Chapter 5

Why is A Discrete Cobb-Douglas Production Function A Numerical Core of Social and Economic Science?

Foreword to Chapter 5

This chapter aims at clarifying the current characteristics of the economic literature. The characteristics are well revealed, starting with the Cobb-Douglas production function and inspecting the endogenous system and its KEWT database for 81 countries by countries, sector (government and private), and year and over years. The literature stays at the price-equilibrium while the KEWT database at the endogenous-equilibrium. A scientific discovery is that the endogenous-equilibrium is a surrogate for the price-equilibrium and reinforces the market principles cooperatively. The literature is exogenous and well-behaved while the endogenous system endogenous and totally measured. Taylor rule inevitably prevails when and where Say's (1803) law of supply= demand is denied as accepted in the literature. Nevertheless, the market results obey author's neutrality of the financial/market assets to the real assets under the endogenous-equilibrium. If the market results become short-sighted and biased, this fact comes from the difference between the micro-based level and the macro aggregated endogenous level. In short, the literature cannot integrate various partial equations with different assumptions while the KEWT database simultaneously integrates all the aspects and equations, with no assumption and beyond ex-post results versus ex-ante causes.

This chapter sums up the essence of the Cobb-Douglas production function, scientifically but not mathematically, starting with Euler's theorem and the compatible market principles. The essence of the author's C-D production function (hereunder, the C-D PF) unites equations and its hyperbola with philosophy that has been discussed separately from equations in the literature. A system of partial=whole realizes the united philosophy and equations in two dimensional hyperbolas. Without money-neutral led by the real assets endogenously, the author could not find the foundation of social/economic science in the *EES*.

Signposts to Chapter 5: Cobb-Douglas, Euler's theorem, discrete, the real assets, the financial/market assets, the market principles, scientific, partial, whole, equations, assumptions, purely endogenous, the relative share of capital, the capital-labor ratio, the capital-output ratio, break-even point (BEP), *EES* ("*Earth Endogenous System*")

1. Questions and Answers to the essence of the C-D production function

The author first questions: (1) Why does the Cobb-Douglas production function (hereunder, the C-D PF) presume well-behaved? (2) Why is the C-D PF for economics, micro and macro, and econometrics, used as a base tool, although homogeneous degree one is simple? (3) Why does the C-D PF, in the literature, solely use the capital-labor ratio, neglecting the capital-output ratio? (4) Why does the C-D PF follow an exogenous rate of technological progress, instead of developing an endogenous rate of technological progress? (5) Is it impossible for the C-D PF to specify and further erase assumptions required for economic equations?

These questions are related to the essence of the C-D PF. The author, in this section, abbreviates the background and construction history and roughly sums up the implication of (1) no assumption and (2) limits of econometrics externally using statistics data. The core is 'purely endogenous with no assumption to endogenous equations' in "*Earth Endogenous System*" (hereunder the *EES*).

First, assumptions are used differently by model/system. Assumptions are used for theory and supporting equations and also for statistics data in estimation, although both belong to economic analyses, economics and econometrics. These assumptions are, strictly speaking, all exogenous. Economists say endogenous but, its essence is not 'purely endogenous' but partial. It implies that a system set up must use no assumption in equations formulated for the system and exclude any use of externals in statistics. It implies that economists cannot express 'no assumption' under the market principles. Conclusively, the *EES* and its KEWT database use the same: KEWT database holds under no assumption and measures the price levels, absolute *P* and relative *p*, whose value accurately equals 1.0000000, and as a result, the elasticity of substitution, $\sigma = -\frac{\Delta k/k}{\Delta(r/w)/(r/w)}$, exactly becomes 1.0000000: $P = p = 1/0000000$ and $\sigma = 1.0000000$, where the marginal rate of substitution (*MRS*=*r*/*w*), *MPL*=*w*, and *MPK*=*r*. It implies that perfect competition under one price per commodity hold, reinforcing and cooperating with the market principles. All of these facts are solely macro-oriented and proved in the *EES* and KEWT, theoretically and empirically.

Second, for limits of econometrics externally using statistics data, the author here selected and inspected four papers in *Econometrics* (see, References) that commonly use *the C-D PF* : (1) Dhrymes, P. J. (1962), Kmenta, J. (1964); (2) Fisk, P. R. (1964, 1966); and (3) Zellner, A., Kmenta, and J. Dreze, J. (1966). These three respectively publish unbiased estimators for the parameters, properties of alternative estimates, estimation of marginal product, and specification and estimation of modeling. Last (4), Marshall, David, A. (1972, 2005), estimates the expected marginal rate of substitution (cf. the above *MRS*). The author pays attention to their key words such as parameters, estimators, properties, marginal product, and specification. This is because the author has suffered from how to express/measure these key words in my life-work. In short, statistics data are consecutive while economic and econometrics analyses, regardless of discrete and continuous, look after rules, hypotheses, and stylized facts in changing results, by year under changing economic and social policies and using waste mixture of micro and macro. Really, the methodology surprisingly progresses over years yet, actual and estimated/forecasted results, never the same as before. Naturally and regrettably, assumptions cannot be strictly specified and Lucas's (1976) critique holds forever.

Then, is there any common connector between the current economic and econometrics analyses that use various s and the *EES* and KEWT database that use a discrete?

For this question, let the author watch typical graphs in four different papers, currently most cited by readers:

(1) Dai and Singleton (427-429, 431, 433,-435, *JFE* 63, 2002): 'Expectation puzzles, time-varying risk premium, and affine models of the term structure.' Seven figures each indicates econometrics equations, where the x axis shows maturity (years) and, the y axis several parameters, unadjusted, and projection coefficients, risk-adjusted, prepared in four tables. These figures correspond with those drawn using endogenous data. It seems no comparable each other yet, an endogenous discovery clarifies that actual data and results are always within a certain range of endogenous data and results. In this respect, the comparison expresses each own results at different frameworks but is based on the same endogenous results.

(2) Gallmeyer, Hollifield, and Zin (947, 948, *JME* 52, 2005): 'Taylor rules, McCallium rules and the term structure of interest rates.' This paper is based on market data, externally. Taylor rules apparently manipulate relationships between market and financial assets but, actually are controlled by the real assets behind. Hitherto the real assets have never been expressed *purely endogenously*. Nevertheless, Taylor rules have resulted in the same results of the real assets, due to the author's

neutrality of the financial/market assets to the real assets (money-neutral). Therefore graphically, 1) Taylor Rule and 2) McCallum Rule, commonly with stochastic volatility and stochastic price of risk are actually shown. The above four figures taken in the y axis do not disperse but converge to zero line, by the level of monetary policy taken in the x axis. It implies that by the author's neutrality, these figures simultaneously hold under the endogenous-equilibrium.

(3) Flood and Rose (962,964, 966-967, *JME* 52, 2005): 'Estimating the expected marginal rate of substitution: A systematic exploitation of idiosyncratic risk,' where 14 time-series figures are shown by data, method, T-bill and expected *MRS*, and firm. Again, expected *MRS* does not enlarge, regardless of good or bad monetary policy since money-neutral works under the endogenous-system. Idiosyncratic risk happens when the situation rapidly runs out of moderate endogenous data.

(4) Palivos (1927-29, *JEDC*, 2001): 'Social norms, fertility and economic development.' Palivos' four figures show the tendency spread between actual statistics data over years, 1967/78/79 to 1986/87; *GDP*, services, consumption, and real fixed investment. These data are ex-post and, do not follow national income equality of 'income=expenditures=output at the real assets' that guarantees Say's law. Nevertheless, differences between statistics and endogenous data do not diverge but converge over years. This is because the actual statistics data always stay within a certain range of endogenous data under money-neutral.

The author now pays attention to an actual ex-post fact that wages divided by profits at enterprises has decreased particularly for the last few years. This actual fact shows policy-makers have failed to maintain dynamic endogenous balances between government and private sectors at the macro level. In this respect, the above market reflections as proved by the above four papers are still short-sighted. Policy makers should not solely rely on statistics and external data and accordingly, Taylor' rules (1993) and market indicators too much. Policy makers consecutively need to remember the equality of $Y = C + \Pi = W + P$, actually hidden in the real assets.

2. Euler's theorem and the C-D PF

2.1. Preliminary common arrangement to the C-D PF

For the initial data: In the literature, the initial data has its own proper implication in the C-D PF, $Y = AK^{\alpha}L^{1-\alpha}$, where A is total factor productivity, $A = TFP$. Let the C-D PF differentiate using d as each statistics (actual) variable's by time: $d\,A/A = d\,Y/Y - \alpha d\,K/K - (1-\alpha)\,d\,L/L$, where Y is *GDP* and K is actual stock of capital, and L is actual labor. Here, the relative share of capital, α, is determined by the initial ratio or an average ratio during the years taken for analysis. Thus, the value/ratio of α depends on the initial actual variables, *GDP*, *K*, and *L*. The author indicates; is K estimated consistently with other variables? Answer is yes, since each variable is independent of each other in the literature.

Compound interest calculation: In the literature, the rate of interest (interest rate) is externally given. Economists do not expect that the rate of return is estimated accurately. A reason is that the C-D PF historically has relied on the capital-labor ratio, $k = K/L$, neglecting the capital-output ratio, $\Omega = K/Y$. $\alpha = \Omega \cdot r$ holds as an identity. Nevertheless, $\alpha = \Omega \cdot r$ has not been tested using Ω and r and also has not been connected with total factor productivity, $\alpha = \Omega \cdot r$, in the literature.

Compound interest rate is simply calculated when deposit-time *T* is infinite. Start with total sum of capital and interest, $(1+r)^T$. If two times per year, $\left(1+\frac{r}{2}\right)^{2T}$; and if *n* times per year, $\left(1+\frac{r}{n}\right)^{nT}$. Suppose $n \to \infty$. The answer is: $(1+r/n)^{\{(n/r)\}} \to e = 2.718 \ldots$, resulting in $(1+r/n)^{\wedge}\{(n/r)rT\} \to e^{\wedge}(rT)$. $(1+r/n)^{\{(n/r)\}} \to e = 2.718 \ldots$

For $(1+1/n)^n$: When $n = 1, (1+1)^1 = 2$; if $n = 2, (1+1/2)^2 = 1 + 2 \times 1/2 + 1/4 = 2.25$; and when $n = 3, (1+1/3)^{\wedge}3 = 1 + 3 \times$ *(13)^2+3×(13)+(133=2.37037037*; and finally, the base of natural (Napierian) logarithm, $e = 2.718281828459$ is obtained.

2.2. Laws of IRC, CRC, DRC: from exogenous to endogenous

Let us analyze IRC (increasing returns to capital), CRC (constant returns to capital), DRC (diminishing returns to capital) using the, $y = Ak^{\alpha}$: Each growth rate by logarithmic differentiation, $g(y) = g(A) + \alpha g(k)$. Differentiate, $y = Ak^{\alpha}$, partially by *k*; the rate of return, $r = \frac{\partial y}{\partial k} = \alpha A k^{-(1-\alpha)}$.

The growth rate of r is $g(r) = g(A) - (1-\alpha)g(k)$. Then, $g(r) = g(y) - g(k)$ holds simply. Therefore,

1. Condition of IRC: $g(r) > 0 \to g(y) > g(k)$.
2. Condition of CRC: $g(r) = 0 \to g(y) = g(k)$.
3. Condition of DRC: $g(r) < 0 \to g(y) < g(k)$.
4. To author's understanding, (1) homogenous degree one, *m*=1.000, presents CRC and under CRS. (2) If *m*>1.000, it presents IRC but, CRS does not hold. (3) If

m<1.000, it presents DRC but, CRS does not hold.

5. Contrarily in the case of the KEWT database, the rate of return, $r = \Pi/K$, is measured as an endogenous equation and this equation, theoretically, empirically, and simultaneously, distinguishes IRC, CRC, and DRC; each under CRS. More clearly, the rate of return function to the ratio of net investment to output clarifies IRC, CRC, and DRC, as a reduced form of the endogenous equation: The less the ratio of net investment to output the higher the rate of return is endogenously. Maximum profits expressed using the parabolic function in the literature is consistent with maximum returns with minimum net investment expressed using the hyperbolic function; the former never needs the origin and the latter, definitely needs the origin and the circle hidden behind the hyperbolic curve. In the case of recursive programming, however, the transitional path follows DRC. Seldom IRC happens, where the rate of return shows minus (see Philippines, 2010). When the rate of return becomes close to zero due to huge deficits and national debts, CRC happens, as in Japan 2010 for the last fifteen years. This sort of CRC cannot be solved so that the market reflects the serious situation directly. This is the true character of deflation. Deflation cannot be attacked unless deficits and debts decrease sharply over years. The financial/market policies are apt to be short-sighted. These policies never exclude the true cause of deflation at the real assets, as shown by the above hyperbolic function. This discovery is empirically proved by author’s neutrality of the financial/market assets to the real assets.

2.3. Euler’s theorem

This sub-section mathematically but generally explains Euler’s theorem by using the C-D PF. The next sub-section further steps into earlier/typical C-D PF equations, formulated by P. H. Wicksteed (1938). In this point of entry, there has been no example to precisely connect C-D PF equations with required assumptions. Literature cannot precise reason for this fact, due to the essence of the market principles.

First, the core of the C-D PF is homogeneous degree one or constant returns to scale. Set homogeneous degree one, $Y = f(K, L)$.

Euler’s theorem is $KF_K + LF_L = Y$, where $F_K = \partial F/\partial K, F_L = \partial F/\partial L$.
Then, $\mathrm{Y} = \mathrm{AK}^{\alpha} L^{1-\alpha}, 0 < \alpha < 1$, proves $KF_K + LF_L = Y$ as follows:
$f_K = \partial Y/\partial K = \alpha AK^{\alpha-1}L^{1-\alpha} = \alpha AK^{\alpha}L^{1-\alpha}K^{-1} = \alpha\, Y/K.$
$f_L = \partial Y/\partial L = (1-\alpha)AK^{\alpha}L^{-\alpha} = (1-\alpha)AK^{\alpha}L^{1-\alpha}L^{-1} = (1-\alpha)\, Y/L.$

These imply that production outcome Y is perfectly distributed to two factors, K and L.

To realize the above Euler's theorem, perfect competition (law of one price) and rational behavior are subjectively required. How to set up the design and system for guaranteeing Euler's theorem?

