Chapter 9

Optimum Functions-measure in Hyperbola[1]

Foreword to Chapter 9

Endogenous equations and reduced hyperbola functions are each expressed by graph in the two-dimensional plane, as proved in the *Earth Endogenous System* (*EES*; 1st edition 2013, 2nd edition 2014), with five attributes: the Horizontal and Vertical asymptotes, the Width, the Shape, and the Curvature. This chapter, from the viewpoint of measure, presents each qualitative level of hyperbola functions, commonly to all the endogenous parameters and variables. In the endogenous-equilibrium, each hyperbola is endowed with an optimum point, while parabola with a maximum or minimum point in the same two-dimensional plane. The market principles divide an amount into qualitative price and quantitative quantity, but vertically by goods and services. The *EES* cannot divide an amount into quality and quantity but, the quality of an amount is measured by the curvature, where quality=quantity=1, like money human has used after birth.

How to measure the qualitative level of curvature by hyperbola functions? This is the core of this chapter. The origin of a plane is an ultimate point of moderation but immeasurable, supposing the vertical asymptote overlaps the *y* axis. An optimum range numerously exists between just close-to-origin points and stable wider range of the *x* axis. It implies that the larger the curvature-value, the more stable the optimum level is. This chapter generally clarifies moderate optimum level by hyperbola function and measures the curvature empirically by country, using the KEWT database, whose original data are from "*International Financial Statistics Yearbook*," IMF, over years.

[1] **Acknowledgements:** I am much obliged to Dr. Yisheng Huang and his family. He and I are happily riding 'one' carriage and 'one' friend through two publications, really beyond space and time. Also I am thankful to EmProf. in mathematics, Dr. Yoshiomi Furuta, who is one of my teachers in Japan for many years. And I feel deep connection with Ishida Shizuko, as one of my broader benefactors. Lastly, I could perceive a fact that librarians in the world are one of invaluable bearers for human progress and original discoveries. This is because I have found original sources for whatever I hit in mind, in Libraries and bookstores, public or private, of many countries in the West or East.

Signposts to Chapter 9: algebraic, geometric, economic, science, human, physics, methodology, the two-dimensional plane, hyperbola, parabola, functions, origin, moderation, diagonal, the golden ratio, the silver ratio, endogenous, equilibrium, qualitative, quantitative, parameter, variable, system, measure, the market principles, partial, whole, optimum, maximum, minimum, essence, empirical, by country, Fermat

1. Introduction

What are application-differences between the parabola in the economic literature and the hyperbola in the *EES* ("*Earth Endogenous System*," 2013)? The substance of both curves is the same in a cone, where each cutting-angle differs only. The parabola algebraically presents a solution for the optimum-value, by having a unique maximum or minimum equation. Contrarily, the author's hyperbola needs to measure a unique optimum-value, by using hyperbola functions and taking advantage of horizontal and vertical asymptotes. Optimum function-measure in hyperbola is a goal in this chapter. The level of curvature expresses qualitative difference, instead of hyperbola graph and diagram.

What are differences between scientific and non-scientific broadly? In physics and element chemistry, several dimensions are taken into consideration, commonly to nano (closest zero) and universe. In social sciences and economic science, however, researchers cannot flow over the three-dimensions existing in the actual world. This chapter strictly remains scientific in economics and econometrics. Question: Is it fruitful to stick to two-dimensions in a plane? Yes, no doubt, as proved in this chapter. Moreover, this chapter originally measures optimum points of hyperbola functions.

Then, what is a connector spreading between and among economic methodologies and their philosophies behind? Human history tells us numerous ideas, thoughts, and philosophies, as mostly accepted by the golden ratio in Greece (i.e., 3: 4: 5, or 1: 1.618). Particularly, the dialectic invented by G. W. Friedrich Hegel shows us the existence of two adverse extreme phenomena. Old China has preserved the Yin and Yang principle for several millenniums. Old Japan has inherited Kaminori or Shinto (i.e., circle or cyclical world, closer to Nature and managed by a few original Gods), where the silver ratio (1: 1.414=$\sqrt{2}$) has originally prevailed up to date. In the case of the *EES*, the hyperbola in the two-dimensions completely unites philosophy, methodology, and all policy variables; commonly under the silver ratio.

Languages are diversified by race in the actual world. Language expressions of one material uniquely express one substance commonly in the world. Thus, words and languages obey the essence. Social/economic science looks for the essence of economic rules and hypotheses using statistics data and in the market principles. If economic science looks for the essence of economic rules using endogenous data, it is much easier to obtain the essence of economic rules. However, statistics data are always within a certain range of endogenous data, as proved in the *EES*. Essences of economic rules are found much easier in endogenous data than actual statistics data, where the market principles and the endogenous-equilibrium cooperate wholly and consistently.

Finally, this chapter explains the essence of the origin existing in various dimensions of natural and social sciences, here broadly and apart from our strict scientific approach. The origin is an ultimate small immeasurable point that expresses an extreme value. For example, see Appendix 1 related to a base for broader scientific dimensions. The origin of a plane is shown at the cross point of the x axis and the y axis or at the zero corner of the diagonal. The origin of the three dimensions is shown similarly to a plane, by adding the z axis to x and y axis. The origin of one dimension never appears in two eyes of human body. Lastly, this chapter enters into the five and six dimensions by introducing Iyono's SUIT (the Supreme Universe Integration Theory, denominated by the author[2]) and, proves that the origin exists anywhere, one to six dimensions, and any time, over years. This chapter proves that the origin of the two-dimensions is the same as the origin of the five and six dimensions.

[2] Iyonoishi is her Pen-name. Her name is Ishida Shizuko. Most broadly and historically over all science fields, she writes and records her ideas, intuitions, diagrams, equations and proofs most close to our daily life, solely using Japanese. At first she declined to accept the author's proposal to translate the essence of her papers and books into English. Finally, she accepted my proposal since her discoveries are useful and able to serve researchers in the actual world. This paper touches the essence of her discoveries only. In the near future, the author of this book intends to summarize her up-dated discoveries, super-universally comparing with the author's in the two dimensional plane, together with others such as aromatherapy and preventive medicine, natural agriculture and botany.

2. Review of the Literature with the First Appearances

What is the difference between optimum (Opt) and maximum or minimum (Max or Min), using the same data? Answer is no difference: Readers can get both using the same data. But, each origin mathematically differs. Opt comes from geometrical; when a function is specific, the results are the same by using different ways. Max or Min comes from algebraically; formulated by quadratic expression and its root. Hyperbola has two dimensions, the 1^{st} and 3^{rd} or, the 2^{nd} and 4^{th}, while parabola one dimension or, the 1^{st} quadrant. Readers understand a fact that discrete and continuous cases each use partial differential to get the same result of MAX (plus) or MIN (minus), by using the same data.

Further, Kamiryo (2013; 2014) correctly finds a distinct between i) exogenous, ii) 'partially endogenous' wrongly stated in the economic literature, and iii) purely endogenous in the *EES* and the *HEU*. For example, suppose the use of the Cobb-Douglas production function. Solow, R. M. (1956; 1957) formulated the rate of technological progress and solved its result; externally or by counting backward to the residual in the Cobb-Douglas production function. The equation related to an external rate of technological progress was formulated, typically, by Barro, Robert, J., and Sala-i-Martin, Xavier (1995, pp.36-39, pp.80-92), starting with Sala-i-Martin, X. (1990a; 1990b). This equation is the same as an equation that uses purely endogenous rate of technological progress, as proved theoretically and empirically in the *EES*. Note, the elasticity of substitutions is always 1.0000 in the *EES*, which corresponds with symmetry in the two-dimensional plane.

Nevertheless, this chapter advocates here that Opt is solved by using hyperbola functions and, Max or Min by using parabola functions. Further, this chapter finds that Opt is solved with eight linear functions and twenty-five hyperbola functions, as Kamiryo (2013h) formulates separately in a sister paper designed for qualitative levels of democracy. Opt is easily measured by eight linear functions but, these linear is always reduced from hyperbola functions.

Historically in economics, the author finds that the first appearance for geometrical analysis was Joan Robinson's (1933) 'imperfect competition' and, that for algebraic analysis was J. R. Hicks' (1932, 1935) 'wage rate,' under the general equilibrium that requires indispensable assumptions. Chapter 2 of 'The Geometry' in Robinson (1933, pp.26-43) presents 16 figures derived from units of output, average cost, total cost, and marginal cost given in advance, and by using related elasticity equations under the market principles.

Graphical curves by figure show concave and convex and are enough general but, hyperbolic curves never appear, which the author confirms. These figures naturally result in asymmetric, since the elasticity of substitutions, σ, is not 1.0, differently from the *EES*. Kamiryo (2013, 1st edition) consistently finds that perfect competition is measured by using endogenous equations with no assumption, where $\sigma = 1.0000$ and symmetric between two quadrants, the 1st and 4th or, the 2nd and 3rd. Earlier unique research related to competition was in Sraffa (1926, pp.535-550).

More recently, McKenzie, Lionel, W. (1956, pp.165-180), Afriat, S. (1972, pp.568-598), Carroll, C. (1997, pp.1-57), Harris, Christopher and David, Laibson (2001, pp.935-957), Laibson, D. (1997, 443-479), Polster, Burkard (2004), Richmond, J. (1974, pp.515-521), Thaler, R. and, H. Shefrin, H. (1981, pp.392-406), Wim, Meeusen and, Julien, van den Broeck (1977, 435-444), are well interrelated.

Particularly, Harris, Christopher and David, Laibson (2001), indicates a natural direction for a stream from static to dynamic under the market principles. Also, McKenzie (1956) clarifies a limitation of specialization and an efficiency in production. When $\sigma = 1.0000$ holds, efficiency= effectiveness holds without specification. For competitiveness, the author never insists on the use of the Cobb-Douglas production function. Yet, Wim Meeusen *et al*. (1977) must be respected as a base.

