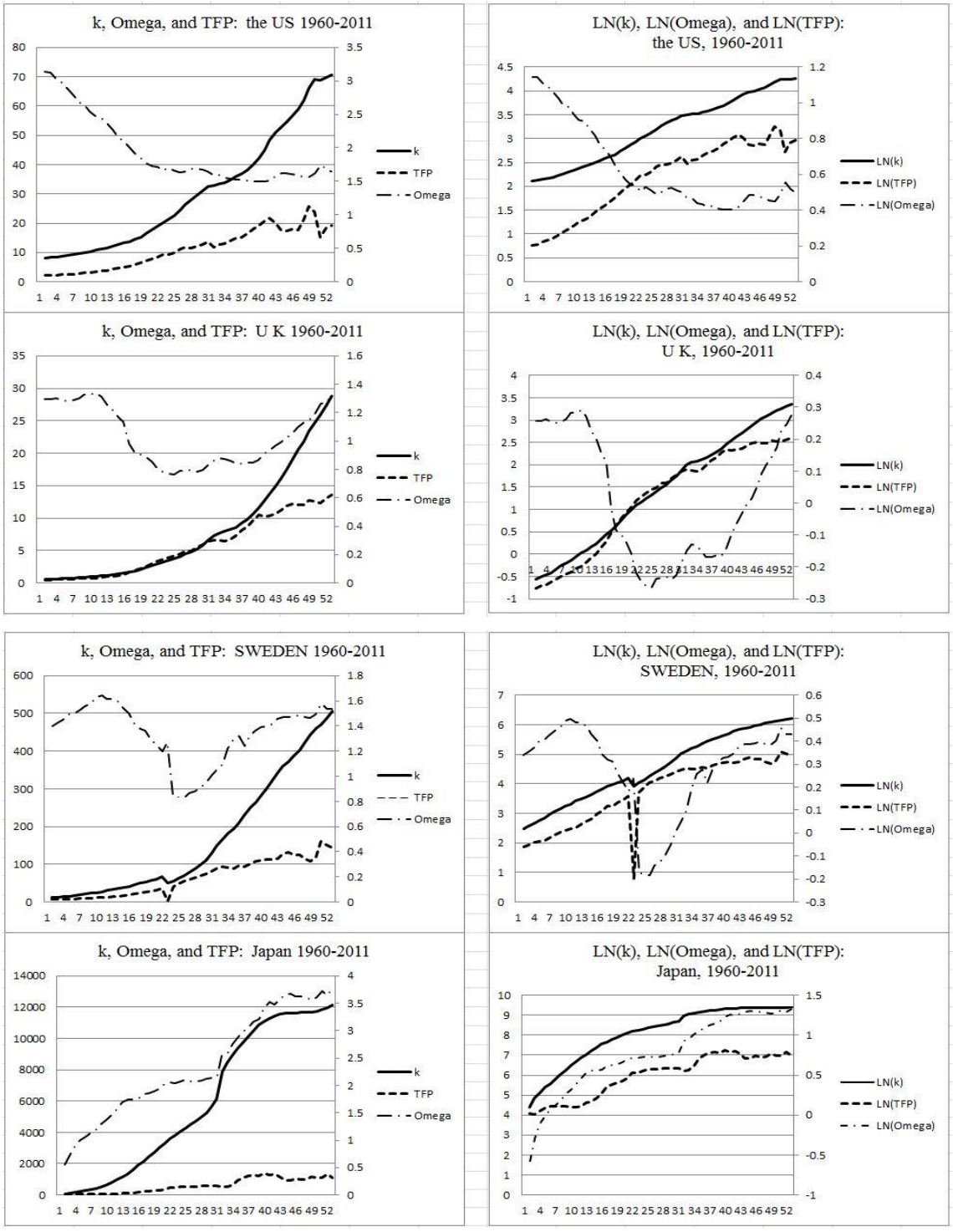
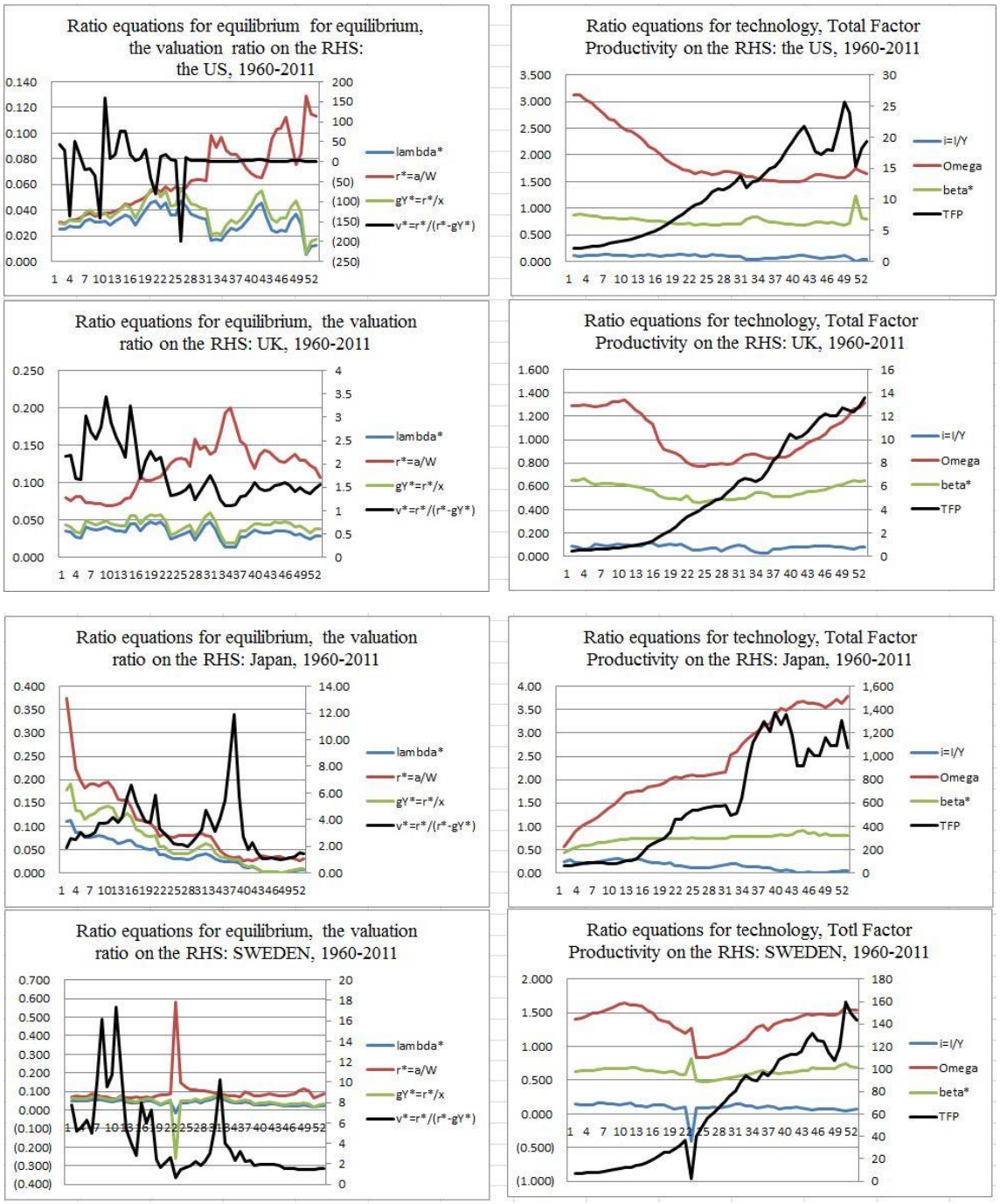


Fig. A1 Results of Table A1 by country, 2011



Data source: The KEWT database, whose original data are from *IFSY*, IMF.

Fig. A2 Trends of the capital-labor ratio, the rate of tech. progress, and the capital-output ratio: real vs. LN measure



Data source: The KEWT database, whose original data are from *IFSY*, IMF.

Fig. A3 Linear hyperbola functions by country: the US, U K, Japan, and Sweden

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