When this system is successfully set up, $PY = Pf_K K + Pf_L L = rK + wL$ holds, where $P = \text{price}, w = \text{nominal wage rate}, r = \text{rate of profits/returns}.$

Suppose, $Pf_L > w$:
The LHS of this inequality is marginal revenue and its RHS is marginal expenses for increase in employment. This phenomenon is rational and so that executed. If the inequality is reversed ($Pf_L > w$), the phenomenon is irrational and so that not executed.

The same applies to capital and the rate of profits/returns, $Pf_K > r$ and $Pf_K < r$.

Therefore, the price-equilibrium holds if and only if $Pf_L = w$ and $Pf_K = r$.

The above proofs applying Euler's theorem to the C-D PF are mathematical so that these are justified mathematically. Note 1: Mathematics holds always even if equations are not whole but partial in any model/system while economics does not holds unless the whole model/system is consistent over years. Note 2: the market principles are the carrier of the price-equilibrium. The market principles show prices by vertically by goods and services, where the above P is externally given and cut the whole consistency of the whole system. In other words, statistics data hold partially by nature. The proofs gouge the limit of economic statistics and its data.

2.4. Historical review of Wicksteed (1938) for Euler's theorem

Euler's theorem was earlier analyzed by Robinson, J. (1934), along with 'static' marginal productivity theory. Euler's theorem has been a common base, among Keynesian (Post, Neo, and New schools), and classical and Neo-classical schools. Euler's theorem robustly holds among static and dynamic; closed and open; and further under perfect and imperfect competition. Euler's theorem thus holds beyond the use of the production function, the price-equilibrium, and the author's endogenous-equilibrium. Euler's theorem will last and never fades away.

Chapter of "Optimum function-measure," chapter 9 in this book, presented to *Annals of Mathematics*, Princeton, in Aug 2014, discusses Euler, Leonhard, from the viewpoint of geometry and the golden ratio versus the silver ratio. Also, the previous sub-section touched Euler, Leonhard, as true founder of the C-D PF and numerous facts.

This sub-section, apart from Euler, Leonhard, solely reviews Philip, H. Wicksteed's (399, 407-408, 412-413, 1938) *Co-ordination of the Laws of Distribution*, and cites Wicksteed's unique equations so as to appeal the requirement of precise interpretation of assumptions in economics.

Wicksteed (ibid., **399** and **407-8**): First let the author cite, "the mathematical form of statement…as a safeguard against unconscious assumptions, and as a reagent that will precipitate the assumptions held in solution in the *verbiage* of our ordinary disquisitions."[1] Wicksteed uniquely formulates the following equations, using $P = f(a, b, c, \ldots)$ as a homogeneous function of the first degree:

$mP = f(ma, mb, mc, \ldots)$,

$P = a\frac{\partial P}{\partial a} + b\frac{\partial P}{\partial b} + c\frac{\partial P}{\partial c} + \cdots$.

While the mathematician has only to set out the generalized form of Euler's theorem in order to show [2] that

$P \lesseqgtr a\frac{\partial P}{\partial a} + b\frac{\partial P}{\partial b} + \cdots$.

According as $mP \lesseqgtr +(ma, mb, \ldots)$.

Wicksteed (ibid., **412-413**): But for the most profitable output, demand curve, which is higher, must have a greater slope than the cost curve.

Let *x* be output, *y* price, and *z* average cost.

Then $y + x\frac{dy}{dx} = z + x\frac{dz}{dx}$ (marginal revenue = marginal cost).

$\therefore$ if $y > z, \frac{dy}{dx} < \frac{dz}{dx}$.

$\therefore$ the negative slope of the demand curve is greater than that of the cost of curve. (In perfect competition—see p. 407 above—we have the special case in which $dy/dx = 0$.

$\therefore$ when $y > z, \frac{dz}{dx}$ must be positive. Since the prices of the factors are constant, this entails diminishing physical returns.

For the above equations, the author first indicates that the above equations are right but, for assumptions required for equations' justification, Wicksteed does not clarify the essence of assumptions under the market principles. The author indicates the following statements:

[1] *Co-ordination*, Prefatory Note, p. 4.

[2] Cf. Wicksell, loc. *the Economic Journal*, Dec1906, p. 189, and Chapman, loc. *the Economic Journal*, Dec1906, p. 526.

(1) In the price-equilibrium and its market principles, it is true that quantities are physical while qualities are prices, as ex-post analyzed by Jorgenson and Griliches (1967). The market principles show prices vertically and equation needs externals such as *CPI*, so that equations need arbitrary assumptions to justify modeling.

(2) By the author's money-neutral, the real assets and financial/market assets are the same within the range of moderate equilibrium. Nevertheless, mathematics cannot distinguish economic assumptions with the case of no assumption in the *EES*. Wicksteed could not naturally explain the existence of indispensable assumptions. This fact is totally applicable to economic literature.

3. Recursive programming of technology FLOW and Technology-STOCK=A=TFP, each by Hicks, Solow, and Harrod

This section empirically and consistently summarizes 'flow=the rate of technological progress' and 'stock=total factor productivity' in technology, by using recursive programming in the transitional path and, comparing three models, Hicks (1932), Solow (1956), and Harrod (1942), that use the same .

An endogenous rate of technological progress is measured by $g_{A(FLOW)} = i(1-\beta)$, simultaneously with the growth rate of capital stock as total factor productivity (*TFP*), $g_{TFP(STOCK)} = k^{1-\alpha}/\Omega$. The author's C-D PF is 'discrete' and all sorts of possible parameters and variables are measured with *seven endogenous parameters*.[3]

Historically, relationships between the rate of technological progress and total factor productivity *TFP* was empirically clarified using Hicks (1932), Solow (1956), and Harrod (1942):

[3] 1. Endogenous net investment to endogenous net income, $i = I/Y$.
2. The rate of change in population, $n_E = n$.
3. The relative share of capital, $\alpha = \Pi/Y$, where $\alpha = \Omega^*/r^*$.
4. The capital-output ratio, $\Omega^* = K/Y$, (or, $\Omega^* = \frac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)(1+n)+n(1-\alpha)}$).
5. The technology coefficient (or the quantitative net investment coefficient), β^*, (or, $\beta^* = \frac{\Omega^*(n(1-\alpha)+i(1+n))}{i(1-\alpha)+\Omega^* \cdot i(1+n)}$).
6. The diminishing returns to capital (DRC) coefficient. $\delta_0 = 1 + LN(\Omega^*)/LN((1-\beta^*)/\beta^*)$.
7. Speed years for convergence, $1/\lambda^*$, the speed coefficient, $\lambda^* = (1-\alpha)n + (1-\delta_0)g_A^*$, and $g_A^* = i(1-\beta^*)$.

(1) Technology-$_{\mathbf{FLOW}}$: the rate of technological progress, m

Set Hicks' $m = g^*_{A(FLOW)}$. Then, Solow's, $m(1-\alpha)$. Harrod's, $m \cdot \alpha$.

As a result, $\text{Hicks}' g^*_{A(FLOW)} = Slow's\ g^*_{A(FLOW)} + Harrod's\ g^*_{A(FLOW)}$.

The relative share of capital determines three differences for an endogenous rate of technological progress, $g^*_{A(FLOW)} = i(1-\beta^*)$.

(2) Technology-$_{\mathbf{STOCK}}$: Total Factor Productivity (A=TFP)

Set Solow's $A = TFP$.

Then, $\text{Hicks}' g^*_{A(TFP)} = \text{Slow}'\text{s}\ g^*_{A(TFP)} \ll \text{Harrod}'\text{s}\ g^*_{A(TFP)}$. For total factor productivity (TFP), $\text{Harrod}'\text{s}\ g^*_{A(TFP)}$ is empirically much higher than $\text{Hicks}' g^*_{A(TFP)} = \text{Slow}'\text{s}\ g^*_{A(TFP)}$.

Why is Harrod's TFP higher than those of Hicks and Solow? It is perfectly proved by one new equation at the author's PhD (Note 5, 2003), $A = TFP = k^{1-\alpha}/\Omega$.[4] The capital-output ratio is much lower than the other two of Hicks and Solow. Why is $\text{Slow}'\text{s}\ g^*_{A(TFP)}$ the same as $\text{Hicks}' g^*_{A(TFP)}$? $k^{1-\alpha} = A \cdot \Omega$ holds under Hicks' $Y = F(AK, AL)$ and Solow's $Y = F(AK, L)$.

Tables 1, **2**, **3** and **Figures 2** and **3** explain the above technology-$_{\text{FLOW}}$ and technology-$_{\text{STOCK}}$. For technology-$_{\text{FLOW}}$, each country has its own character uniquely. The same pattern never happens even in endogenous data. For technology-$_{\text{STOCK}}$, each country, accordingly, has its own unique character over times/years in the transitional path. Watch the rate of technological progress and the growth rate of A=TFP, by time and by the point of convergence, with the speed years for convergence by country. Relationship between the speed yeas, technology-$_{\text{FLOW}}$, and technology-$_{\text{STOCK}}$ are more complicated and it is difficult these movements orally. Even under the same number of the speed years technology-$_{\text{FLOW}}$ and technology-$_{\text{STOCK}}$ move differently. It implies that consumption is independent of technology, as shown by consumption-neutral.

---

[4] Partial differentials calculated by the C-D PF differ from $\frac{\partial Y}{\partial A} = 1.00000$ or A=Y here.

1. Hicks': $\frac{\partial Y}{\partial A} = K^\alpha L^\beta = 1L\left(\frac{K}{L}\right)^\alpha$, where partial difference is 1.0000 under $1 = \alpha + \beta$.
2. Solow's: $\frac{\partial Y}{\partial A} = \alpha A^{\alpha-1} K^\alpha L^\beta = 2A^{\alpha-1} L\left(\frac{K}{L}\right)^\alpha$, where partial difference is $2A^{\alpha-1}$ under $1 = \alpha + \beta$.
3. Harrod's $\frac{\partial Y}{\partial A} = \beta A^{\beta-1} K^\alpha L^\beta = \beta A^{\beta-1} L\left(\frac{K}{L}\right)^\alpha$, where partial difference is $\beta A^{\beta-1}$ under $1 = \alpha + \beta$.

4. Asymmetry between the capital-labor ratio And the capital-output ratio

This section discusses asymmetry issue hidden in the C-D PF. The economic literature discusses asymmetric issues more broadly, without sticking to the C-D PF. This section will conclusively suggest some answer to broader asymmetric problems in macroeconomics. The capital-labor ratio and the capital-output ratio are asymmetric in the transitional path, as this section empirically proves. Why? This is because the elasticity of substitution is constant in the C-D PF applied to the *EES*, as empirically shown soon below. Endogenous equations with no assumption are symmetric, as proved by reduced forms of endogenous equations in two-dimension plane. This is because essentially hyperbolas are each symmetric.

Thus first, this section empirically summarizes asymmetric movements between the capital-labor ratio and the capital-output ratio by using recursive programming for the transitional path by year. Second, this section takes some researches related to asymmetric issue and suggests conclusive answers.

For the above first research, the author presents *three explanations*, using (1) **Figure 4** (Simulation of elasticity of the capital-output ratio and the capital-labor ratio by country, with each speed years), (2) **Figure 5** (Structure analyses of seven endogenous parameters in recursive programming), and (3) **Figure 6**, **7**, and **8** (Growth rates of the capital-labor ratio and the capital-output ratio: Base area, Euro area, and Asian area).

Underlying question is: Why does the C-D PF discussed in the literature, solely use the capital-labor ratio, neglecting the capital-output ratio? The author's answer is: well-acceptance and also convenience so that the s can retrieve externals in the literature.

Figures 4 and Figures 6, 7, 8 empirically compare the capital-labor ratio with the capital-output ratio by country and by area. These empirical analyses are impossible if the author's so called macro-utility $((\rho/r)(C/Y))$ was not found between consumption and technology. For example, Feng Wang (190-191, 2007 based on 2005) shows figures 11.5, 11.6, and 11.7, with assumptions estimated based on statistics data. The author indicates that if assumptions differ, results change and that his finding between population and consumption significantly differs from endogenous proofs (see Chapter 15, the *EES*). However, the author's intention is not directed to the above numerical indications but to his global fact-finding that population decreases with consumption. His global finding is consistent with the author's finding that statistics data exist always within a certain range of endogenous data, as proved by KEWT database.

Now concretely, the above (1) explanation with Figure 4: The simulation of elasticity clarifies a finding that we should avoid using fixed elasticity values. This finding in Figure 4 proves that in recursive programming has changing values of elasticity in the capital-labor ratio, $k = K/L$, and the capital-output ratio, $\Omega = K/Y$, over times/years. The related equations are set as follows:

$y = k\hat{}q_k$ and $q_k = LN(y)/LN(k)$.
$y = \Omega\hat{}q_\Omega$ and $q_\Omega = LN(y)/LN(\Omega)$.

Values of elasticity are calculated each by

$\eta_k = (\Delta K/K)/(\Delta L/L)$ and $\eta_\Omega = (\Delta K/K)/(\Delta Y/Y)$
(cf, the case of *MRS*=*sigma*).

In short, in $k = K/L$ and $\Omega = K/Y$, elasticity value of $\eta_k = (\Delta K/K)/(\Delta L/L)$ does not express a fixed value over times/years in the transitional path, here regardless of the use of the capital-output ratio additionally. It is impossible for one to prove this finding, unless capital and national disposable net income are measured purely endogenously.

Second, for the above (2) with Figure 5: Structure analyses of seven endogenous parameters were already precisely proved, as shown in Chapters 7 and 8 in the *EES*. Figure 5 is similar and presented for readers' image-help. In short, seven endogenous parameters determine all the parameters and variables simultaneously in the author's discrete C-D PF. Particularly, the capital-output ratio works an important role in the discrete production function: One must pay attention to the movements of two decisive parameters (i.e., the technology coefficient (or the quantitative net investment coefficient), β^*, and the diminishing returns to capital coefficient), with the level of net investment. Results are more severely shown when the total economy is shown by sector (the G and PRI sectors).

Third, for the above (3) with Figures 6, 7, and 8: Focusing on the capital-labor ratio and the capital-output ratio, each growth rate is compared by area, Base area, Euro area, and Asian area. Each country again shows various movements according to national taste, consumption, and technology, as clarified in the previous section.

Here the author casts a question: Could one wholly answer Paul Krugman's (797 words, Nov 4 2013, New York Times) 'Those Depressing Germans'? No, no one can answer. Why? Statistics data differ from theoretical data, which may be far from theoretical data and no one can verify this fact. The author here fairly answers the author's question.