Lastly back to Hicks, J. R. (1932 & 1935; pp.232-246, Appendix), his Appendix is related to elasticity equations and composed of the following four items: i). The Co-ordination of the Laws of Distribution, 233; ii). Increasing returns, 239; iii). The Elasticity of Derived Demand, 241; and iv). The Distribution of the National Dividend, 246. Values of elasticity of substitutions are complicated due to asymmetric character under the market principles. E. F. Ramsey' mathematical theory of saving is immovable as a base even today. The author confirms that Ramsey, E. F. (1928), Hicks, J. R. (1932 & 1935), and Robinson, J. (1933) completed each analysis and seldom hitherto, under the same market principles. This chapter does not step into these reviews any more. These essentials are separately stated and discussed, by focusing on a few aspects.

3. Circle and the Silver Ratio

Historically this section originally assembles and finds essentials of the golden ratio and the silver ratio. The results are ultimately culture-oriented by country and, civilization-oriented by wider geographical area. There is no economic study of hyperbola functions, in the author's origin investigations repeatedly hitherto. The versions were wholly clarified by the *EES* so that the author sums up important contents in this section by using BOX 1 and BOX 2, and by supplementing new fact-findings after publication of the *EES*.

Appendix of the *EES* (476-523, 2013; 480-527, 2014, seldom change and everlasting; see two Boxes soon below) wholly clarifies the character of hyperbola. This section inspects the Appendix and sums up the character of the hyperbola, adds new arrangements and, presents a base for the circle and the silver ratio. This section explains preliminary steps for the following sections that concentrate on essentials of the golden ratio and the silver ratio. Historically, the author himself presents the first appearance of geometrical new essence of the silver ratio.

BOX 1 Character of hyperbola: elements, attributes, and measure of the curvature

Four elements, a, b, c, and d for hyperbola standard type, the case of no zero:

Additionally, $e = \frac{c}{d}$. $f = d - \frac{b \cdot c}{a}$. $\frac{f}{a} = \frac{1-b \cdot c}{a^2}$.

$y = h(x)$. by setting $h(x) = \frac{cx+d}{ax+b}$. $y = \frac{c}{a} + \frac{d-\frac{b \cdot c}{a}}{ax+b} = \frac{c}{a} + \frac{f}{ax+b}$.

Five attributes: $VA = \frac{-b}{a}$. $HA = \frac{c}{a}$. $Width = \sqrt{\left\lfloor \frac{f}{a} \right\rfloor}$. $Sharpe = \sqrt{2\left\lfloor \frac{f}{a} \right\rfloor}$. $Curvature = \frac{1}{Sharpe}$. $Curvature = 1/\sqrt{2\left\lfloor \frac{f}{a} \right\rfloor}$. $Curvature = 1/\sqrt{2\left\lfloor \frac{1-b \cdot c}{a^2} \right\rfloor}$.

Measure of the curvature:

$x \cdot y = 1.0000$ or $y = 1.0000/x$. $1 = cx + d$. If c=0, d=1.0. If a=1.0 and b=0, $ax + b = x$. As a result, $y = 1.0000/x$ holds under a=1.0, b=0, c=0, and d=1.0.

$Curvature = 1/\sqrt{2\left\lfloor \frac{1-b \cdot c}{a^2} \right\rfloor} = 1/\sqrt{2\left\lfloor \frac{1-1}{1} \right\rfloor} = 1/\sqrt{2\left\lfloor \frac{0}{1} \right\rfloor}$. It implies Curvature=0 or Curvature overlaps the origin of the plane.

The author shows and interprets a new fact-finding hidden in the curvature (hereunder no large letter, replacing C by c): If elements c and d

are zero, the curvature disappears, where $x \cdot y = 1.0000$ or $y = 1.0000/x$. Can one find the origin and the curvature in $y = 1.0000/x$? This question makes us stimulate a whole answer to the origin and the curvature within the range of scientific approach.

3.1 Character of the hyperbola

Before getting into detail, this sub-section summarizes two types of character in the hyperbola by using two Boxes; **Box 1** shows the general or standard type and **Box 2** the specified type.

BOX 2 Specific types of each element=0 and also two elements=0

i) a=0, the linear type: $y = \frac{cx+d}{b}$ or $y = \frac{c}{b}x + \frac{d}{b}$. **To** 2-3 $r^*(n)$.
2-4 $n(r^*)$.

ii) b=0: $y = \frac{cx+d}{ax}$ and $y = \frac{c}{a} + \frac{d}{ax}$. **To** 2-1 $r^*(i)$. 3-4 $n(\Omega^*)$. 6-1 $\beta^*(i)$.
6-2 $\widetilde{\beta}^*(i)$.

iii) c=0 and d=1: $y = \frac{1}{ax+b}$. **To** speed(i). 1.3 speed(n).

iv) d=0: $y = \frac{cx}{ax+b}$ and $y = \frac{c}{a} + \frac{-\frac{b\cdot c}{a}}{ax+b}$. **To** 3-1 $\Omega^*(i)$. 3-2 $i(\Omega^*)$.
4-1 i(n). 4-2 n(i). 4-3 $\Omega^*(\beta^*)$. 4-4 $\beta^*(\Omega^*)$. 6-4 $\widetilde{\alpha}(i)$.

v) c=0: $y = \frac{d}{ax+b}$. **To** 2-2 $i(r^*)$. 3-3 $\Omega^*(n)$.

vi) No zero, the standard type: $y = \frac{cx+d}{ax+b}$ and $y = \frac{c}{a} + \frac{d-\frac{b\cdot c}{a}}{ax+b}$.
To 5-1 $\beta^*(n)$. 5-2 $n(\beta^*)$. 5-3 $\widetilde{\beta}^*(n)$. 5-4 $n(\widetilde{\beta}^*)$. 6-3 $\alpha(i)$.
6-3-2$\alpha(n)$.

vii) $y = \frac{cx+1}{ax}$, $b = 0$ and $d = 1$ (correctly adding):
To 1.2 i(speed). 1.4 n(speed).

It is impossible for us to draw the two types using the *x* and *y* axes in the same two-dimensional plane. Why can no one draw standard and specified types of character simultaneously? This is because the curvature is only calculated by a combination of its determinants. A hyperbola function with four elements, a, b, c, and d, solely produces the general type. No one can calculate two different curvatures by using one combination of four elements.

When one or two elements among four are zero, the corresponding hyperbola function produces the specified type, as shown in Box 2. There are '25' specified types of character, where the author finds 'two' additional specified types as shown in the bottom of Box 2.

The measure of the curvature (see Box 1 above): The curvature's essence remains unchanged: The relation between the Width, the Sharpe, and the Curvature is the same. The Width and the Sharpe serve to the Curvature. The *EES* tried to use the Width and the Sharpe directly. This chapter tries to calculate the Curvature directly, apart from the Width and the Sharpe. This is because the essence of these three is the same.

Conclusively the origin prevails in the two-dimensions, similarly to the five and six dimensions. The essence of the origin is the same. Accordingly the essence of the curvature reflects the same, regardless of the number of dimensions. Therefore empirical results of hyperbola functions in the two dimensions essentially reflect those in the five and six dimensions and, supported by a lucky fact-finding that statistics data are always within a certain range of endogenous data.

Endogenous equations: Researchers have as many as endogenous equations consistently by country, sector, and year and over years, once the seven endogenous parameters are measured. These endogenous equations are geometrically all absorbed into 25 specified functions by type. Accordingly each curvature is measured and drawn in the two-dimensional plane. The general/standard type of character is included in the sixth type in Box 2. Readers will notice that the standard type is limited to most fundamental endogenous parameters, equations, and their reduced forms; the net investment to the national disposable net income (*NDI*=*Y*), $i = I/Y$, the rate of change in population= the growth rate of population, $n_E = n$,the relative share of capital, $\alpha = \Pi/Y$, and the technology coefficient, β^*. These parameters are policy parameters in the *EES*. These parameters manage the endogenous-equilibrium, beyond space and time; static and dynamic, and balanced and unbalanced, discrete and continuous, and moderate and immoderate.

The origin equals an ultimate moderation but immeasurable. A point of close to the origin is countable and measurable and, managed by hyperbola functions and corresponding curvatures. The sharper the curvature the closer to moderation the hyperbola function is; where the Yin and Yang principle prevails. The author emphasizes that the Yin and Yang principle in olden China is now expressed numerically using the hyperbola functions and that

this new finding is proved originally based on old Japanese Kaminori or Shinto.

3.2 Circle and the silver ratio

The circle and the silver ratio have been observed in Japanese culture and civilization over two millenniums. The *EES* proves the circle and the silver ratio by using the two dimensions in the plane. This section empirically and generally shows fundamental relations between the circle and the curvature, by using Fig. 1 for mild and sharp curvatures; Fig. 2 for convex curvature; Fig. 3 for the character of curvature; and Fig. 4 Empirical results of standard-type curvature-measure: generalization.

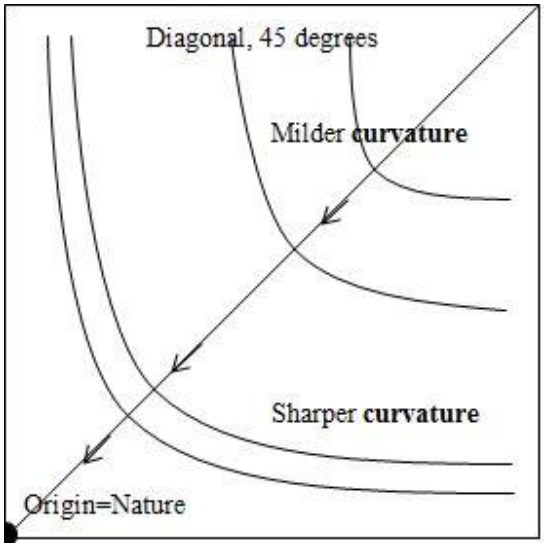


Fig. 1 Mild and sharp curvature using the diagonal and the origin: generalization

Notes: (1) Hyperbolic curvatures and their functions are measured in the *EES*, in the two-dimensional plane. (2) Hyperbolic curvatures are shown in static-state using diagram, as drawn above here. (3) Hyperbolic curvatures are unique such that without using the vertical and horizontal asymptotes, the VA and the HA, hyperbolic curvature is measured and illustrated. (4) Therefore, the curvature constitutes a bridge to connect the *EES* and diagram. (5) This is a reason why the author first raises this Fig. 8, before showing up a variety of geometrical diagrams in the two-dimensional plane.