(1). Money-neutral: Decision-makers cannot manipulate the exchange rate by country.

(2). The real rate of return=0, so that the nominal growth rate of *GDP* equals the rate of inflation/deflation (minus inflation).

(3). The balance of payments is shown by $BOP = (S - I) = \Delta D + (S - I)_{PRI}$. Net investment turns to a low level if deficit, $\Delta D = (S - I)_G$, and debts, $D = \Delta D / r_{M(10\,yrs)}$, where $r_{M(10\,yrs)}$ is ten year debt yield and, used for one of three tests of money-neutral.

(4). Crowd-out of the PRI sector is a result of huge deficits and debts. This does not improve by decision-makers' manipulation, the same as ten year debt yield.

(5). True cause of high unemployment, enlarging inequality,[5] low growth rates of output and profits, and low net investment or crowd-out remain aggravating results. Politics-neutral prevails under the market principles. Then, which is worse, Germany and Republicans?

Turning to the second issue and look up Dhrymes, P. J. (1962); Kmenta, J. (1964); Fisk, P. R. (1964, 1966); Zellner, A., Kmenta, and J. Dreze, J. (1966); Marshall, David, A. (1972, 2005); Meeusen, W., and J. Vandenbroeck (1977); and Bliss Christopher (1999, 2000). Bliss Christopher (1999, 2000) indicates that Freedman, M. (1992) and Cannon, E., and Duck, N. W. (2000) are all right and that empirically growth regressions do no tell us an answer. Symmetry is strictly defined as the condition that the two curves are overlapped into one when one curve is rotated by 180 degrees. Why is econometrics analysis not possible to obtain a definite answer? Conclusively the market principles make econometrics analysis to get no answer, due to vertical price formation by goods and service. This is shown in Conclusion that summarizes wholly this chapter. Then, why is the *EES* and its KEWT database definitely possible to obtain a robust answer? What cause determines the asymmetry between $k = K/L$ and $\Omega = K/Y$ by country? The KEWT database shows that few countries each have symmetry between the two ratios (due to close to CRC) but all other countries each falls into asymmetry between the two ratios.

Let the author here indicate relationship between KEWT database and its

[5] Economists recall a fact that nominal consumption is stable or that by increasing consumption/sales tax rate, real consumption decreases so that Diffusion Index will aggravate. This remains partial interpretation. Simply, endogenous facts are: Tax cut and increase-subsidies are too short-sighted and unexpectedly destroy business trends. The wage rate turn adversely lower and spread inequality between rich and poor, resulting in unexpected shrimp of consumption. Growth and profit ratios never recover, as proved in the KEWT database series. Here dynamic balances hold, instantly, minute by minute, and always under the market principles.

recursive programming and by sector that is one of keys to open a robust door. KEWT database is measured robustly with three sectors (government and private, G & PRI and aggregated amount by item). The total number of countries is 81 and includes steady African countries. The statistical quality of African countries has gradually improved by IMF staff's education endeavor yet, the author avoids using many African countries in this chapter. This is because basic endogenous equations fluctuate repeatedly. The recursive programming and the KEWT database are consistent by country, sector, and years and over years, under no assumption. Empirical results of recursive programming for the transitional path by time, *t*, catches delicate changes more clearly than those of the KEWT database. Yet, the KEWT database simultaneously determines thousand variables & parameters or values & ratios by country and, harmonizes and unites statistics data, actual data (statistics data+ externals), and endogenous data, ex-post and ex-ante, causes and results and space and time, just like holography.[6]

5. Measuring break-even point (BEP) in the C-D PF

The C-D PF produces broad expressions. As one of expressions such as the real rate of return =zero (*RRR*=0), this section briefly states the author's proof of life-time employment in Japan. According to Atsuo Ueda (2009), Drucker had continued to appeal the importance of life-time employment, particularly in Japan, until his death in Nov 2005. The author's expression of Drucker's proof (hereunder, *Drucker's expression*) is the following:

The author (350, 1965) expressed the formula of break-even point (BEP), $X = F/(1-v)$, by a hyperbola, $1 = \frac{f}{1-v}$, where $v = V/X$ and $f = 1 - v$. The BEP hyperbola is an identity and no one cannot deny. BEP hyperbola prevails not only in corporate analysis in enterprises but also in macro analysis. A System of National Accounts (SNA, 1993, 2008), however, cannot introduce the BEP hyperbola into the SNA. When national disposable net income is measured by $Y = C + S = W + \Pi$ over years, the BEP hyperbola holds, as proved in this section. The difference between micro and macro is external expenses, $E = X - Y$. In the case of life-time employment, $F = W$ and $V = E$ hold so that $X = W + E + \Pi = F + V + \Pi$. In the case of part-timer economy, $X = (W + E) + \Pi = V + \Pi$ holds:

[6] Gabor, Dennis. (1971). Nobel Lecture, December 11, 1971. Social scientific discoveries apparently differ from natural scientific discoveries but, to the author's understanding, 'money' by country (as a world common scale) and the exchange rate prove 'holography' in the actual world.

For example, the BEP rises up to 1.5837 if $f = 0.4500 \gg 1.0000$ and, further rises up to 7.9185 if $f = 0.0900 \gg 1.0000$. Let us compare **Tables 4-1** (the US) and **5-1** (Japan) before adjustment with **Tables 4-2** (the US) and **5-2** (Japan) after adjustment of the variable expenses.

The above results indicate a fact that the higher the fixed expenses to net sales, the less the rate of return. This fact is another expression of value added per capita=the ratio of value added to net sales multiplied by net sales per capita; productivity or value added per capita $= y = \frac{Y}{L} = \frac{Y}{X} \cdot \frac{X}{L}$. In the case of the macro level, the same $\mathrm{y} = Y/L$ holds due to no reduction or $Y = X$.

The author finds three endogenous principles in the expressions of the *Drucker BEP* under purely endogenous as follows:

1st: There is no measure difference between micro and macro since external expenses are offset, as shown above.

2nd: There is no measure difference between with per capita and without per capita. This is because the number of population/workers is wholly offset in the *Drucker BEP*.

3rd: In the transitional path, average=marginal or productivity as value added per capita is constant by time/year, before and after the convergence point of time, as proved by recursive programming in a separate paper.

When actual data (double-entry bookkeeping accounts, statistics data, and external data) are used for Drucker's expression, the 2nd principle does not hold (see a paper presented to IAES Conference, Madrid, 2014). Nevertheless, actual data are always within a certain range of endogenous data, as proved purely endogenously with no assumption and under perfect competition. There is much room for cooperative researches between macro and micro (enterprises), including the application of *Drucker's expression* to enterprise sector after redistribution of taxes. For example, the relationship between average and marginal of economic activities is often shown illustratively in the literature, often using combination of two different parabolas. This illustration is numerically clarified when listed company data are using for the *Drucker's expression*.

6. Conclusions

This chapter develops foundation of the C-D PF in economic and econometrics analyses. The essence of the *EES* (theory) and its KEWT database (practice, where theory=practice) is robust and strictly clarifies true meaning of assumptions in the literature. Indispensable limit is shown not only in Bliss Christopher (1999, 2000) but also in any of economic and econometrics analyses. Economics methodology under the market differs from mathematics, setting aside behavior science and politics.

Among others, this chapter essentially develops unsolved but indispensable problems lying between Euler's theorem and the C-D Production Function. Nevertheless, the author's true intention is never against new progresses in econometrics but respects their fruits. The author sincerely proposes cooperative joint work between 'economic/ econometric statistics data analyses and methodology' and 'the *EES* and KEWT database under no assumption.'

One more is this: Economic analysis has developed with the C-D PF, with the stream of micro to macro and based on individual utility, which was boldly reviewed by Paul Samuelson before and after in the 1940s. If assumptions had been strictly applied to statistics analysis, a brave consensus that macro must be a base for micro had prevailed earlier. The *EES* proposed macro-utility as the relative discount rates functioning between consumer and producers goods. The author developed this fact more precisely in a separate chapter.

Table 1 Recursive programming of technology-$_{\text{stock=A*=TFP*}}$ and the speed years for convergence: 12 countries

country	1. the US	2. Japan	3. Australia	4. France	5. Germany	6. the UK	7. China	8. India	9. Brazil	10. Mexico	11. Russia	12. S Africa
speed years	110.19	107.87	28.27	62.98	85.77	123.38	24.81	21.56	20.53	20.31	(2.89)	25.19
	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*	A^*=TFP*
	23.98	**55810**	**61.01**	**34.08**	**109.83**	**21.18**	**8.02**	**44.90**	**0.96**	**74.54**	**73.25**	**68.73**
time, t												
0	18.247	1322	32.817	16.829	18.677	11.811	2.517	22.558	0.221	39.682	73.248	32.402
1	18.255	1340	33.137	16.911	18.953	11.828	2.544	22.799	0.241	40.175	73.248	32.773
2	18.264	1358	33.483	16.996	19.235	11.845	2.583	23.068	0.262	40.730	73.248	33.184
3	18.272	1376	33.858	17.083	19.523	11.862	2.633	23.369	0.284	41.354	73.248	33.639
4	18.281	1395	34.262	17.174	19.816	11.880	2.695	23.707	0.307	42.053	73.248	34.143
5	18.290	1415	34.698	17.268	20.116	11.898	2.769	24.085	0.331	42.836	#NUM!	34.699
6	18.300	1435	35.169	17.365	20.422	11.916	2.857	24.508	0.357	43.711	#NUM!	35.314
7	18.310	1456	35.677	17.465	20.735	11.935	2.957	24.981	0.384	44.688	#NUM!	35.991
8	18.320	1477	36.224	17.568	21.054	11.954	3.071	25.511	0.412	45.778	#NUM!	36.737
9	18.331	1499	36.812	17.675	21.381	11.974	3.200	26.104	0.442	46.993	#NUM!	37.557
10	18.342	1522	37.445	17.786	21.714	11.994	3.345	26.768	0.473	48.348	#NUM!	38.459
11	18.353	1546	38.125	17.901	22.055	12.015	3.505	27.510	0.506	49.859	#NUM!	39.450
12	18.365	1570	38.855	18.019	22.403	12.036	3.683	28.340	0.541	51.543	#NUM!	40.536
13	18.377	1595	39.640	18.141	22.758	12.057	3.879	29.270	0.578	53.422	#NUM!	41.727
14	18.390	1620	40.481	18.268	23.122	12.079	4.094	30.311	0.617	55.518	#NUM!	43.032
15	18.403	1647	41.383	18.398	23.493	12.102	4.329	31.478	0.658	57.861	#NUM!	44.462
16	18.416	1674	42.351	18.534	23.873	12.125	4.585	32.787	0.701	60.479	#NUM!	46.026
17	18.430	1702	43.387	18.673	24.262	12.148	4.864	34.256	0.746	63.411	#NUM!	47.738
18	18.444	1731	44.498	18.818	24.659	12.172	5.167	35.907	0.795	66.698	#NUM!	49.611
19	18.459	1761	45.688	18.967	25.066	12.196	5.494	37.765	0.846	70.390	#NUM!	51.660
20	18.474	1791	46.962	19.122	25.481	12.221	5.846	39.860	0.900	74.542	#NUM!	53.902
21	18.490	1823	48.327	19.281	25.907	12.247	6.225	42.226	0.957	79.223	#NUM!	56.355
22	18.506	1856	49.788	19.446	26.342	12.273	6.632	44.904	1.018	84.512	#NUM!	59.040
23	18.523	1890	51.353	19.616	26.787	12.300	7.066	47.940	1.083	90.502	#NUM!	61.979
24	18.540	1925	53.028	19.793	27.243	12.327	7.529	51.392	1.151	97.303	#NUM!	65.199
25	18.558	1961	54.822	19.975	27.710	12.355	8.020	55.326	1.224	105.05	#NUM!	68.727
26	18.577	1998	56.743	20.163	28.188	12.384	8.541	59.822	1.302	113.89	#NUM!	72.595
27	18.595	2037	58.802	20.357	28.677	12.413	9.090	64.978	1.384	124.03	#NUM!	76.840
28	18.615	2077	61.007	20.558	29.178	12.442	9.667	70.908	1.472	135.68	#NUM!	81.501
29	18.635	2118	63.370	20.766	29.692	12.473	10.271	77.755	1.565	149.13	#NUM!	86.622
30	18.656	2161	65.903	20.981	30.218	12.504	10.902	85.687	1.665	164.71	#NUM!	92.255
31	18.677	2205	68.620	21.203	30.757	12.536	11.558	94.916	1.771	182.84	#NUM!	98.455
32	18.699	2251	71.534	21.432	31.310	12.568	12.236	105.70	1.884	204.01	#NUM!	105.29
33	18.722	2298	74.662	21.669	31.877	12.601	12.936	118.35	2.004	228.86	#NUM!	112.82
34	18.745	2347	78.019	21.914	32.458	12.635	13.654	133.28	2.133	258.16	#NUM!	121.15
35	18.769	2398	81.626	22.167	33.054	12.670	14.388	150.96	2.271	292.90	#NUM!	130.35
36	18.794	2451	85.502	22.428	33.665	12.705	15.135	172.04	2.418	334.29	#NUM!	140.53
37	18.819	2505	89.670	22.699	34.293	12.741	15.892	197.30	2.575	383.91	#NUM!	151.82
38	18.845	2562	94.153	22.978	34.936	12.778	16.656	227.76	2.743	443.73	#NUM!	164.34
39	18.872	2621	98.980	23.267	35.597	12.815	17.422	264.72	2.923	516.31	#NUM!	178.26
40	18.900	2682	104.179	23.565	36.276	12.854	18.189	309.88	3.115	604.98	#NUM!	193.75
41	18.928	2745	109.783	23.874	36.973	12.893	18.951	365.46	3.321	714.06	#NUM!	211.00
42	18.957	2811	115.827	24.193	37.689	12.933	19.707	434.40	3.542	849.27	#NUM!	230.26
43	18.987	2879	122.350	24.522	38.424	12.974	20.452	520.62	3.778	1018	#NUM!	251.78
44	19.018	2950	129.395	24.863	39.180	13.016	21.184	629.38	4.032	1231	#NUM!	275.87
45	19.049	3023	137.011	25.215	39.958	13.058	21.899	767.90	4.304	1502	#NUM!	302.88
46	19.082	3100	145.248	25.579	40.757	13.102	22.596	946.10	4.595	1850	#NUM!	333.21
47	19.115	3180	154.166	25.956	41.579	13.146	23.271	1178	4.908	2301	#NUM!	367.32
48	19.149	3263	163.827	26.345	42.425	13.192	23.922	1483	5.245	2892	#NUM!	405.76
49	19.185	3350	174.303	26.748	43.295	13.238	24.549	1889	5.606	3678	#NUM!	449.15
50	19.221	3440	185.672	27.164	44.191	13.285	25.149	2438	5.994	4734	#NUM!	498.22
51	19.258	3534	198.021	27.594	45.114	13.334	25.721	3190	6.411	6174	#NUM!	553.81
52	19.296	3632	211.446	28.039	46.065	13.383	26.265	4237	6.860	8166	#NUM!	616.91
53	19.335	3735	226.055	28.500	47.045	13.433	26.780	5721	7.342	10968	#NUM!	688.67
54	19.375	3842	241.966	28.976	48.055	13.485	27.265	7866	7.862	14978	#NUM!	770.44
55	19.416	3953	259.313	29.469	49.096	13.537	27.722	11028	8.421	20828	#NUM!	863.82
56	19.458	4070	278.243	29.979	50.171	13.590	28.150	15802	9.023	29541	#NUM!	970.67
57	19.501	4192	298.921	30.507	51.279	13.645	28.550	23192	9.671	42820	#NUM!	1093
58	19.545	4320	321.532	31.053	52.424	13.701	28.922	34959	10.370	63575	#NUM!	1234
59	19.590	4454	346.282	31.618	53.606	13.758	29.267	54294	11.123	96927	#NUM!	1396
60	19.637	4594	373.402	32.203	54.827	13.816	29.587	87194	11.936	152199	#NUM!	1583
61	19.684	4741	403.150	32.808	56.089	13.875	29.882	145429	12.812	246993	#NUM!	1800
62	19.733	4895	435.819	33.435	57.394	13.935	30.153	253195	13.757	415917	#NUM!	2051
63	19.783	5057	471.734	34.084	58.743	13.997	30.402	462957	14.777	730168	#NUM!	2344
64	19.834	5227	511.264	34.756	60.140	14.060	30.630	895504	15.878	1343835	#NUM!	2684
65	19.886	5406	554.821	35.453	61.587	14.124	30.838	1848608	17.067	2609964	#NUM!	3083

Data source: KEWT 7.13, 1960/90-2011 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

Table 2 Recursive programming of technology-$_{FLOW}$ and technology-$_{STOCK=A=TFP}$, each by Hicks, Solow, and Harrod, in the transitional path: the US 2010

speed years	FLOW	three sorts of tech Neutrality			STOCK	three sorts of tech Neutrality		
110.19	the US	Hick	Solow	Harrod	the US			
at converge	0.00590	0.00590	0.00476	0.00114	23.982	23.982	51.239	27.257
	$m=g_{A(FLOW)}$	$N_{EUT}=m$	$N_{EUT}=m(1-\alpha)$	$N_{EUT}=m\alpha$	Hicks's A	Solow's A	Harrod'A	differ=H-S
0	0.00318	0.00318	0.00256	0.00061	18.247	18.247	36.522	18.275
1	0.00320	0.00320	0.00259	0.00062	18.255	18.255	36.542	18.287
2	0.00323	0.00323	0.00261	0.00062	18.264	18.264	36.563	18.299
3	0.00325	0.00325	0.00263	0.00063	18.272	18.272	36.584	18.312
4	0.00328	0.00328	0.00265	0.00063	18.281	18.281	36.606	18.325
5	0.00331	0.00331	0.00267	0.00064	18.290	18.290	36.629	18.339
6	0.00333	0.00333	0.00269	0.00064	18.300	18.300	36.653	18.353
7	0.00336	0.00336	0.00271	0.00065	18.310	18.310	36.678	18.368
8	0.00338	0.00338	0.00273	0.00065	18.320	18.320	36.703	18.383
9	0.00341	0.00341	0.00275	0.00066	18.331	18.331	36.730	18.399
10	0.00344	0.00344	0.00277	0.00066	18.342	18.342	36.757	18.415
11	0.00346	0.00346	0.00279	0.00067	18.353	18.353	36.785	18.432
12	0.00349	0.00349	0.00281	0.00067	18.365	18.365	36.814	18.449
13	0.00351	0.00351	0.00284	0.00068	18.377	18.377	36.844	18.467
14	0.00354	0.00354	0.00286	0.00068	18.390	18.390	36.876	18.486
15	0.00356	0.00356	0.00288	0.00069	18.403	18.403	36.908	18.505
16	0.00359	0.00359	0.00290	0.00069	18.416	18.416	36.941	18.525
17	0.00362	0.00362	0.00292	0.00070	18.430	18.430	36.976	18.546
18	0.00364	0.00364	0.00294	0.00070	18.444	18.444	37.011	18.567
19	0.00367	0.00367	0.00296	0.00071	18.459	18.459	37.048	18.589
20	0.00369	0.00369	0.00298	0.00071	18.474	18.474	37.086	18.612
21	0.00372	0.00372	0.00300	0.00072	18.490	18.490	37.125	18.635
22	0.00374	0.00374	0.00302	0.00072	18.506	18.506	37.166	18.659
23	0.00377	0.00377	0.00304	0.00073	18.523	18.523	37.207	18.684
24	0.00379	0.00379	0.00306	0.00073	18.540	18.540	37.250	18.710
25	0.00382	0.00382	0.00308	0.00074	18.558	18.558	37.295	18.737
26	0.00384	0.00384	0.00310	0.00074	18.577	18.577	37.340	18.764
27	0.00387	0.00387	0.00312	0.00075	18.595	18.595	37.388	18.792
28	0.00390	0.00390	0.00314	0.00075	18.615	18.615	37.436	18.821
29	0.00392	0.00392	0.00316	0.00076	18.635	18.635	37.486	18.851
30	0.00395	0.00395	0.00319	0.00076	18.656	18.656	37.538	18.882
31	0.00397	0.00397	0.00321	0.00077	18.677	18.677	37.591	18.914
32	0.00400	0.00400	0.00323	0.00077	18.699	18.699	37.646	18.947
33	0.00402	0.00402	0.00325	0.00078	18.722	18.722	37.702	18.981
34	0.00405	0.00405	0.00327	0.00078	18.745	18.745	37.761	19.015
35	0.00407	0.00407	0.00329	0.00079	18.769	18.769	37.820	19.051
36	0.00410	0.00410	0.00331	0.00079	18.794	18.794	37.882	19.088
37	0.00412	0.00412	0.00333	0.00080	18.819	18.819	37.945	19.126
38	0.00415	0.00415	0.00335	0.00080	18.845	18.845	38.010	19.165
39	0.00417	0.00417	0.00337	0.00080	18.872	18.872	38.077	19.206
40	0.00420	0.00420	0.00339	0.00081	18.900	18.900	38.146	19.247
41	0.00422	0.00422	0.00341	0.00081	18.928	18.928	38.217	19.289
42	0.00425	0.00425	0.00343	0.00082	18.957	18.957	38.290	19.333
43	0.00427	0.00427	0.00345	0.00082	18.987	18.987	38.365	19.378
44	0.00430	0.00430	0.00347	0.00083	19.018	19.018	38.442	19.425
45	0.00432	0.00432	0.00349	0.00083	19.049	19.049	38.521	19.472
46	0.00435	0.00435	0.00351	0.00084	19.082	19.082	38.603	19.521
47	0.00437	0.00437	0.00353	0.00084	19.115	19.115	38.686	19.571
48	0.00440	0.00440	0.00355	0.00085	19.149	19.149	38.772	19.623
49	0.00442	0.00442	0.00357	0.00085	19.185	19.185	38.860	19.676
50	0.00445	0.00445	0.00359	0.00086	19.221	19.221	38.951	19.730
51	0.00447	0.00447	0.00361	0.00086	19.258	19.258	39.044	19.786
52	0.00450	0.00450	0.00363	0.00087	19.296	19.296	39.139	19.844
53	0.00452	0.00452	0.00365	0.00087	19.335	19.335	39.237	19.903
54	0.00455	0.00455	0.00367	0.00088	19.375	19.375	39.338	19.963
55	0.00457	0.00457	0.00369	0.00088	19.416	19.416	39.441	20.025
56	0.00460	0.00460	0.00371	0.00089	19.458	19.458	39.547	20.089
57	0.00462	0.00462	0.00373	0.00089	19.501	19.501	39.655	20.155
58	0.00465	0.00465	0.00375	0.00090	19.545	19.545	39.767	20.222
59	0.00467	0.00467	0.00377	0.00090	19.590	19.590	39.881	20.291
60	0.00470	0.00470	0.00379	0.00091	19.637	19.637	39.998	20.362
61	0.00472	0.00472	0.00381	0.00091	19.684	19.684	40.118	20.434
62	0.00475	0.00475	0.00383	0.00092	19.733	19.733	40.241	20.508
63	0.00477	0.00477	0.00385	0.00092	19.783	19.783	40.368	20.585
64	0.00479	0.00479	0.00387	0.00092	19.834	19.834	40.497	20.663
65	0.00482	0.00482	0.00389	0.00093	19.886	19.886	40.630	20.743

Data source: KEWT 7.13, 1960/90-2011 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

Table 3 Recursive programming of technology-$_{FLOW}$ and technology-$_{STOCK=A=TFP}$, each by Hicks, Solow, and Harrod, in the transitional path: Japan 2010

speed years	FLOW	three sorts of tech Neutrality			STOCK	three sorts of tech Neutrality		
107.87	Japan	Hick	Solow	Harrod	Japan			
at converge	0.01028	0.01028	0.00929	0.00099	55810	55810	178550	122740
	$m=g_{A(FLOW)}$	$N_{EUT}=m$	$N_{EUT}=m(1-\alpha)$	$N_{EUT}=m\alpha$	Hicks's A	Solow's A	Harrod'A	differ=H-S
0	0.00471	0.00471	0.00426	0.00045	1322	1322	2841	1518
1	0.00477	0.00477	0.00431	0.00046	1340	1340	2882	1543
2	0.00482	0.00482	0.00436	0.00046	1358	1358	2925	1567
3	0.00488	0.00488	0.00441	0.00047	1376	1376	2969	1593
4	0.00493	0.00493	0.00446	0.00047	1395	1395	3015	1620
5	0.00499	0.00499	0.00451	0.00048	1415	1415	3062	1647
6	0.00504	0.00504	0.00456	0.00048	1435	1435	3110	1675
7	0.00510	0.00510	0.00461	0.00049	1456	1456	3160	1704
8	0.00515	0.00515	0.00466	0.00050	1477	1477	3212	1734
9	0.00521	0.00521	0.00470	0.00050	1499	1499	3265	1765
10	0.00526	0.00526	0.00475	0.00051	1522	1522	3320	1797
11	0.00531	0.00531	0.00480	0.00051	1546	1546	3376	1830
12	0.00537	0.00537	0.00485	0.00052	1570	1570	3434	1865
13	0.00542	0.00542	0.00490	0.00052	1595	1595	3495	1900
14	0.00548	0.00548	0.00495	0.00053	1620	1620	3557	1937
15	0.00553	0.00553	0.00500	0.00053	1647	1647	3621	1974
16	0.00559	0.00559	0.00505	0.00054	1674	1674	3687	2013
17	0.00564	0.00564	0.00510	0.00054	1702	1702	3756	2054
18	0.00569	0.00569	0.00515	0.00055	1731	1731	3826	2096
19	0.00575	0.00575	0.00519	0.00055	1761	1761	3899	2139
20	0.00580	0.00580	0.00524	0.00056	1791	1791	3975	2184
21	0.00586	0.00586	0.00529	0.00056	1823	1823	4053	2230
22	0.00591	0.00591	0.00534	0.00057	1856	1856	4134	2278
23	0.00596	0.00596	0.00539	0.00057	1890	1890	4218	2328
24	0.00602	0.00602	0.00544	0.00058	1925	1925	4304	2379
25	0.00607	0.00607	0.00549	0.00058	1961	1961	4394	2433
26	0.00612	0.00612	0.00553	0.00059	1998	1998	4486	2488
27	0.00618	0.00618	0.00558	0.00059	2037	2037	4582	2545
28	0.00623	0.00623	0.00563	0.00060	2077	2077	4682	2605
29	0.00628	0.00628	0.00568	0.00060	2118	2118	4785	2667
30	0.00634	0.00634	0.00573	0.00061	2161	2161	4892	2731
31	0.00639	0.00639	0.00577	0.00061	2205	2205	5003	2797
32	0.00644	0.00644	0.00582	0.00062	2251	2251	5117	2867
33	0.00649	0.00649	0.00587	0.00062	2298	2298	5237	2938
34	0.00655	0.00655	0.00592	0.00063	2347	2347	5360	3013
35	0.00660	0.00660	0.00597	0.00063	2398	2398	5489	3091
36	0.00665	0.00665	0.00601	0.00064	2451	2451	5622	3172
37	0.00671	0.00671	0.00606	0.00064	2505	2505	5761	3256
38	0.00676	0.00676	0.00611	0.00065	2562	2562	5905	3343
39	0.00681	0.00681	0.00616	0.00066	2621	2621	6055	3435
40	0.00686	0.00686	0.00620	0.00066	2682	2682	6211	3530
41	0.00692	0.00692	0.00625	0.00067	2745	2745	6373	3629
42	0.00697	0.00697	0.00630	0.00067	2811	2811	6542	3732
43	0.00702	0.00702	0.00635	0.00068	2879	2879	6718	3840
44	0.00707	0.00707	0.00639	0.00068	2950	2950	6902	3952
45	0.00712	0.00712	0.00644	0.00069	3023	3023	7093	4070
46	0.00718	0.00718	0.00649	0.00069	3100	3100	7292	4192
47	0.00723	0.00723	0.00653	0.00070	3180	3180	7501	4320
48	0.00728	0.00728	0.00658	0.00070	3263	3263	7718	4455
49	0.00733	0.00733	0.00663	0.00071	3350	3350	7945	4595
50	0.00738	0.00738	0.00667	0.00071	3440	3440	8182	4742
51	0.00744	0.00744	0.00672	0.00072	3534	3534	8430	4896
52	0.00749	0.00749	0.00677	0.00072	3632	3632	8689	5057
53	0.00754	0.00754	0.00681	0.00073	3735	3735	8960	5226
54	0.00759	0.00759	0.00686	0.00073	3842	3842	9245	5403
55	0.00764	0.00764	0.00691	0.00073	3953	3953	9543	5589
56	0.00769	0.00769	0.00695	0.00074	4070	4070	9855	5785
57	0.00775	0.00775	0.00700	0.00074	4192	4192	10183	5991
58	0.00780	0.00780	0.00705	0.00075	4320	4320	10527	6207
59	0.00785	0.00785	0.00709	0.00075	4454	4454	10888	6434
60	0.00790	0.00790	0.00714	0.00076	4594	4594	11268	6674
61	0.00795	0.00795	0.00719	0.00076	4741	4741	11667	6926
62	0.00800	0.00800	0.00723	0.00077	4895	4895	12088	7193
63	0.00805	0.00805	0.00728	0.00077	5057	5057	12531	7474
64	0.00810	0.00810	0.00732	0.00078	5227	5227	12998	7771
65	0.00815	0.00815	0.00737	0.00078	5406	5406	13490	8085