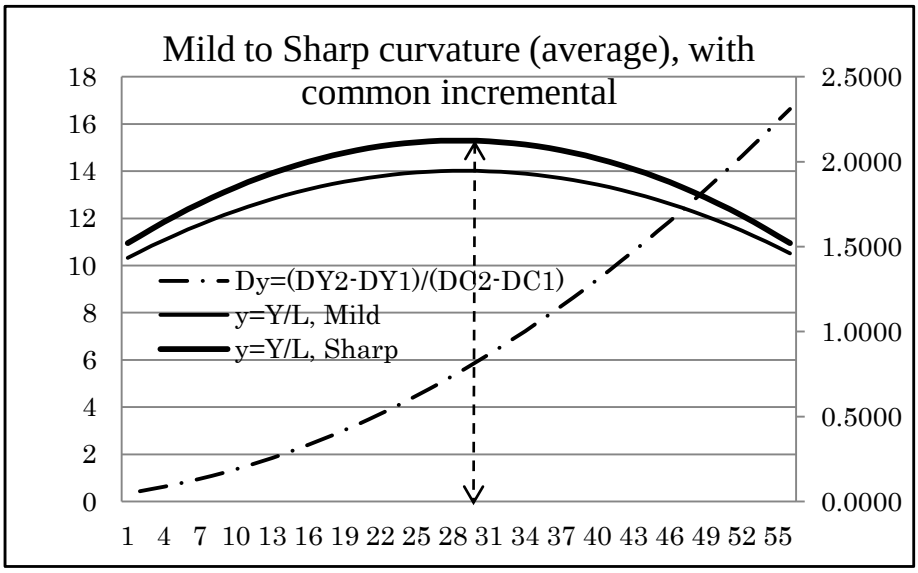


Fig. 2 Convex curvature: generalization

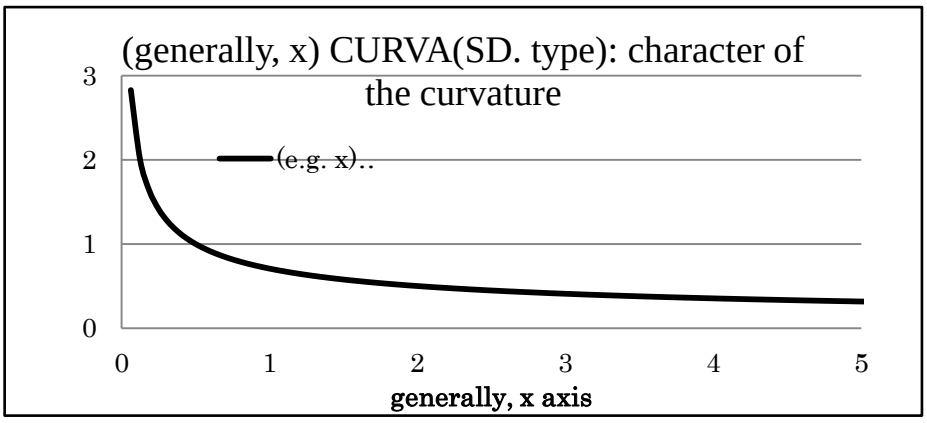


Fig. 3 Standard-type curvature-measure: generalization

For curvature-measure, if an independent variable is fixed, the value of the curvature is fixed generally. Suppose in **Fig.1**; mild and sharp curvatures have each asymptote overlapping the *y* axis. A mild curvature has room for approaching its asymptote more closely than a sharp curvature has. It indicates that optimum range of hyperbola function with a mild curvature is wider than that with a sharp curvature. These curvatures are solely drawn each by convex, as shown in **Fig. 2**. Rotate Fig. 2 by one and half quadrants (i.e., 135 degrees) to the left, with a diagonal supposed. Two curvatures after rotating correspond with those in Fig. 1. Fig. 2 was separately proved in another paper for individual life-time productivity to the next generations (see References at the end).

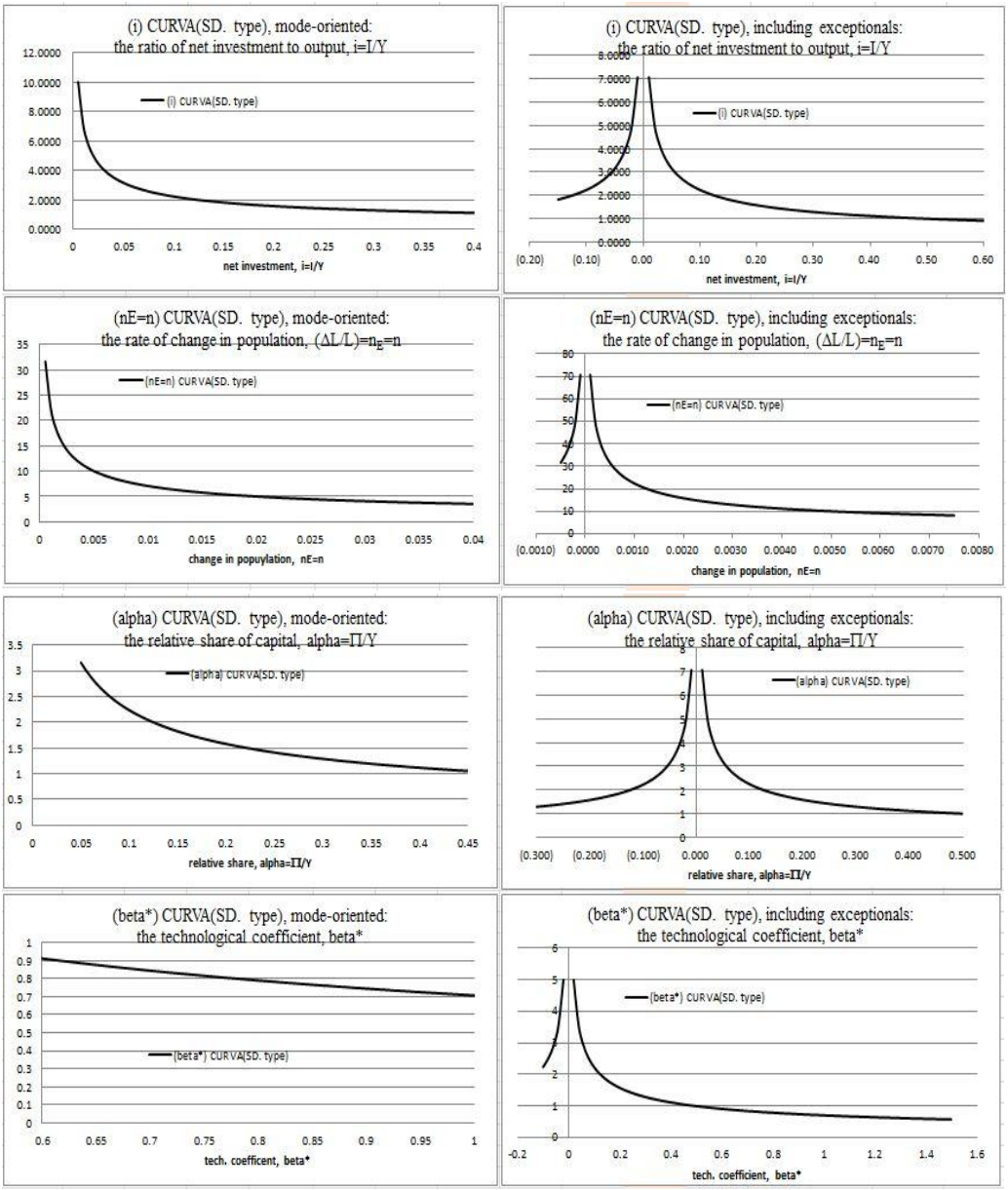


Fig. 4 Empirical results of standard-type curvature-measure: generalization

Fig. 3 generally illustrates the character of curvature, mild and sharp. The more the number of an independent variable, the milder the curvature is and, vice versa. The character of curvatures depends on the number of the independent variables on the *x* axis. It indicates that the curvature changes above zero and less than 1.0 in the 1st quadrant and, the zero point must be immeasurable. This fact was proved by using the 2 Weights, W_1 and W_2, between average and incremental, as discussed in a separate paper (see References at the end).

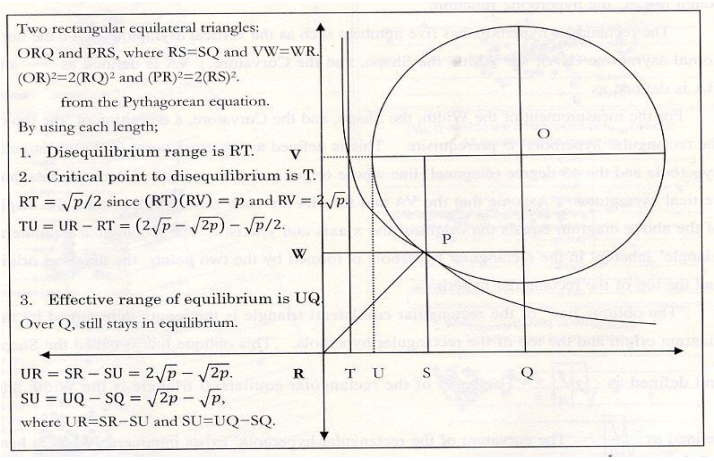
Fig. 4 empirically shows standard-type curvature-measure, where four parameters are used. On the LHS of Fig. 4 each shows one quadrant while on the RHS each shows two quadrants, the 1st and the 3rd. Why does each parameter, $i = I/Y$, $n_E = n$, $\alpha = \Pi/Y$, and the technology coefficient, β^*, has a minus value? The author exceptionally takes in two parameters, $\alpha = \Pi/Y$ and β^*. These imply that there is the origin and Yin and Yang always adjoin each other. A whole picture by human enlarges from small plane to large plane, philosophically and numerically towards the universal.