Data source: KEWT 7.13, 1960/90-2011 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

Table 4-1 BEP in the C-D PF, from macro to micro: the US, before adjustment

11 MACRO BEP→MICRO BEP				X=Y/0.9					BEP=f/(1-v)			
the US 2010			from MACRO	Y/X								to MICRO
		Life-Time employment		0.9	0.5	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	0.7919	14246	1425	11577	0.9744	0.1000	0.8798	0.8127	2669	0.18733
1	12950	10254	0.7919	14389	1439	11693	0.9744	0.1000	0.8798	0.8127	2695	0.18733
2	13080	10357	0.7919	14533	1453	11810	0.9744	0.1000	0.8798	0.8127	2722	0.18733
3	13211	10461	0.7919	14678	1468	11929	0.9744	0.1000	0.8798	0.8127	2750	0.18733
4	13343	10565	0.7919	14825	1483	12048	0.9744	0.1000	0.8798	0.8127	2777	0.18733
5	13476	10671	0.7919	14974	1497	12169	0.9744	0.1000	0.8798	0.8127	2805	0.18733
6	13611	10778	0.7919	15123	1512	12290	0.9744	0.1000	0.8798	0.8127	2833	0.18733
7	13747	10886	0.7919	15275	1527	12413	0.9744	0.1000	0.8798	0.8127	2861	0.18733
8	13885	10995	0.7919	15427	1543	12537	0.9744	0.1000	0.8798	0.8127	2890	0.18733
9	14024	11105	0.7919	15582	1558	12663	0.9744	0.1000	0.8798	0.8127	2919	0.18733
10	14164	11216	0.7919	15738	1574	12789	0.9744	0.1000	0.8798	0.8127	2948	0.18733
				X=Y/0.5								
				Y/X	0.5							
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	**0.7919**	25643	12822	22974	0.8838	0.5000	**1.5837**	0.8959	2669	0.10407
1	12950	10254	**0.7919**	25900	12950	23204	0.8838	0.5000	**1.5837**	0.8959	2695	0.10407
2	13080	10357	**0.7919**	26159	13080	23437	0.8838	0.5000	**1.5837**	0.8959	2722	0.10407
3	13211	10461	**0.7919**	26421	13211	23671	0.8838	0.5000	**1.5837**	0.8959	2750	0.10407
4	13343	10565	**0.7919**	26685	13343	23908	0.8838	0.5000	**1.5837**	0.8959	2777	0.10407
5	13476	10671	**0.7919**	26952	13476	24147	0.8838	0.5000	**1.5837**	0.8959	2805	0.10407
6	13611	10778	**0.7919**	27222	13611	24389	0.8838	0.5000	**1.5837**	0.8959	2833	0.10407
7	13747	10886	**0.7919**	27494	13747	24633	0.8838	0.5000	**1.5837**	0.8959	2861	0.10407
8	13885	10995	**0.7919**	27769	13885	24879	0.8838	0.5000	**1.5837**	0.8959	2890	0.10407
9	14024	11105	**0.7919**	28047	14024	25128	0.8838	0.5000	**1.5837**	0.8959	2919	0.10407
10	14164	11216	**0.7919**	28328	14164	25379	0.8838	0.5000	**1.5837**	0.8959	2948	0.10407
					X=Y/0.1							
					Y/X	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	0.7919	128216	115394	125547	0.8087	0.9000	7.9185	0.9792	2669	0.02081
1	12950	10254	0.7919	129500	116550	126804	0.8087	0.9000	7.9185	0.9792	2695	0.02081
2	13080	10357	0.7919	130796	117717	128074	0.8087	0.9000	7.9185	0.9792	2722	0.02081
3	13211	10461	0.7919	132105	118895	129356	0.8087	0.9000	7.9185	0.9792	2750	0.02081
4	13343	10565	0.7919	133427	120085	130650	0.8087	0.9000	7.9185	0.9792	2777	0.02081
5	13476	10671	0.7919	134762	121286	131957	0.8087	0.9000	7.9185	0.9792	2805	0.02081
6	13611	10778	0.7919	136110	122499	133277	0.8087	0.9000	7.9185	0.9792	2833	0.02081
7	13747	10886	0.7919	137472	123725	134611	0.8087	0.9000	7.9185	0.9792	2861	0.02081
8	13885	10995	0.7919	138847	124962	135957	0.8087	0.9000	7.9185	0.9792	2890	0.02081
9	14024	11105	0.7919	140235	126212	137317	0.8087	0.9000	7.9185	0.9792	2919	0.02081
10	14164	11216	0.7919	141638	127474	138690	0.8087	0.9000	7.9185	0.9792	2948	0.02081

Data source: KEWT 7.12, 1960/90-2010 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

Table 4-2 BEP in the C-D PF, from macro to micro: the US, after adjustment

22 MACRO BEP→MICRO BEP				X=Y/0.9					BEP=f/(1-v)			
the US 2010			from MACRO	Y/X								to MICRO
	Half-Life-Time employment			0.9	0.5	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	0.7919	14246	1425	11577	0.9744	0.1000	0.8798	0.8127	2669	0.18733
1	12950	10254	0.7919	14389	1439	11693	0.9744	0.1000	0.8798	0.8127	2695	0.18733
2	13080	10357	0.7919	14533	1453	11810	0.9744	0.1000	0.8798	0.8127	2722	0.18733
3	13211	10461	0.7919	14678	1468	11929	0.9744	0.1000	0.8798	0.8127	2750	0.18733
4	13343	10565	0.7919	14825	1483	12048	0.9744	0.1000	0.8798	0.8127	2777	0.18733
5	13476	10671	0.7919	14974	1497	12169	0.9744	0.1000	0.8798	0.8127	2805	0.18733
6	13611	10778	0.7919	15123	1512	12290	0.9744	0.1000	0.8798	0.8127	2833	0.18733
7	13747	10886	0.7919	15275	1527	12413	0.9744	0.1000	0.8798	0.8127	2861	0.18733
8	13885	10995	0.7919	15427	1543	12537	0.9744	0.1000	0.8798	0.8127	2890	0.18733
9	14024	11105	0.7919	15582	1558	12663	0.9744	0.1000	0.8798	0.8127	2919	0.18733
10	14164	11216	0.7919	15738	1574	12789	0.9744	0.1000	0.8798	0.8127	2948	0.18733
				X=Y/0.5								
				Y/X	0.5							
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	**0.4500**	25643	12822	22974	0.5023	0.5000	**0.9000**	0.8959	2669	0.10407
1	12950	10254	**0.4500**	25900	12950	23204	0.5023	0.5000	**0.9000**	0.8959	2695	0.10407
2	13080	10357	**0.4500**	26159	13080	23437	0.5023	0.5000	**0.9000**	0.8959	2722	0.10407
3	13211	10461	**0.4500**	26421	13211	23671	0.5023	0.5000	**0.9000**	0.8959	2750	0.10407
4	13343	10565	**0.4500**	26685	13343	23908	0.5023	0.5000	**0.9000**	0.8959	2777	0.10407
5	13476	10671	**0.4500**	26952	13476	24147	0.5023	0.5000	**0.9000**	0.8959	2805	0.10407
6	13611	10778	**0.4500**	27222	13611	24389	0.5023	0.5000	**0.9000**	0.8959	2833	0.10407
7	13747	10886	**0.4500**	27494	13747	24633	0.5023	0.5000	**0.9000**	0.8959	2861	0.10407
8	13885	10995	**0.4500**	27769	13885	24879	0.5023	0.5000	**0.9000**	0.8959	2890	0.10407
9	14024	11105	**0.4500**	28047	14024	25128	0.5023	0.5000	**0.9000**	0.8959	2919	0.10407
10	14164	11216	**0.4500**	28328	14164	25379	0.5023	0.5000	**0.9000**	0.8959	2948	0.10407
					X=Y/0.1							
					Y/X	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	**0.0900**	128216	115394	125547	0.0919	0.9000	**0.9000**	0.9792	2669	0.02081
1	12950	10254	**0.0900**	129500	116550	126804	0.0919	0.9000	**0.9000**	0.9792	2695	0.02081
2	13080	10357	**0.0900**	130796	117717	128074	0.0919	0.9000	**0.9000**	0.9792	2722	0.02081
3	13211	10461	**0.0900**	132105	118895	129356	0.0919	0.9000	**0.9000**	0.9792	2750	0.02081
4	13343	10565	**0.0900**	133427	120085	130650	0.0919	0.9000	**0.9000**	0.9792	2777	0.02081
5	13476	10671	**0.0900**	134762	121286	131957	0.0919	0.9000	**0.9000**	0.9792	2805	0.02081
6	13611	10778	**0.0900**	136110	122499	133277	0.0919	0.9000	**0.9000**	0.9792	2833	0.02081
7	13747	10886	**0.0900**	137472	123725	134611	0.0919	0.9000	**0.9000**	0.9792	2861	0.02081
8	13885	10995	**0.0900**	138847	124962	135957	0.0919	0.9000	**0.9000**	0.9792	2890	0.02081
9	14024	11105	**0.0900**	140235	126212	137317	0.0919	0.9000	**0.9000**	0.9792	2919	0.02081
10	14164	11216	**0.0900**	141638	127474	138690	0.0919	0.9000	**0.9000**	0.9792	2948	0.02081

Data source: KEWT 7.12, 1960/90-2010 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

Table 5-1 BEP in the C-D PF, from macro to micro: Japan, before adjustment

11 MACRO BEP→MICRO BEP				X=Y/0.9					BEP=f/(1-v)			
Japan 2010			from MACRO	Y/X								to MICRO
		Life-Time employment		0.9	0.5	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	414521	374658	0.9038	460579	46058	420716	0.9895	0.1000	1.0043	0.9134	39863	0.08655
1	419994	379605	0.9038	466660	46666	426271	0.9895	0.1000	1.0043	0.9134	40390	0.08655
2	425626	384695	0.9038	472918	47292	431987	0.9895	0.1000	1.0043	0.9134	40931	0.08655
3	431422	389933	0.9038	479357	47936	437869	0.9895	0.1000	1.0043	0.9134	41489	0.08655
4	437387	395325	0.9038	485985	48599	443923	0.9895	0.1000	1.0043	0.9134	42062	0.08655
5	443527	400874	0.9038	492808	49281	450155	0.9895	0.1000	1.0043	0.9134	42653	0.08655
6	449848	406587	0.9038	499831	49983	456570	0.9895	0.1000	1.0043	0.9134	43261	0.08655
7	456356	412470	0.9038	507063	50706	463176	0.9895	0.1000	1.0043	0.9134	43886	0.08655
8	463059	418528	0.9038	514510	51451	469979	0.9895	0.1000	1.0043	0.9134	44531	0.08655
9	469962	424767	0.9038	522180	52218	476985	0.9895	0.1000	1.0043	0.9134	45195	0.08655
10	477073	431194	0.9038	530081	53008	484202	0.9895	0.1000	1.0043	0.9134	45879	0.08655
				X=Y/0.5								
				Y/X	0.5							
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	414521	374658	**0.9038**	829042	414521	789179	0.9495	0.5000	**1.8077**	0.9519	39863	0.04808
1	419994	379605	**0.9038**	839989	419994	799599	0.9495	0.5000	**1.8077**	0.9519	40390	0.04808
2	425626	384695	**0.9038**	851252	425626	810321	0.9495	0.5000	**1.8077**	0.9519	40931	0.04808
3	431422	389933	**0.9038**	862843	431422	821355	0.9495	0.5000	**1.8077**	0.9519	41489	0.04808
4	437387	395325	**0.9038**	874773	437387	832711	0.9495	0.5000	**1.8077**	0.9519	42062	0.04808
5	443527	400874	**0.9038**	887054	443527	844401	0.9495	0.5000	**1.8077**	0.9519	42653	0.04808
6	449848	406587	**0.9038**	899696	449848	856435	0.9495	0.5000	**1.8077**	0.9519	43261	0.04808
7	456356	412470	**0.9038**	912713	456356	868826	0.9495	0.5000	**1.8077**	0.9519	43886	0.04808
8	463059	418528	**0.9038**	926117	463059	881586	0.9495	0.5000	**1.8077**	0.9519	44531	0.04808
9	469962	424767	**0.9038**	939924	469962	894729	0.9495	0.5000	**1.8077**	0.9519	45195	0.04808
10	477073	431194	**0.9038**	954145	477073	908267	0.9495	0.5000	**1.8077**	0.9519	45879	0.04808
					X=Y/0.1							
					Y/X	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	414521	374658	0.9038	4145210	3730689	4105346	0.9126	0.9000	9.0383	0.9904	39863	0.00962
1	419994	379605	0.9038	4199943	3779948	4159553	0.9126	0.9000	9.0383	0.9904	40390	0.00962
2	425626	384695	0.9038	4256261	3830635	4215330	0.9126	0.9000	9.0383	0.9904	40931	0.00962
3	431422	389933	0.9038	4314217	3882796	4272729	0.9126	0.9000	9.0383	0.9904	41489	0.00962
4	437387	395325	0.9038	4373867	3936481	4331805	0.9126	0.9000	9.0383	0.9904	42062	0.00962
5	443527	400874	0.9038	4435268	3991741	4392615	0.9126	0.9000	9.0383	0.9904	42653	0.00962
6	449848	406587	0.9038	4498479	4048631	4455219	0.9126	0.9000	9.0383	0.9904	43261	0.00962
7	456356	412470	0.9038	4563564	4107207	4519677	0.9126	0.9000	9.0383	0.9904	43886	0.00962
8	463059	418528	0.9038	4630587	4167528	4586056	0.9126	0.9000	9.0383	0.9904	44531	0.00962
9	469962	424767	0.9038	4699618	4229656	4654423	0.9126	0.9000	9.0383	0.9904	45195	0.00962
10	477073	431194	0.9038	4770727	4293654	4724848	0.9126	0.9000	9.0383	0.9904	45879	0.00962

Data source: KEWT 7.12, 1960/90-2010 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

Table 5-2 BEP in the C-D PF, from macro to micro: Japan, after adjustment

22 MACRO BEP→MICRO BEP				X=Y/0.9					BEP=f/(1-v)			
Japan 2010			from MACRO	Y/X								to MICRO
	Half-Life-Time employment			0.9	0.5	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	0.7919	14246	1425	11577	0.9744	0.1000	0.8798	0.8127	2669	0.18733
1	12950	10254	0.7919	14389	1439	11693	0.9744	0.1000	0.8798	0.8127	2695	0.18733
2	13080	10357	0.7919	14533	1453	11810	0.9744	0.1000	0.8798	0.8127	2722	0.18733
3	13211	10461	0.7919	14678	1468	11929	0.9744	0.1000	0.8798	0.8127	2750	0.18733
4	13343	10565	0.7919	14825	1483	12048	0.9744	0.1000	0.8798	0.8127	2777	0.18733
5	13476	10671	0.7919	14974	1497	12169	0.9744	0.1000	0.8798	0.8127	2805	0.18733
6	13611	10778	0.7919	15123	1512	12290	0.9744	0.1000	0.8798	0.8127	2833	0.18733
7	13747	10886	0.7919	15275	1527	12413	0.9744	0.1000	0.8798	0.8127	2861	0.18733
8	13885	10995	0.7919	15427	1543	12537	0.9744	0.1000	0.8798	0.8127	2890	0.18733
9	14024	11105	0.7919	15582	1558	12663	0.9744	0.1000	0.8798	0.8127	2919	0.18733
10	14164	11216	0.7919	15738	1574	12789	0.9744	0.1000	0.8798	0.8127	2948	0.18733
				X=Y/0.5								
				Y/X	0.5							
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	**0.4500**	25643	12822	22974	0.5023	0.5000	**0.9000**	0.8959	2669	0.10407
1	12950	10254	**0.4500**	25900	12950	23204	0.5023	0.5000	**0.9000**	0.8959	2695	0.10407
2	13080	10357	**0.4500**	26159	13080	23437	0.5023	0.5000	**0.9000**	0.8959	2722	0.10407
3	13211	10461	**0.4500**	26421	13211	23671	0.5023	0.5000	**0.9000**	0.8959	2750	0.10407
4	13343	10565	**0.4500**	26685	13343	23908	0.5023	0.5000	**0.9000**	0.8959	2777	0.10407
5	13476	10671	**0.4500**	26952	13476	24147	0.5023	0.5000	**0.9000**	0.8959	2805	0.10407
6	13611	10778	**0.4500**	27222	13611	24389	0.5023	0.5000	**0.9000**	0.8959	2833	0.10407
7	13747	10886	**0.4500**	27494	13747	24633	0.5023	0.5000	**0.9000**	0.8959	2861	0.10407
8	13885	10995	**0.4500**	27769	13885	24879	0.5023	0.5000	**0.9000**	0.8959	2890	0.10407
9	14024	11105	**0.4500**	28047	14024	25128	0.5023	0.5000	**0.9000**	0.8959	2919	0.10407
10	14164	11216	**0.4500**	28328	14164	25379	0.5023	0.5000	**0.9000**	0.8959	2948	0.10407
					X=Y/0.1							
					Y/X	0.1						
t	Y	W	f=W/Y	X	Z=X-Y	E=W+Z	e/f	v=Z/X	BEP=f/(1-v	e=E/X	$P_{(X)}$=X-E	$p_{(P/X)}$=P/X
0	12822	10153	**0.0900**	128216	115394	125547	0.0919	0.9000	**0.9000**	0.9792	2669	0.02081
1	12950	10254	**0.0900**	129500	116550	126804	0.0919	0.9000	**0.9000**	0.9792	2695	0.02081
2	13080	10357	**0.0900**	130796	117717	128074	0.0919	0.9000	**0.9000**	0.9792	2722	0.02081
3	13211	10461	**0.0900**	132105	118895	129356	0.0919	0.9000	**0.9000**	0.9792	2750	0.02081
4	13343	10565	**0.0900**	133427	120085	130650	0.0919	0.9000	**0.9000**	0.9792	2777	0.02081
5	13476	10671	**0.0900**	134762	121286	131957	0.0919	0.9000	**0.9000**	0.9792	2805	0.02081
6	13611	10778	**0.0900**	136110	122499	133277	0.0919	0.9000	**0.9000**	0.9792	2833	0.02081
7	13747	10886	**0.0900**	137472	123725	134611	0.0919	0.9000	**0.9000**	0.9792	2861	0.02081
8	13885	10995	**0.0900**	138847	124962	135957	0.0919	0.9000	**0.9000**	0.9792	2890	0.02081
9	14024	11105	**0.0900**	140235	126212	137317	0.0919	0.9000	**0.9000**	0.9792	2919	0.02081
10	14164	11216	**0.0900**	141638	127474	138690	0.0919	0.9000	**0.9000**	0.9792	2948	0.02081

Data source: KEWT 7.12, 1960/90-2010 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

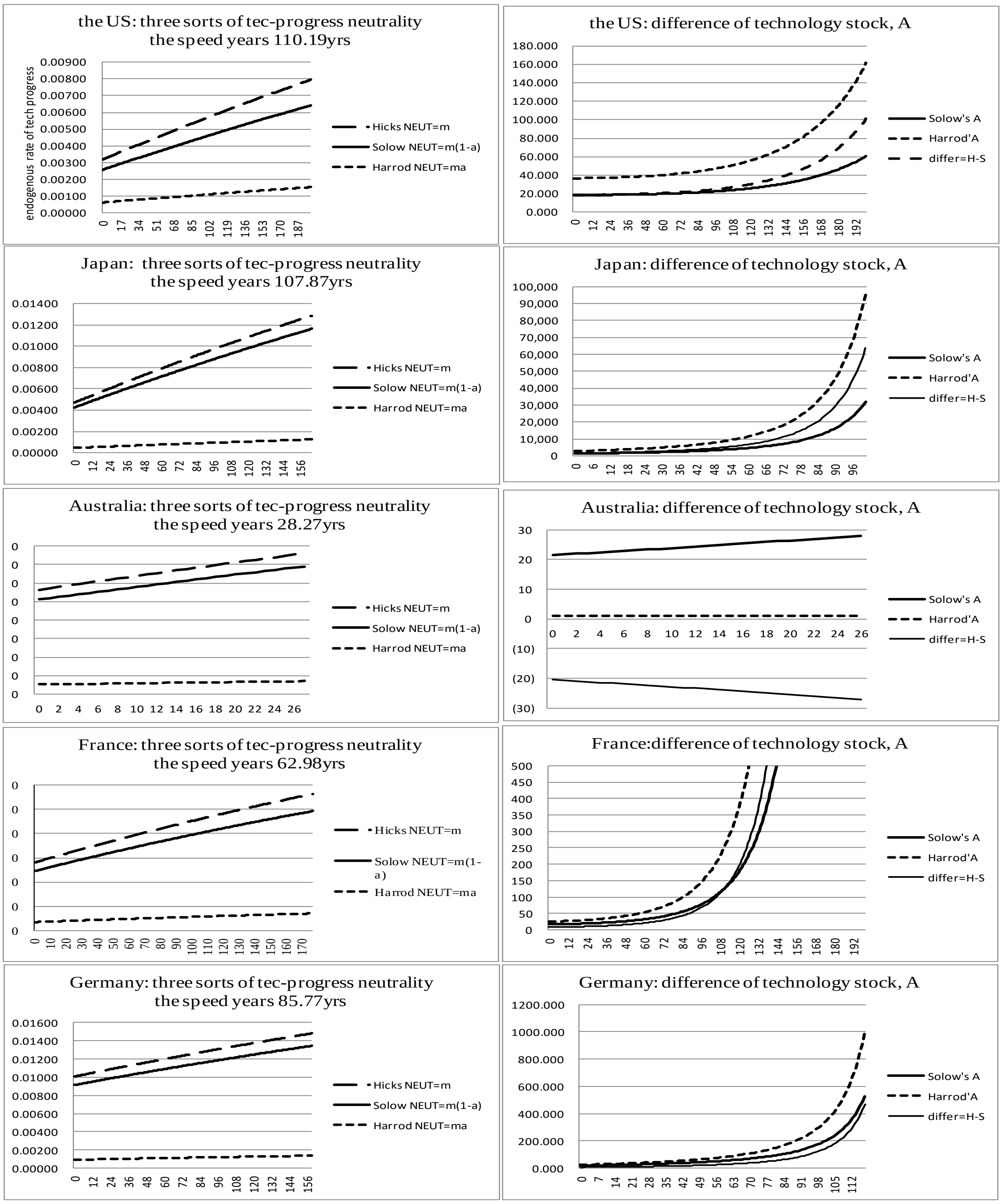

Figure 1 Recursive programming of technology-$_{\text{FLOW}}$ and technology-$_{\text{STOCK=A=TFP}}$, each by Hicks, Solow, and Harrod, in the transitional path: five countries 2010

Data source: KEWT 7.13, 1960/90-2011 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

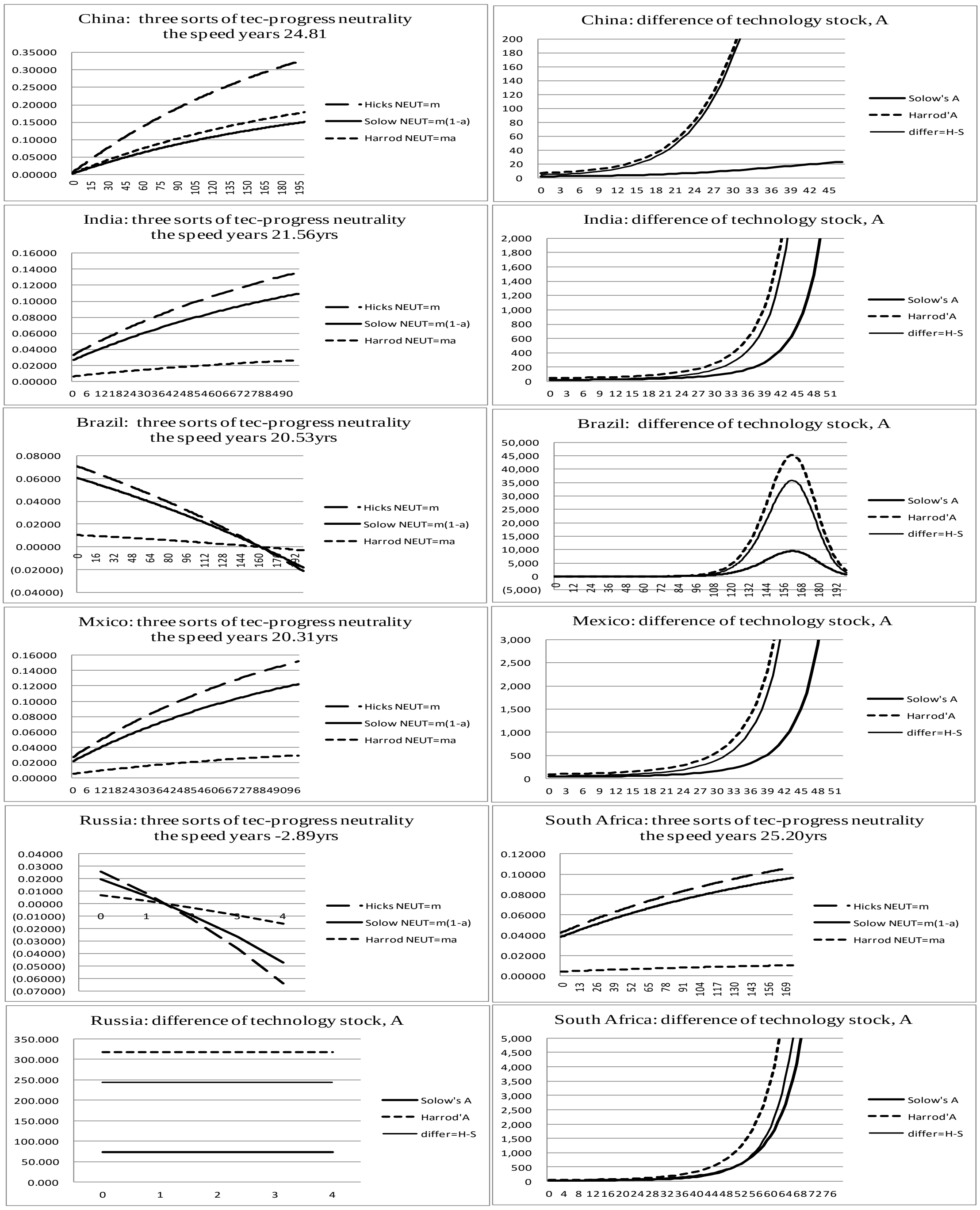

Figure 2 Recursive programming of technology-$_{\text{FLOW}}$ and technology-$_{\text{STOCK=A=TFP}}$, each by Hicks, Solow, and Harrod, in the transitional path: six countries 2010

Data source: KEWT 7.13, 1960/90-2011 by sector, for 86 countries, whose original data are from *IFSY*, IMF.