4. Implications and Proofs: Uniting the Golden Ratio with the Silver Ratio

What is the implication uniting the golden ratio in the ellipse with the silver ratio in the ellipse? It is that western and orient culture and civilization have the same essence in the world, historically and up to date. The author confirms that united harmony leads to sustainable society in this world; far from against to cooperative. Society, people and consumers, are by nature destined to be peaceful, based on individual' consciousness; step by step, family, town, city, country, area, continent, and the world, towards the universal. The essence of uniqueness becomes broader and deeper with human history, as our ancestors have experienced hundred millenniums.

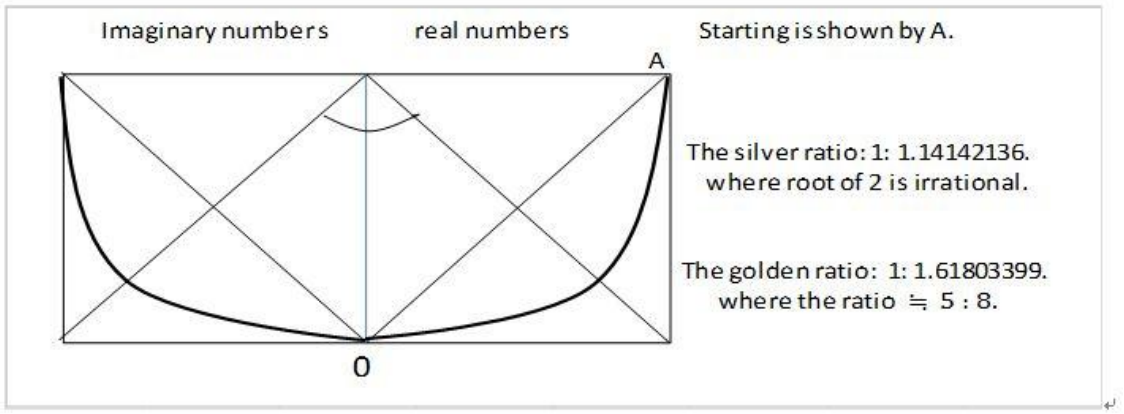
Scientifically, it is a new fact-finding that the golden ratio completely overlaps the silver ratio, each preserving its own originality. The author has confirmed this fact repeatedly at worldwide specified libraries, together with specific librarians. Some part of this fact-finding was partially proved and published already in the *EES* and a few papers for eight new fact-findings soon after the *EES*. Therefore, this chapter proves the whole of the fact-finding conclusively, citing some part of proofs published.

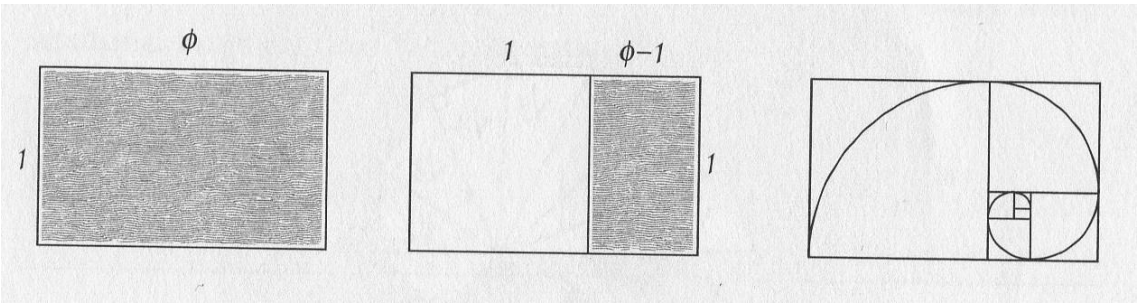
For proofs: The author prepares three figures to highlight uniqueness of our proofs. **Fig. 5** shows the hyperbola in the 1st quadrant and cites our original idea to the circle. The hyperbola in the literature shows two hyperbolas by rotating Fig. 5 by 45 degrees to the right. Why does the literature express two hyperbolas facing each other with no quadrant? The author supposes that two hyperbolas facing each other are original and the one hyperbola closed in the 1st quadrant is our challenge for new ideas. 'Hyperbolic functions' in the literature express six trigonometric functions. The two hyperbolas never schedule the specified type that produces 25 'hyperbola functions.'



Source: Page 470, the *EES*, by Kamiryo (2013a)

Fig. 5 Basic relations between the origin, the diagonal, the circle, and the hyperbola





Sources: Above, cited from Ishida's original (see References) and, bottom cited from Polster, Burkard (2004, pp.41, *Q. E. D: Beauty in Mathematical Proof*).

Note: Surely, a typo in the silver ratio should be: 1:1.4142136.

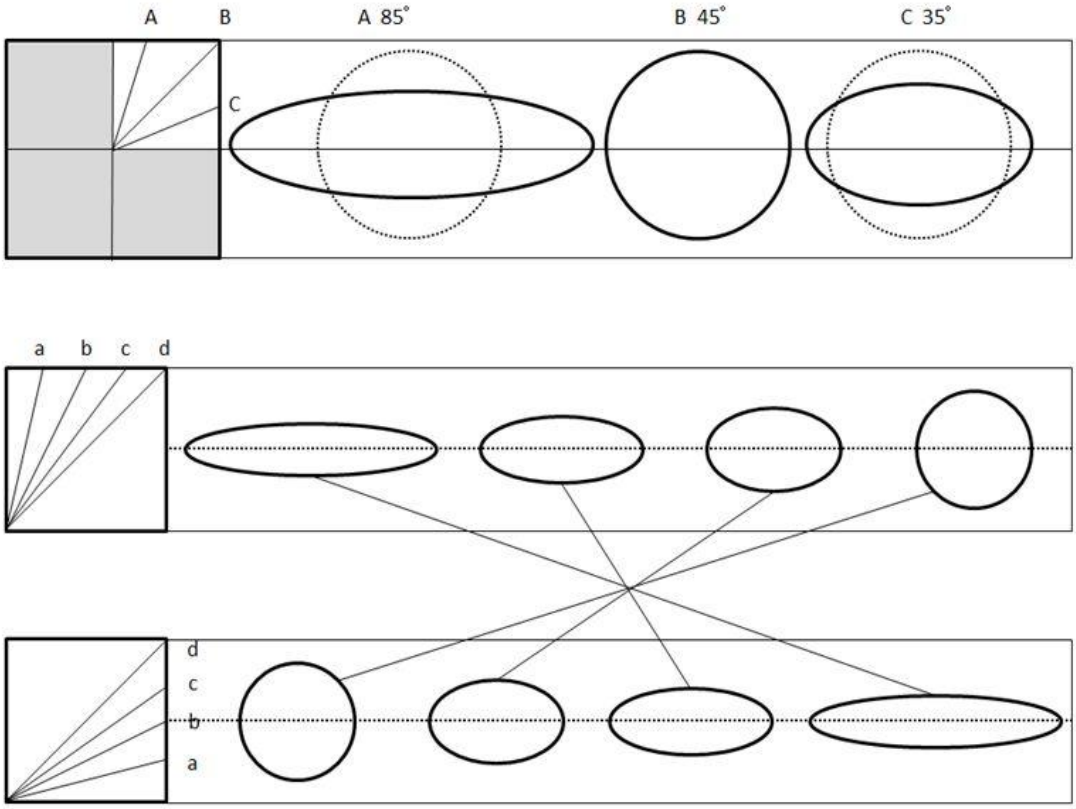
Fig. 6 Definite difference of the curve of the hyperbola between Ishida and the literature in terms of 'close to the origin point'

The upper figure in **Fig. 6** cites Ishida's new discovery, where the origin is shown by 0 (zero) and the starting point is A, indicating each range of real and imaginary numbers and referring the golden ratio and the silver ratio. Fig. 6 broadly proves that Ishida's frame of the origin is similar to that in physics and element chemistry. A definite difference, however, is found in each hyperbolic curve; Ishida's curve is sharp more than the curve common to natural sciences. One reason is traced back to the difference of the total angle in each pair of triangles, 108 degrees versus 105 degrees (confer red frame with white frame in Fig. A1 in Appendices). Ishida newly proves a delicate difference, by repeating hundreds of familiar experiments.

The author sensitively pays attention to the combination of the golden and silver ratios in Fig. 6. The author confirms that no one proves why the two ratios overlap each other. Combinations of the golden and silver ratios are connected with the ellipse and the circle, as illustrated by Fig. 7.

The Top of **Fig. 7** finds that the diagonal presents a base for the ellipse and the circle. Watch the point B or 45 degrees of the diagonal.

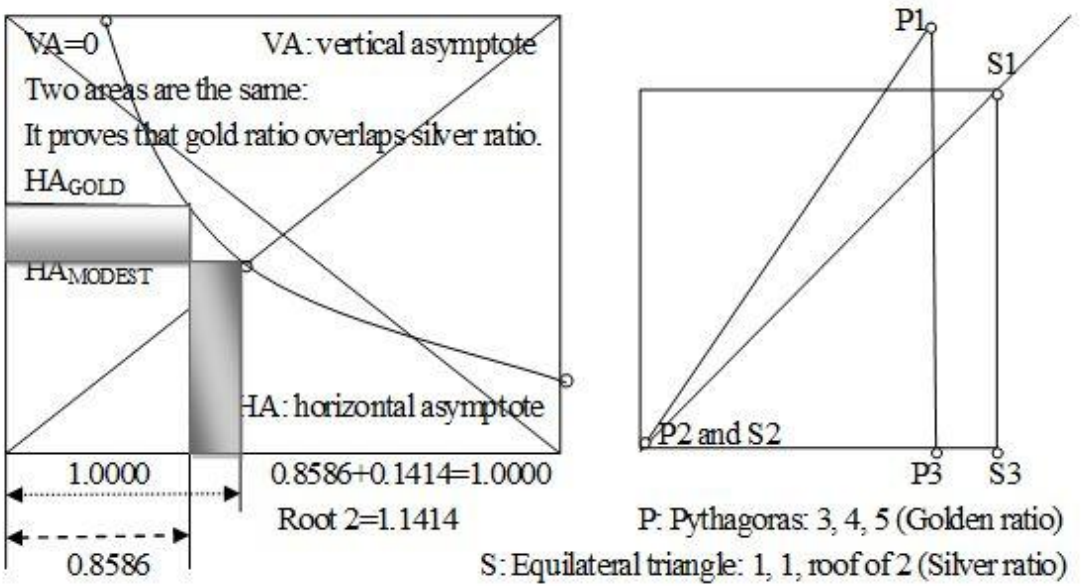
In the Middle and Bottom of **Fig. 7**, four different levels of overlapping are each shown using the center line. The author finds that the order along with the center line is reverse; the right faces the left and, left faces the right. The author states; the *y* axis of the middle and the *x* axis of the bottom each are immeasurable, which corresponds with the origin immeasurable.



Source: Original illustration in this chapter. The author repeatedly confirms this fact as the first appearance worldwide; similar to Fig. 8.

Fig. 7 Relations between the ellipse and the circle, starting with the diagonal

Fig. 8 on the next page shows a definite fact that a complete overlapping of the golden ratio and the silver ratio holds, by calculating each total area and also, each incremental area, by ratio. The incremental area by ratio is shown by gray color. The author understands that the Western culture and civilization represented by the golden ratio and the Japan culture and civilization have a common root or the essence of consciousness. Fig. 8 starts with an individual who is distinguished by human consciousness or perception and, with each family as a base for society. Ideally, the real assets determine consumption and wealth, with the author's politics-neutral and spirituality- neutral.



Source: Cited from a full-manuscript (2013) presented to *Royal Economic Society* Conference, Manchester, 7-9 April, 2014.
Note: square root of 2 is about 1.4142, instead of 1.1414; How comes the 0.8586 there, for confirmation.

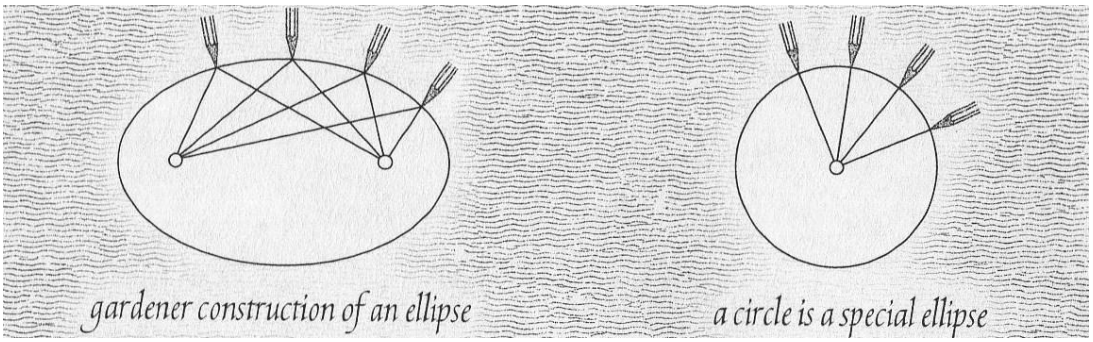
Fig. 8 Overlap-proof of the golden ratio to the silver ratio, in the same two-dimensional plane

5. Diagrams from Static-situation to Dynamics, Under the Same Balanced-state

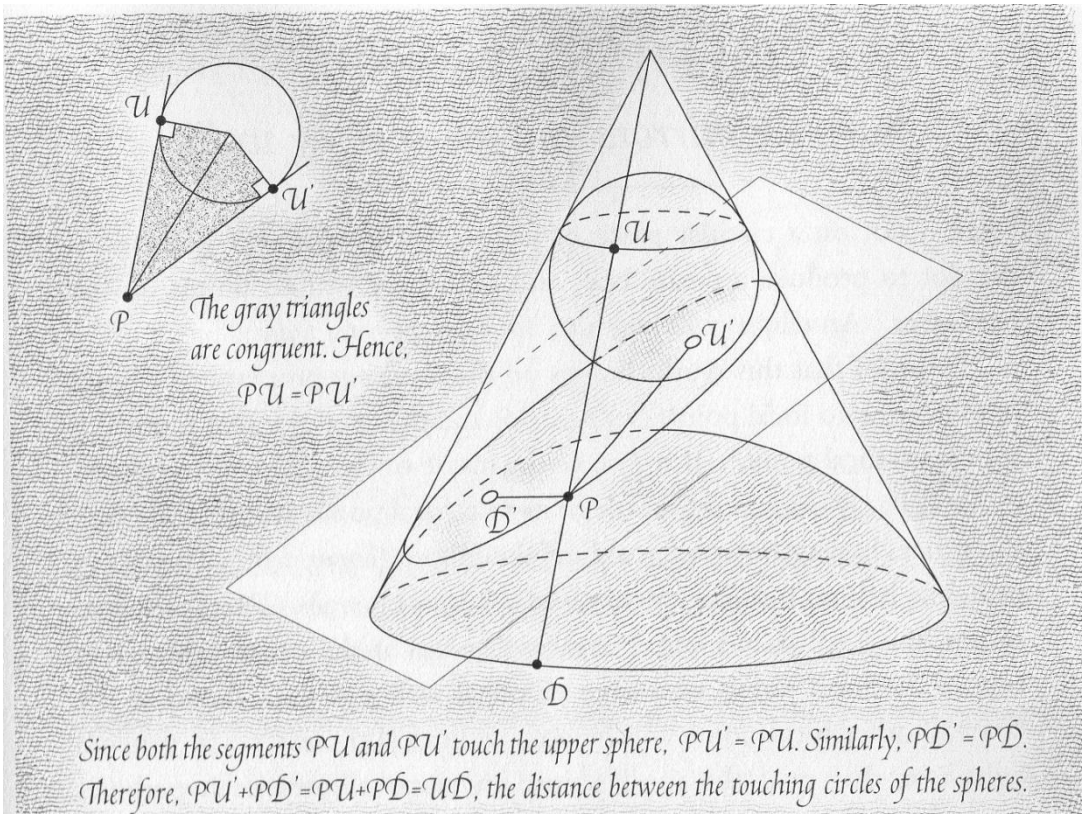
This section summarizes relationship between static-situation and non-static situation; or, dynamics taking into consideration movements over time. First, the author shows original diagrams in the literature and second, the author's own original diagrams, in the same two-dimensional plane.

First, original diagrams are the following **Fig. 9** and **Fig. 10**.[3]

[3] The author had historically investigated original diagrams in several libraries at dozen university libraries and national libraries, in Europe and the US. The last trip to historical investigations was held in Oct 2013, as concretely stated in 'Acknowledgements' to the author's manuscript presented to *Royal Economic Society* in Nov 2013. The author wanted to discuss reproducible evidences in this manuscript at *Royal Economic Society* Conference, Manchester, on 7, 8, and 9th April 2014. However, perhaps due to



Source: *Q. E. D.* by Polster, B. (2004), N.Y.: Walker & Co., page 26.



Source: *Q. E. D.* by Polster, B. (2004), N.Y.: Walker & Co., page 27.

Notes: (1) Each essence of circle and ellipse is well illustrated in the above Top and Bottom diagrams. (2) With understanding of Fig. 9, the author's originals were drawn, as diagramed in Fig. 10 and Fig. 11.

Fig. 9 Development of the relationship between circle and ellipse

severe competition and also purely endogenous data or, risk-aversion common to academics, this manuscript was not selected but rejected in Dec 2013. The author could perceive the current stream of the literature more concretely in Conference, Manchester.

Chapter 9, *HEU*

Martin, Gardner's (1957, revised) construction of ellipse will last forever. The author respects their life-time contributions by Gardner and Polster on geometrical diagrams. But, why didn't Gardner and Polster show interest in cultural differences between a circle and an ellipse? Or, why were they not possessed with the essence of a circle itself? Does the golden ratio no doubt exist in the real world since old Greece? Geometrically, what is the meaning of the origin in the two-dimensional plane? Is this origin measurable, estimated, or forecasted? Is mathematics technical-oriented? Is mathematics out of philosophy? Who proves mathematics? Are individuals each human? What is the relationship between body and mind in human life? Please take it easy for a moment.

The *EES* is free from politics and spirituality, as proved by politics-neutral and spirituality-neutral. The algebra *EES* and its KEWT database series correspond with the geometry *HEU*. A whole system easily unites algebra with geometry under the market principles and perfect competition. Repeating; why? Reinforced by the character of money supply such that money=quantity=quality=1.0000000, the *EES* holds so easily, differently from mathematics and natural sciences. In endogenous equations and corresponding diagrams, each measures or illustrates the level of philosophy, theories, practices, and history.

Therefore, evidences are spread from Figures 9 and 10 to Figures 11 and 12. Figures have 'Note' soon below: Please, first digest each 'Note.' Fig. 9 is historically originated while Fig. 10 is the author's bliss point, diagramed by dialogue, hyperbola, and circle as a base.