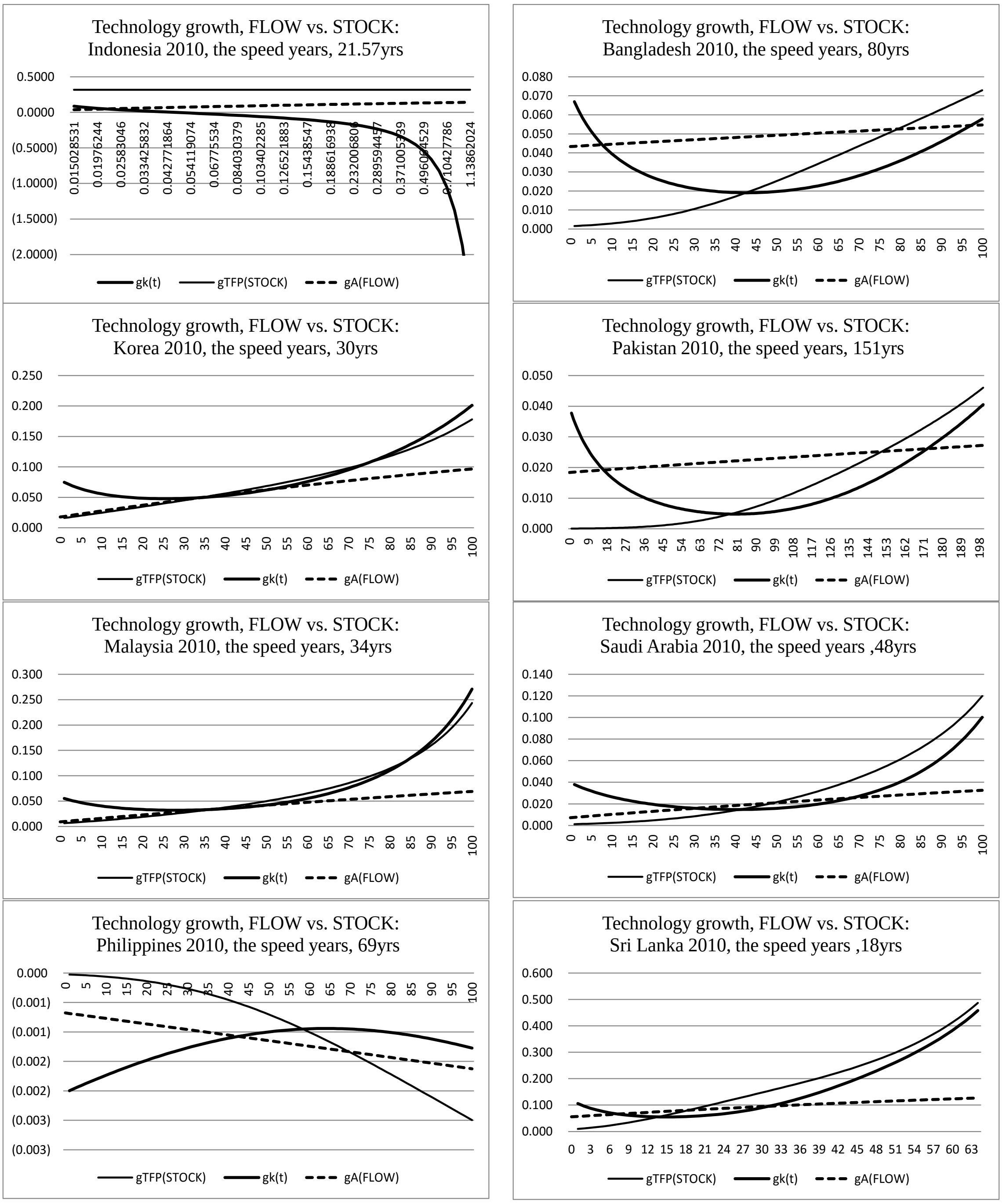

Figure 3 Growth rates of technology (STOCK) & (FLOW) and the capital-labor: 8 Asian countries, 2010

Data source: KEWT 6.12, 1990-2010, 17 Asian countries, whose original data are from *IFSY*, IMF.

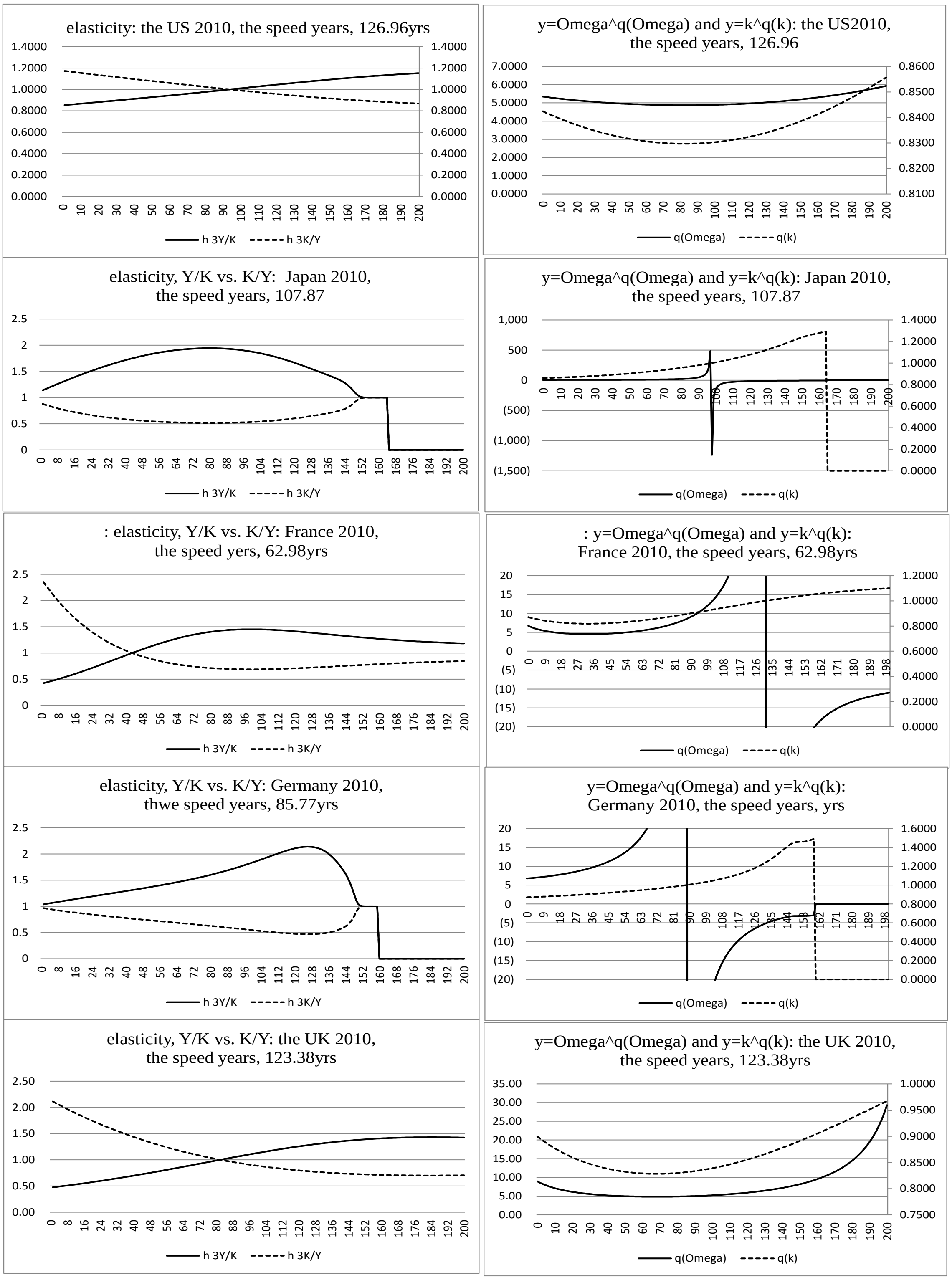

Figure 4 Simulation of elasticity of the capital-output ratio and the capital-labor ratio by country, with each speed years

Data source: Recursive programming by KEWT 6.12, 1990-2010, whose original data are from *IFSY*, IMF.

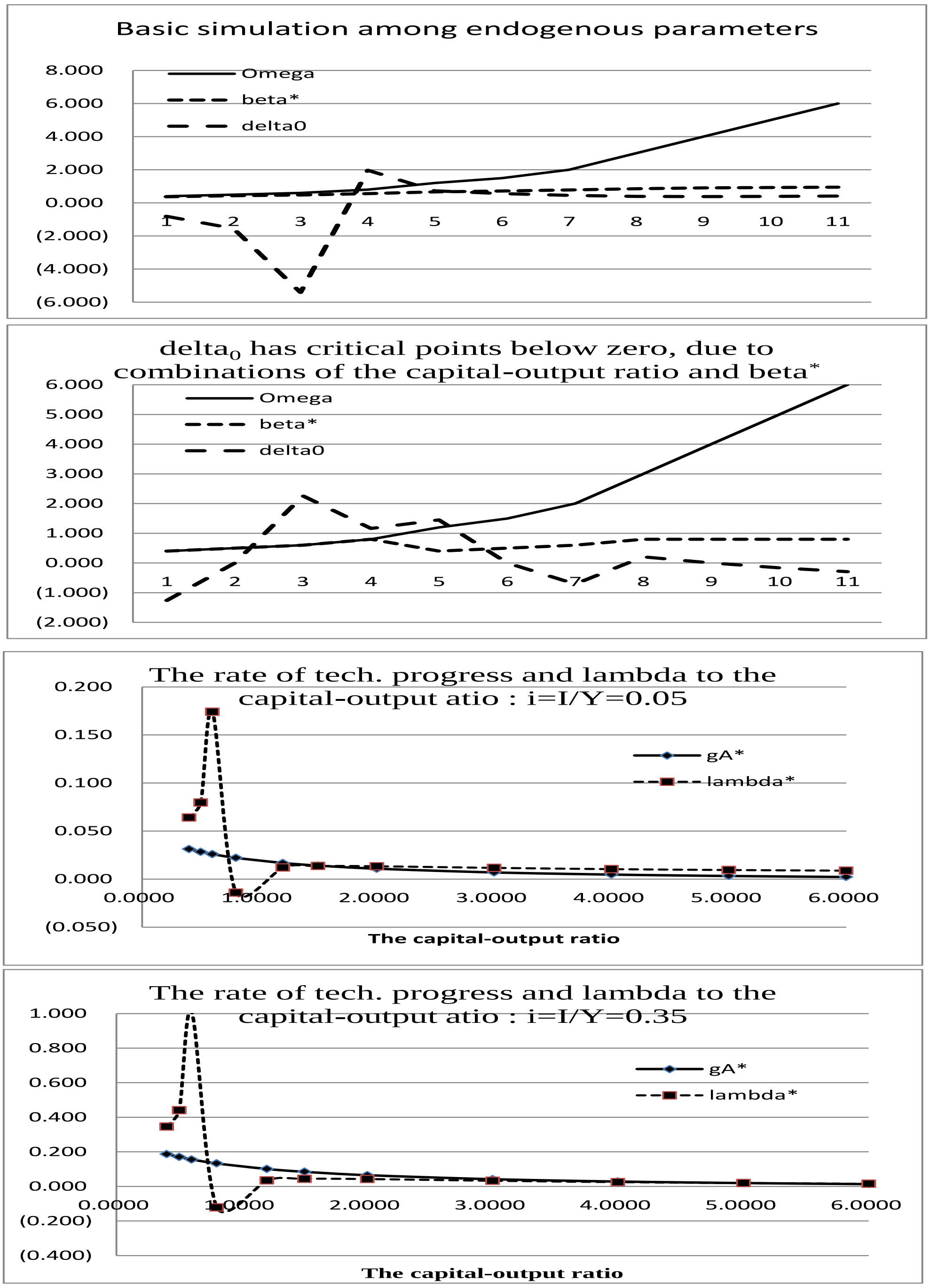

Figure 5 Structure analyses of seven endogenous parameters in recursive programming

Data source: KEWT 6.12, 1960/90-2010, for 81 countries, whose original data are from *IFSY*, IMF.

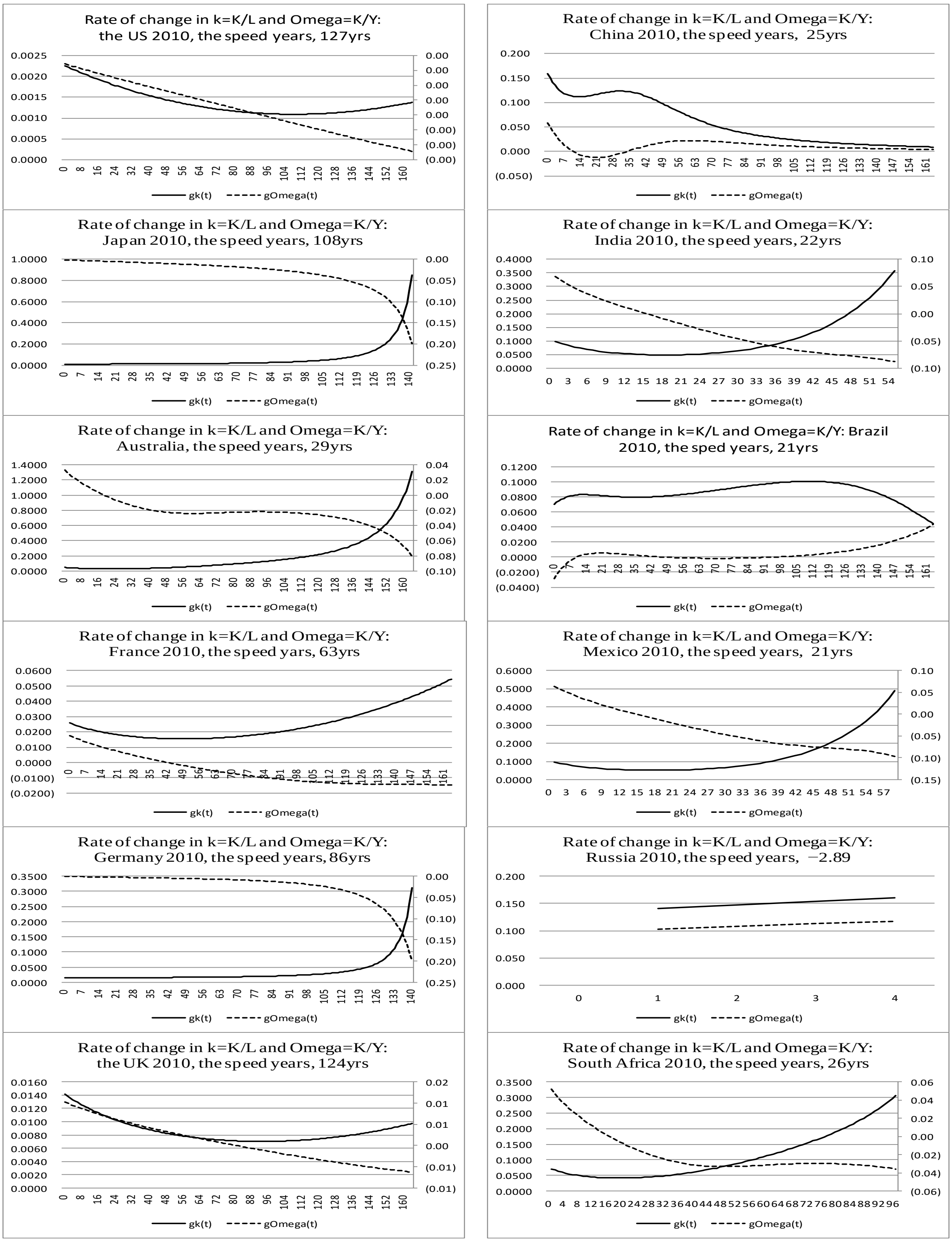

Figure 6 Growth rates of the capital-labor ratio and the capital-output ratio: Base area

Data source: Recursive programming of KEWT 6.12, whose original data from International Financial Statistics yearbook, IMF.