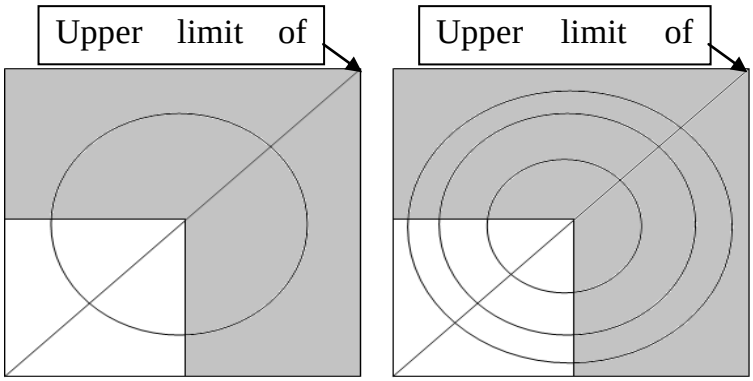
Let the author here clarify Fig. 11 and Fig. 12, simply and shortly and consistently with diagrams.

A circle has its own diagram independently from an ellipse. Japan has preserved its culture with agri-culture (i.e., agriculture) in small inlands, blessed with natural beauty and mild four seasons fitted for rice cropping. People in the world today deeply perceive, accept, and evaluate this fact. Conclusively, it implies the silver ratio civilization, beyond space and time.

An ellipse is able to unite it with a circle by using diagonal. It implies that the Western civilization and agriculture are harmoniously united. No one denies this fact in this diagram. Delicate balances between narrow balances and unbalances are always preserved in the static-situation. One confirms this fact using statistics actual data and also endogenous real data for 86 countries, 1960/90 to 2013, in the case of KEWT 9.15. This is because

'the *true value*' of Fisher (1906, pp. 84) corresponds with each of endogenous data, where simultaneously, results equal causes and causes equal result. Note, in the statistics data causes are not shown but results only vertically under the market principles. Endogenous data exist under perfect competition and with no assumption nor probability.

It is true that the author's 'dynamics' manages the above static-situation with dynamics itself. All of these facts match the Yin and Yang principle in olden China and for several millenniums but, it has been impossible for anyone to measure this principle purely endogenously.



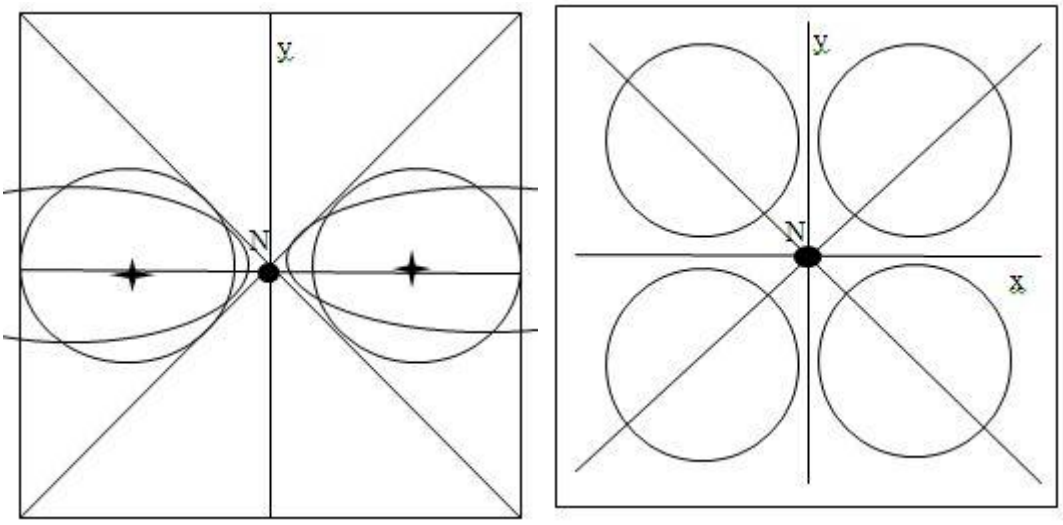
Note: The *y* axis shows individual or macro utility, while the *x* axis individual value or macro output. For simplicity, this figure does not show the hyperbola curve. The hyperbola curve each crosses the point of intersection of the circle on the diagonal.

Fig. 10 Corrected bliss point of diagonal and circle

Next, the author's own originals in diagrams are the following Fig. 11 and Fig. 12, in the same two-dimensional plane. In the case of three-dimensions, the circle turns a sphere, globe, or ball, perfectly round. But, this chapter purely stays at the circle.

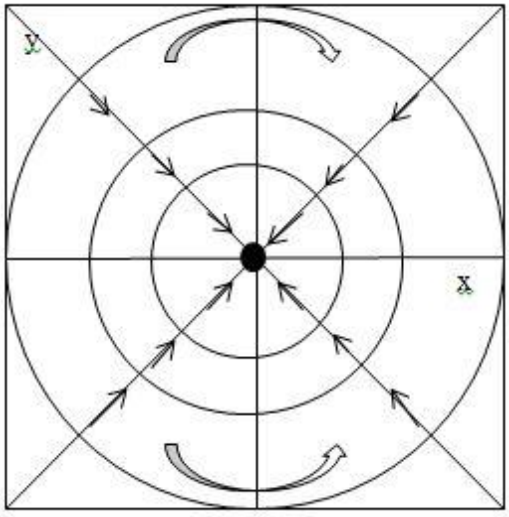
Fig. 11 shows a base for Combinations of circles-ellipses, uniting Western and Japan civilization, which is clarified in Appendix 2 at the end of this chapter. Let the author explain a core why geometry and macroeconomics are so tightly connect with each. Apparently, culture by country and civilization by area in the world are not within the range of macroeconomics. Truth is reverse. National taste (preferences, culture, and history) is independent of technological progress measured purely endogenously. Misleading cause is traced back from individual utility that cannot be estimated in early 1940s, as clarified by Samuelson (see Kamiryo, 2013, AEJ-D-13-00049R1 for 'consumption-neutral'; revised on19Jan.2014).

Once macro-utility is consistently and accurately measured by year and over years, with no correction later, national taste is measured, where culture by country is expressed by the relative discount rate of consumers' goods to producer s' goods. Simultaneously, the rate of technological progress as flow and the growth rate of total factor productivity (*TFP*) as stock are endogenously measured. Therefore, Fig. 11 is meaningful. This chapter illustrates Combinations of circles-ellipses, uniting Western and Japan civilization in Fig. A2, in Appendix 2. **Fig. A2** shows four principal combinations. Yet, each combination changes according to national taste.



Notes: (1) The diagram of the RHS solely shows circles and hyperbolas, for diagonals, while the diagram of the LHS shows circles, ellipse, and hyperbolas, for the same diagonals. (2) In both the LHS and RHS, the origin is Nature (N) itself and extraterritoriality. (3) In the RHS, the distance between the center of circle and that of ellipse differs when the ellipse's shape is flattened. (4) This distance implies that of the Western and Oriental culture by country. (5) A circle is peculiar solely in Japan; the silver ratio has, originally and historically, been brought up in Japan. It suggests a modest way to world unity.

Fig. 11 Static-state diagram to unite balanced and unbalanced in the two-dimensional planes



Notes: (1) This diagram shows close to Nature movements in the levels of moderation, where ultimate-point is the origin. The origin is Nature= immeasurable moderation and, human science, plus and minus, directs for the origin.
(2) In reality, arrows show both close to Nature movements and against to Nature.
(3) This diagram shows a right-handed (CW) rotation and non-clockwise (NCW). CW and NCW imply the same rotation.

Fig. 12 Dynamic-state diagram uniting static- and dynamic-state

6. Conclusions

Does economic science progress along with the transition of time? The author is confident in a fact that if and when economic science is closer to Nature or not. Economic science has apparently progressed over years historically but, to our understanding, sometimes far from Nature and, other times closer to Nature. Here Nature indicates the universal rules prevailing in this world, like mathematics, physics and element chemistry that never suffer from human decision-making but follow the wise providence of Nature or Heaven. It means that the time and years do not influence the progress of economic science. Why are the time and years indifferent of economics progress?

For example, look at Index to the *Review of Economic Studies*, (II), 1934-1935, where the author is surprised to find two dozens of Notes, Comments, and Reply, each the highest level. Already more than eighty years ago, the author understands, all of these researches had basically reached a final step of theories, apart from empirical measurements. The author finds, each fundamental base is alive vividly even at this moment, carefully with strict assumptions. These are historical facts. By what reasons do readers justify the above Index as everlasting base of economic science and economics?

This is because their fact-findings are the same as those discovered in both the *EES* and new fact-findings after the *EES*, where six Nature-Neutrals are proved using the KEWT database, 1960-90 to 2011, by country, area, and year and over years. 'Purely endogenous' means that thousands of endogenous equations are simultaneously formulated with no assumption and no corrections later and, that even perfect competition is numerically measured under the same market principles. It implies that purely endogenous is closest to Nature or natural sciences, where human behavior is set aside. 'The real' remains unchanged, as inspected and proved in a sister paper. In section 2, of course, some fact-findings in the literature were cited and interpreted over years.

Back to hyperbola functions: Endogenous equations are determined only by the seven endogenous parameters, which simultaneously determine all the other parameters and all the variables. Hyperbola functions are each reduced form of endogenous equations. Fundamental equations among hundreds of hyperbola functions are divided into hyperbola functions and linear functions. Hyperbola function has its origin but immensurable yet, the origin expresses an ultimate small point of moderation and is connected with optimum range or moderate levels for optimum. Endogenous optimum essentially differs from the optimum for solving maximum or minimum in the literature, based on statistics and mathematics analyses. Further, a parabola needs no origin but a hyperbola needs the two-dimensional plane to have sharp curvature moderate. A hyperbola will give a key for stable society by individual, family, town, city, country, area, continent, and the world, towards the universal.

This chapter sums up the whole methodology of numerable measure with new fact-findings, citing the first appearances in the *EES* and the KEWT database and, stepping into geometrical expressions in the two-dimensional plane. Appendix touches broader scientific approach acceptable in the near future. This chapter is one of two sister chapters; the other sister chapter deepens qualitative levels of hyperbola functions, applying the curvature to qualitative levels of democracy.