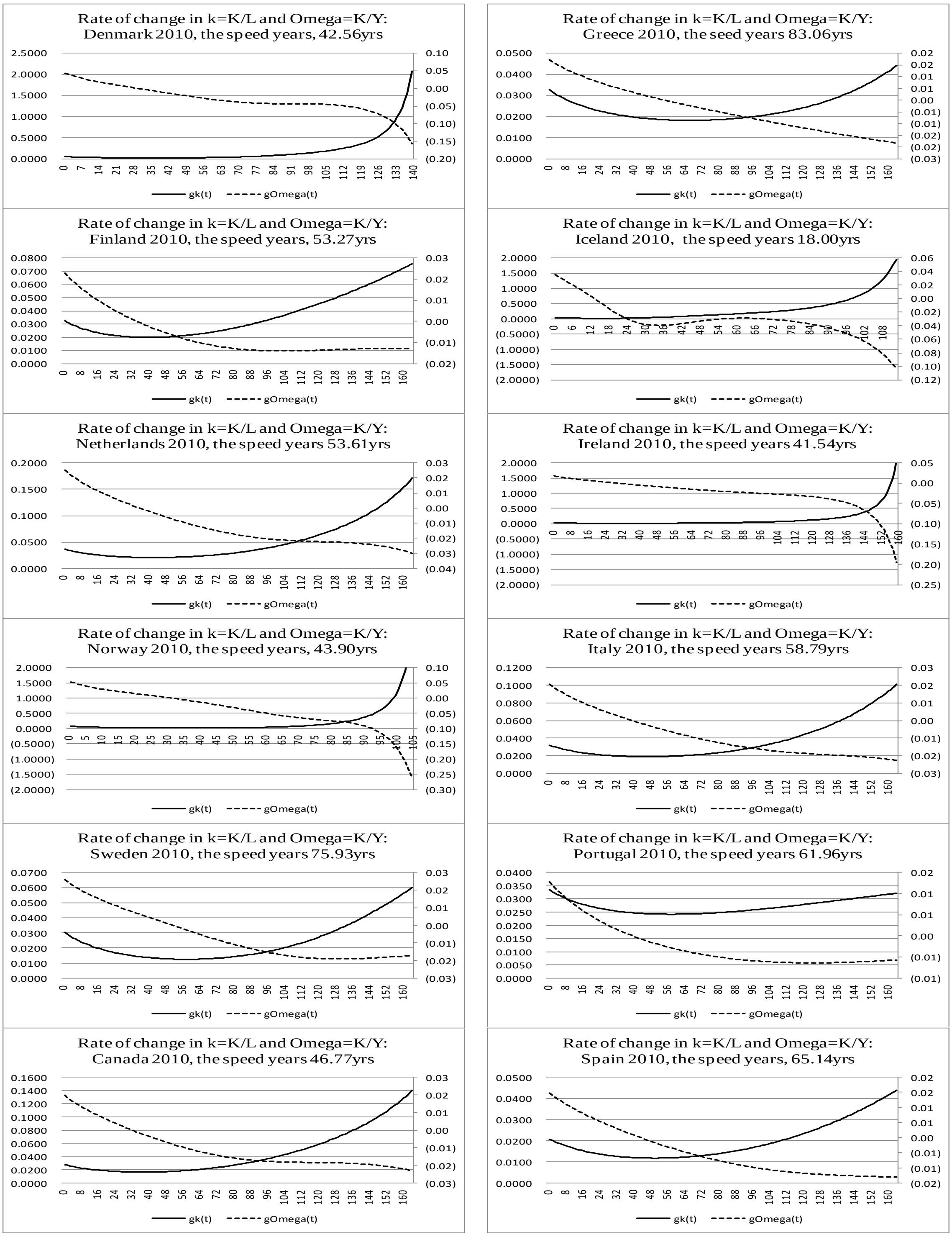

Figure 7 Growth rates of the capital-labor ratio and the capital-output ratio: Euro area

Data source: Recursive programming of KEWT 6.12, whose original data from International Financial Statistics yearbook, IMF.

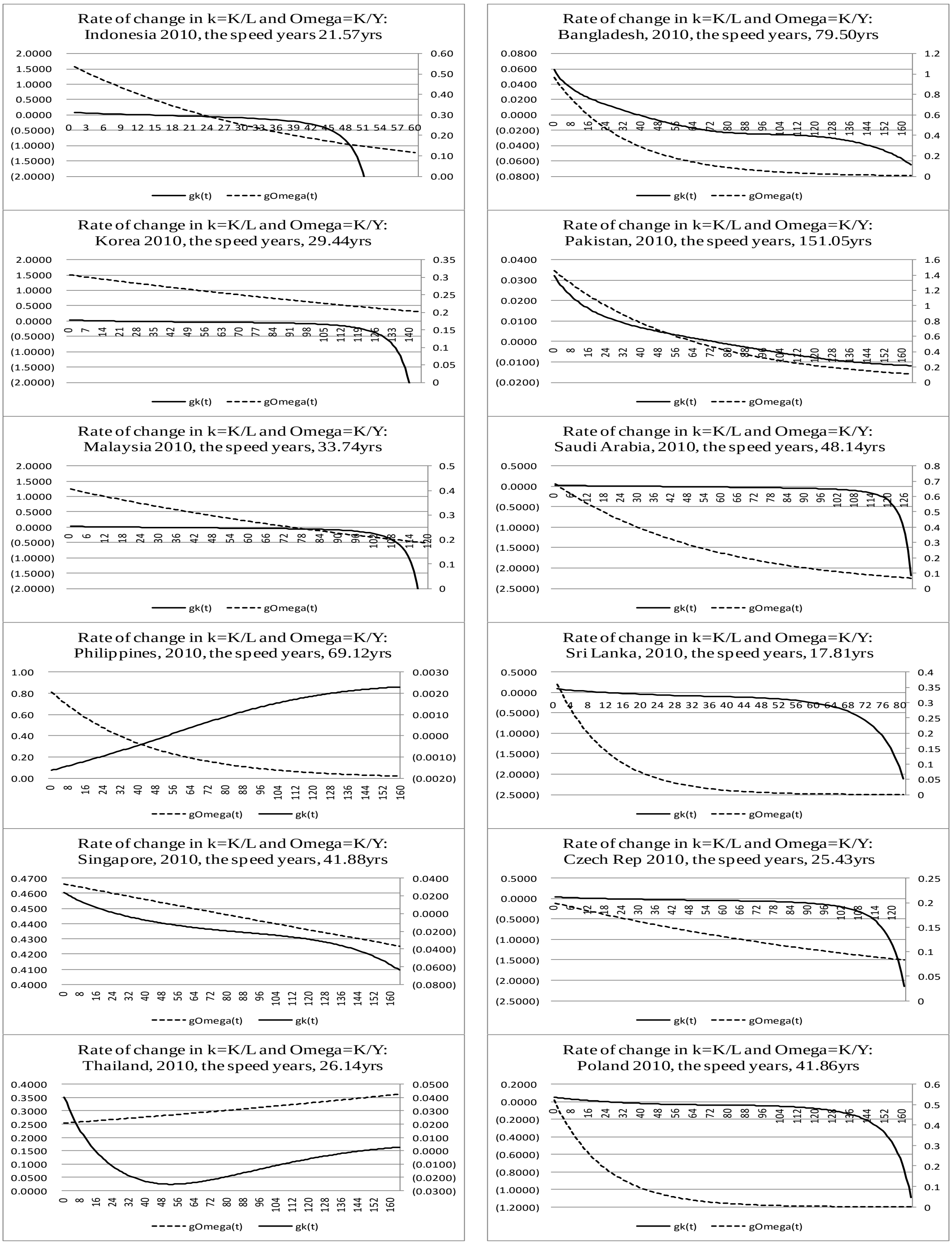

Figure 8 Growth rates of the capital-labor ratio and the capital-output ratio: Asian area

Data source: Recursive programming of KEWT 6.12, whose original data from International Financial Statistics yearbook, IMF.

References

1. Alain, D. Ayong Le Kama. (2001). Sustainable Growth, Renewable Resources and Pollution. *Journal of Economic Dynamics and Control* 25 (Dec, 12): 1911-1918.
2. Bliss Christopher. (1999). Galton's Fallacy and Economic Convergence. *Oxford Economic Papers* 51 (1999): 4-14.
3. Bliss Christopher. (2000). Galton's Fallacy and Economic Convergence: A Reply to Cannon and Duck. *Oxford Economic Papers* 52 (2000): 420-422.
4. Dhrymes, P. J. (1962). On Devising Unbiased Estimators for the Parameters of the Cobb-Douglas Production Function. *Econometrica* 30 (2): 297-304.
5. Feng Wang and Andrew Mason. (2007). Population Ageing: Challenges, Opportunities, and Institutions, 177-196. In: *Transition and Challenge: China's Population at the Beginning of the 21st Century*. Oxford and New York: Oxford University Press. xiv, 348p.
6. Feng Wang, and Andrew Mason. (2005). Demographic Dividend and Prospects for Economic Development in China, 1-18. *United Nations Expert Group Meeting on Social and Economic Implications of changing Population Age Structures*, Mexico (UN/POP/PD/2005/5).
7. Fisk, P. R. (1964). Estimation of Marginal Product from A Cobb-Douglas Production Function. *Econometrica* 32 (3): 451-451.
8. Fisk, P. R. (1966). Estimation of Marginal Product from A Cobb-Douglas Production Function. *Econometrica* 34 (1): 162-172.
9. Gabor, Dennis. (1971). Nobel Lecture, December 11, 1971. http://www.nobelprize.org/nobel_prizes/physics/laureates/1971/gabor-lecture.pdf .
10. Gallmeyer, Michael, F.; Hollifield, Burton; and Zin, Stanley, E. (2005). Taylor Rules, McCallum Rules and the Term Structure of Interest rates. *Journal of Monetary Economics* 52 (July, 5): 921-950.
11. Henry, George. (1898). *The Land Question: What It Involves, and How Alone It Can Be Settled*. New York: Doubleday & McClure Co. 109 (The land Question) +74 (Property in Land) +151 (The Condition of Labor)=334.
12. Jorgenson, D.W., and Griliches, Z. (1967). The Explanation of Productivity Change. *Review of Economic Studies* 34 (June, 3): 249-283.
13. Jorgenson, Dale W. (1963). Capital Theory and Investment Behavior. *American Economic Review*: 53 (May, 2): 247-259.
14. Jorgenson, Dale W. (1966). The Embodiment Hypothesis. *Journal of Political Economy* 74 (Feb, 1): 1-17.
15. Kmenta, J. (1964). Some Properties of Alternative Estimates of the Cobb-Douglas Production Function. *Econometrica* 32 (1-2): 183-188.
16. Krugman, Paul. (4Nov2013). Those Depressing Germans (797 words). *NY Times* 3. http://www.nytimes.com/2013/11/04/opinion/krugman-those-depressing-germans.html

17. Meeusen, W., and J. Vandenbroeck. (1977). Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error. *International Economic Review* 18 (2): 435-445.
18. Marshall, David, A. (2005). Comment on: "Estimating the expected marginal rate of substitution." *Journal of Monetary Economics* 52: 971-979.
19. Palivos. Theodore. (2001). Social norms, fertility and economic development. *Journal of Economic Dynamics & Control* 25: 1919-1934.
20. Phelps, Edmund S. (1961). The Golden Rule of Accumulation, *American Economic Review*, 51 (Sep, 4): 638-643.
21. Phelps, Edmund S. (1965). Second Essay on the Golden Rule of Accumulation, *American Economic Review*, 55 (Sep, 4): 793-814.
22. Qiang, Dai, and Singleton, Kenneth, J. (2002). Expectation Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure. *Journal of Financial Economics* 63 (March, 3): 415-441.
23. Salant, Walter, S. (1975). Introduction to William A. Salant's "Taxes, the Multiplier and the Inflationary Gap." *History of Political Economy* 7 (Spring, 1): 3-18.
24. Salant, Walter, S. (1975). The Balanced Budget Multiplier as the Sum of an Infinite Series. *History of Political Economy* 7 (Spring, 1): 28-31.
25. Samuelson, P. A. and Solow, R. M. (1956). A Complete Capital Model Involving Heterogeneous Capital Goods. *Quarterly Journal of Economics* 70 (Nov, 4): 537-562.
26. Samuelson, Paul, A. (1936-37). A Note on Measurement of Utility. *Review of Economic Studies* 4 (Feb., 2): 155-161.
27. Samuelson, Paul, A. (1967). Irving Fisher and the Theory of Capital, 17-37. In: *Ten Economic Studies in the Tradition of Irving Fisher*. New York, London, Sydney: John Wiley & Sons, 284p.
28. Samuelson, Paul, P. (1975). The Balanced-Budget Multiplier: A Case Study in the Sociology and Psychology of Scientific Discovery. *History of Political Economy* 7 (Spring, 1): 43-55.
29. Wicksteed, Philip, H. (1938). *The Common Sense of Political Economy and Selected Papers and Reviews on Economic Theory* (II), 734-753 for review of Jevons's theory. London: George Routledge & Sons, Ltd. 401-871.
30. Zellner, A., Kmenta, and J. Dreze, J. (1966). Specification and Estimation of Cobb-Douglas Production Function Modells. *Econometrica* 34 (4): 784-795.