At the end, the *EES* is solely policy-oriented, just before redistribution of all taxes. Strategy-oriented, actually reinforces policy-oriented, vertically under the market principles, from the viewpoint of households and enterprises, just after redistribution of all taxes. Just before all taxes and just after cooperate naturally.

Hoping a record to future generations here specified acknowledgements is stated to the editors of Annals of Mathematics. They advised and encouraged the author from time to time. To be plainly, Annals of Mathematics is purely mathematics while this chapter is macroeconomic-oriented. Thankfully listening to their advice, the author could select a journal, pertinent to such contents as following the sequence of events. The author understands their courtesy and rewrites some parts of this chapter more stepping into macroeconomics. Macroeconomics has surprisingly developed over years and with the use of statistics data. Statistics data exist essentially within a certain range of endogenous data, as the author proved endogenously using the KEWT database. It implies that either side pursues the same essence of macroeconomics, with no contradiction and, unitedly and cooperatively. The author advocates here that we researchers develop macroeconomics confidently towards a united macroeconomics by following the same use of the current statistics data. Results are in reality; no inflation/deflation with full employment, without asset-bubbles anymore in the macro level and, surely increasing a real wage rate after inflation in the micro level.

Appendices 1, 2 and **Compilations 1, 2,** and **3**

Appendix 1 is a direct citation of Shizuko, Ishida's original discoveries.
Appendix 2 is Combinations of circles-ellipses, as the unity of Western and Japan civilization.

Compilation 1: The market principles and, two ways for interpreting the current situation.
Compilation 2: Biological references historically related to Fermat' Last Theorem.
Compilation 3: The Golden ratio in Greece versus the Silver ratio in Japan

Appendix 1 A Citation of Shizuko, Ishida's original discoveries

Appendix 1 broadly shows basic new discoveries by Shizuko, Ishida (pen name, Iyonoishi), citing original records published: Tables **A1** and **A2** were presented to the CMI (Clay Mathematics Institute, Cambridge) before knowing that the author must present a paper not directly but using one of worldwide math journals, together with Fig. A1. **Fig. A1** compares Ishida's originals covering physics and element chemistry with those discoveries in the literature.

Table A1 Related to the hole of Riemann hypothesis

Data source: Table 1 Material-passage way shifting back and forth between imaginary and real number areas. This table shows proof-equations of everything molded, Also, see Table 2 (iv, 17 July 2012) below.

Table A2 Related to Neutrino's imaginary and real numbers

ν (neutrino). e (electron).

(C) electron neutrino real number rotary.

$(\nu + e\boldsymbol{i})(\nu - e\boldsymbol{i}) = 0$ (1)

$\nu^2 - e^2\boldsymbol{i}^2 = 0 \quad (\boldsymbol{i}^2 = -1)$ (2)

$\nu^2 + e^2 = 0$ (3)

(D) electron neutrino imaginary real number rotary.

$(e + \nu\boldsymbol{i})(e - \nu\boldsymbol{i}) = 0$ (4)

$\nu^2 - e^2\boldsymbol{i}^2 = 0 \quad (\boldsymbol{i}^2 = -1)$ (5)

$\nu^2 + e^2 = 0$ (6)

Substitute (3) and (6) for $X^2 + Y^2 = Z^2$,

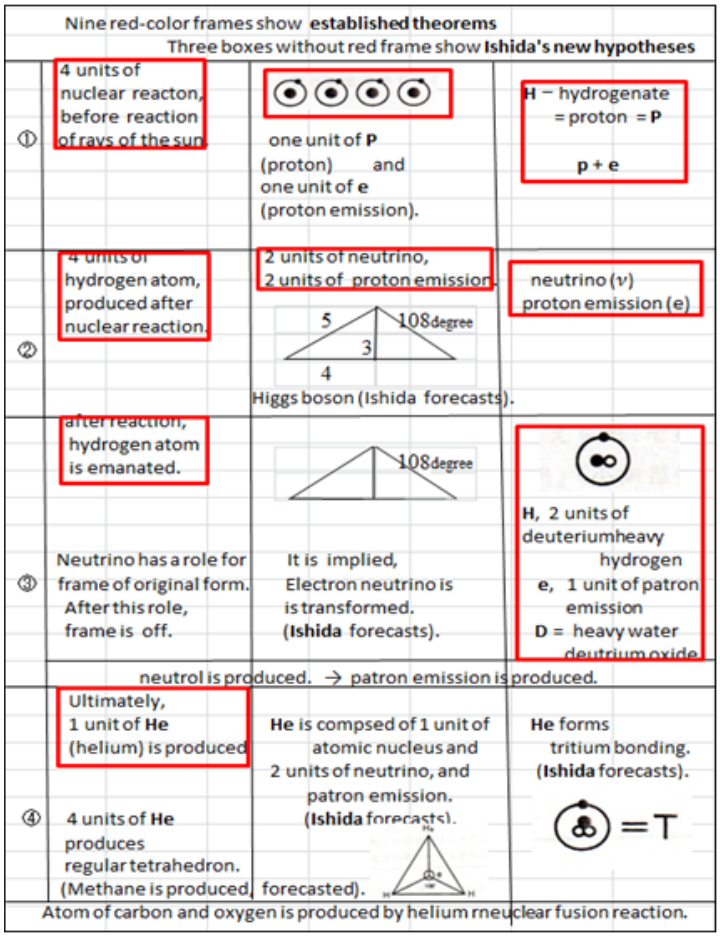
C. $\nu^2 + e^2 = Z^2$. Real spatial geometry of electron neutrino appears.

D. $e^2 + \nu^2 = Z^2$. Imaginary real geometry that appears in real spatial

Source: Table 2 Neutrino's imaginary and real number complex conjugate equations. Cited from Shizuko Ishida (iv, 17 July 2012) in Japanese. C. and D. are expressed by $3^2+4^2=5^2$. Suppose v=4, e=3. Right and left symmetry equilateral triangles appear with 108 degree vertex.

Inspective illustration:

Ishida versus established theories for sunbeam hydrogen atomic nuclear fusion



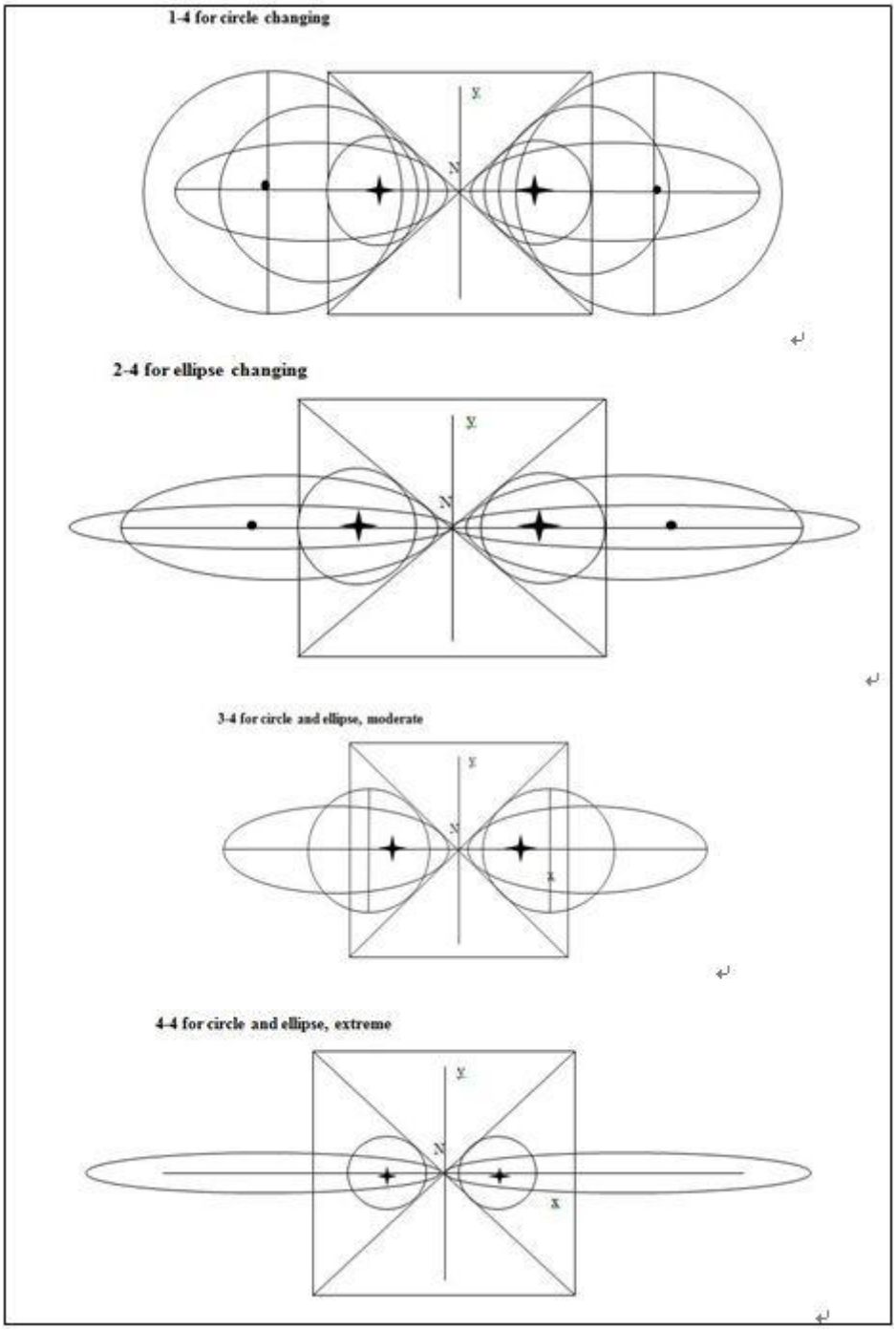
Note: Three frames belong to Ishida's new discoveries regarding to the CMI's seven questions raised to be solved affirmatively.[4]

Fig. A1 Shizuko Ishida's new discoveries (white frames) vs. literature's (red frames) discoveries

[4] **Notes:** (1)The second co-author, Kamiryo, added the original above Figure 17 (Inspective illustration: Ishida versus established theories for sunbeam hydrogen atomic nuclear fusion) on page 19 of Ishida (13 Oct 2013) to her figures, after getting her letter dated on 27Aug 2013. Ishida was eager to add this figure. Figure 17, at a glance, clarifies her new discoveries beyond other scientists. Frameworks marked red are already published in the world, while no red marked boxes belong to her new discoveries.

(2)Latest corrected on 13 May 2015.

Appendix 2 Combinations of circles-ellipses, uniting Western and Japan civilization



Source: Kamiryo (2014), submitted to *Annals of Mathematics*, Princeton.
Note: Each country has its own historical diagram due to national taste and culture.

Fig. A2 Four combinations between Circle and Ellipse

Compilations 1, 2, and 3

Compilation 1 The market principles and two ways for interpreting the current situation

It is true that the market principles behave just like Nature or God in the economic world. The market principles, however, hold vertically by good, service, or software. It implies that the market principles do not show causes but results and that the principles cannot hold as a system in an economy since a system solely holds when vertical threads are waved with weft yarns, wholly as an organ.

There are two ways to interpret the current situation by economy, although both ways are attributed to two-one or the same essence. One way actually follows statistic-data based. The other ways follows the real-assets data based. Statistics-data are results and differently interpreted along with transitions of the current situation. The real-assets data are also results, similarly to statistics-data but, never have unstable axis. The financial-assets in statistics-data reflect the real-assets yet, sometimes the financial assets are wrong-sighted stirred up by the market principles. Any artificial policy cannot lead to wrong-sighted results more than half a year. Concretely, for example, capital is not the capital calculated in corporate accounting but the capital accurately measured in the SNA (1993, 2010).

Why not? This is because the essence of data is ever the real-assets and no other, and because evidences historically prove the neutrality of money. A severe fact for the essence of money measures, purely endogenously, quality=quantity=1.0000000; typically, the elasticity of substitution between capital K and population L, *sigma*, is 1.0000000, and also, the price levels, relative and absolute, $p=P$ is 1.0000000, as always measured in Kamiryo (2013, 2014) by country.

As a result, Marginal productivity of capital equals the rate of return and, marginal productivity of labor/population equals the wage rate under perfect competition with no assumption to thousand equations; *MPK*=the rate of return and *MPL*=the wage rate. Economic activities are not based on micro (households and enterprise) but government and private sectors just before redistribution of all the taxes in the SNA.

Conclusively, in reality, the methodology as an container ensure no unemployment with no inflation/deflation, positively producing the rate of technological progress; by country among many countries and, regardless of

pessimistic transition for national character and population in future. The author stresses a unique fact that the real-assets is not micro-oriented but macro-oriented. Otherwise, sustainable circulation of net investment to consumption per capital never works robustly. Any country has dynamically and instantly balances, beyond space and time. The author continues to present how to solve economic problems soon after all kinds of questions.

Compilation 2 Biological references historically related to Fermat' Last Theorem

Direct proofs for 'Fermat's Last Theorem':

1. Goro Shimura (1971). *Introduction to the arithmetic theory of automorphic functions*. Iwanami Shoten Publishers, and Princeton University Press. 267p.
2. Wiles, Andrew (1995). Modular elliptic curves and Fermat's Last Theorem. *Annals of Mathematics*, Vol. 141, No.3: 443-551.
3. Taylor, Richard and Wiles, Andrew (1995). Ring-theoretic properties of certain Hecke algebras. *Annals of Mathematics*, Vol. 141, No.3: 553-572.
4. Fellmann, Emil A. (1995). *Leonhard Euler*. Hamburg: Rowohlt Taschenbuch Verlag, 156 p.

Essential Researches towards 'Fermat's Last Theorem'

(Here abbreviating translations in Tokyo)

(1) Aczel, Amir D. (1996). *Fermat's Last Theorem: Unlocking the Secret of an Ancient Mathematical Problem*. New York and London: Four Walls Eight Windows. 147p.
(2) Stewart, Ian (2001). *Flatterland*. Cambridge, MA: Perseus Publishing. 301p.
(3) Lines, Malcolm E. (1994). *On the Shoulders of Giants*. Bristol, England, and Philadelphia: Institute of Physics Publishing. 288p.
(4) Bell, Temple (1951). *Mathematics: Queen and Servant of Science*. New York, Toronto, and London: McGraw-Hill Book Co, Inc. 437p.
(5) Gardner, Martin (1957, revised). *Fads and Fallacies: in the name of science*. New York: Dover Publications, Inc. 363p.
(6) Krauss, Lawrence M. (1993). *Fear of Physics: a Guide for the Perplexed*. New York: Basic Book. 206p.
(7) Singh, Simon (1997). *Fermat's Last Theorem: the story of a riddle that confounded the world's greatest minds for 358 years*. London: Fourth Estate. 361p.

Short notes on the above seven references:

1. From the viewpoint of geometry, (2) Stewart to dimensions, (3) Lines o hyperbola and ellipse, and (4) Bell for two dimensions are preferable.
2. From the viewpoint of Fermat's Last Theorem itself, (1) Aczel and (7) Singh, in addition to direct perpetual records of 1. to 4.
3. (5) Gardner, plain strategies; (2) Stewart and (6) Krauss, deep for physics.

Compilation 3 The Golden ratio in Greece versus the Silver ratio in Japan

The author is not a specialist for mathematics and physics but remains a citizen and an outsider. The author here sincerely hopes that his summing up must be clearly understandable to citizens and amateurs with no equations and graphs.

Using Questions and Answers (**Q&A**), the author indicates and confirms the following facts found in the above B. biological references.

The author's range in ***Compilations***

1. Discusses the author's hyperbola originally drawn in the two dimensions or plane, narrowly limiting to plan (hereunder simply, Hyperbola). The heartland of Hyperbola is a circle. This circle is peculiar to national taste, preferences, culture and history in Japan. The silver ratio is well known particularly in Japan. Nevertheless, the author deepens the relationship between Hyperbola and the circle and uniquely extends this relationship towards another unique relationship between the golden ratio in old Greece and the Silver ratio in Japan.

2. Although the author remains a strictly scientific plane for Hyperbola, he does not deny a fact found in preventive medicine and cerebrum (limbic system and hippocampus). Let the author state the essence of human, in a word. We human individually have five senses (hearing, sight, touch, smell, and taste). Smell is understood by another expression of Fragrant, whose character is the same as Nature; no room for human thoughts prevailing in the above four senses (hearing, sight, touch, and taste). Living things (animals and plants), despite, never have these four senses, or these living things completely follow Nature. How to interpret the above human characteristics in the real-assets in the SNA by country author? Conclusively, the author advocates that human needs no rigid distinction between Fragrant and other four senses peculiar to human, or 'purely endogenous' organic system for the SNA is free from human characteristics.

3. The author, even in history of man, finds a new fact that in the literature there is no unity among philosophy, theory, and practice; particularly, between philosophy and practice/measurement. Human has succeeded invaluable philosophy since the 10^{th} to 20^{th} century BC. For example, the Negative and Positive Principle in old China is still alive today. Hyperbola made it possible to accurately measure the level of philosophy, consistently with theory=practice. Theory is true when theory is united with practice through learning by doing. Why is it possible so, rejecting armchair theory? This is traced back to an everlasting fact that statistics-data are always within a certain range of 'purely endogenous' data and, results and causes are overlap in two ways, due to money character (see, a few short paragraphs of A.).

Questions and Answers (**Q&A**) *in these* ***Compilations***

For Q&A, the author selected nine essential Fermat's books for biological references in the above B. The author must apologize if his selection was insufficient for Q&A here. The following Q&A might be inadequate but, the author finds these Q&A, originally after investigating each reference in B or the first appearances.

1. There is no reference for expressing any word of the golden ratio even in 'ellipse' or olden Greece researches repeatedly raised in B.

2. There is no reference for expressing any word of hyperbola/Hyperbola in B. Of course, hyperbola appears commonly in the textbooks and the literature in mathematics, compared with parabola. However, the form differs from the author's. The author's form exceptionally holds by rotating 90 degree to the right or to the left in the plane (for graphs, see, Appendix, pp. 480-527, 2^{nd} edition).

3. To the author's understanding, mathematics is much more severe in proofs, since mathematics is proof itself, where partial always equal to whole and no evidence is required. Social sciences including economics essentially differ from mathematics and plainly present solutions, as long as social sciences use human money or M2 in economics. For example, the level of democracy is accurately measured by country if externals are all given into statistics- data, as in "International Financial Statistics Yearbook," IMF; as a result, by country, by sector (Total, Government, and Private), and by year and over years.

4. Reinforced by the above 3 (the last paragraph soon above), the author's forecast might be boldly derived as a unique system in mathematics, even if mathematicians dislike to accept a concept of organic system.

Q&A 1: Fermat's Last Theorem and *Euler, Leonhard*, life-worked by Fellmann, Emil, Alfred (1995) are united, cooperatively and consistently with the literature.

Q&A 2: There is no difference between Euclidean Geometry and non-Euclidean geometry.

Q&A 3: Hyperbola is another expression of thousand equations with no assumption, wholly as a system-cloth made of warp and woof, where *'warp' is reinforced by the market principles, as vertical-striped, while 'woof' in such that no woof to connect with warp designed for integration* (see page iv, 2nd edition).

Acknowledgements to **Compilations**
The author is thankful to librarians and software-consultants in the author's university, Hiroshima Shudo University, and universities in the West and the East; in 1956 for assets-revaluation and in 1968 for Break-Even Point hyperbola equations in accounting.

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